ERRATA

1. VOLUME I: Page 222 (Fig. 209) is missing
(Email JC Turner to get this page).

2. VOLUME IV. At the end of Appendix 2009 (see heading CWH KNOTS) there are three pages numbered 197, i and ii.
Disgard these three pages, as they do not belong in the book.

THE BRAIDER

VOLUME I : Issue Numbers 1–15

ALBERTUS GEORG SCHAAKE

— February 1995 to August 1998 —

PREFACE

This is the first of four volumes on the Theory and Practice of Braiding. It brings together the first fifteen issues of a Journal called simply *THE BRAIDER.*

Albertus Georg Schaake was born in Holland in 1933. He studied Engineering at Delft Technical University, before emigrating to New Zealand in 1962. His career has been spent first as a Public Works Engineer, and then as a Tutor in Engineering subjects at the Waikato Polytechnic in Hamilton.

For the last thirty years he has devoted countless hours to the study and practice of braiding : to the creation of decorative braids in particular. Towards the end of the 1980s, he began a project to publish his discoveries, occasionally collaborating with other authors, to produce a stream of books, pamphlets, articles and research papers on a variety of topics in braiding theory. These publications range from books on Regular Knots, Fiador Knots and Herringbone Knots through to extensive pamphlets on the braiding of Wheelknots. For a lengthy account of the progress and philosophy of this work up to 1995, the reader is referred to chapter twelve of *History and Science of Knots* edited by J C Turner and P van de Griend, and published by World Scientific (1995).

In 1995, such was the large output of Schaake's ideas on the subject, that he decided to produce a quarterly Journal which would summarize and extend the description of his work. He called the Journal *The Braider*; and in the next eleven years, virtually single-handedly, he produced 60 issues. Such was his industry and application, that when he stopped his researches, due to illness in 2006, he had already prepared quarterly issues which would take the series up to November 2009.

In total, some 1600 pages of ideas, philosophies, theories, and instruction on design and production of decorative braids, have flowed from his computer and have been included in the issues of *The Braider.* In all the issues there are numerous diagrams which illustrate the theories and techniques. All of these were produced by Schaake by means of a computer-aided design package.

Now, in 2007, it is my privilege to bring together all sixty of the issues, unchanged and unedited, into a collection of four spirally-bound Volumes. They will be available for future generations of braiders to study and further the great body of work that Schaake has initiated.

Georg Schaake may truly be said to be the father of Modern Braiding Theory; very little of consequence existed before he began his researches.

The four-volume collection may be taken as a text for all braiders, from beginners to the most advanced. It is essential reading for all serious decorative braiders.

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A quarterly publication for the braiding artisan

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This publication is available to braiding artisans only.

Copies may be obtained from:

A.G. Schaake,
21 Sundown Cresc.,
Hamilton,
New Zealand.

THE BRAIDER — A new quarterly publication

This is the first issue of a new serial publication for the braiding artisan to whom the availability of it is restricted. In 1980, hence 15 years ago, the first steps were taken in developing an **applied** braiding theory. This new braiding theory will in due course change the craft of braiding; in fact it will enable braiding to be elevated to a fully fledged craft-form.

In previous times the great majority of even the most experienced braiders had an extremely limited amount of braiding knowledge, and this situation is still the norm today. This in turn led, and still leads, to the so called "trade secrets" in the braiding arena. Although an 'expert' braider could produce first class braid-work, its nature was nevertheless severely restricted by the small amount of braiding knowledge he or she possessed. Consequently there was little if any significant variation in the braid-work he or she produced. As an overall result, braiding remained through the ages a stagnant craft-form. Although more "recently" several attempts have been made to 'enliven' the braiding scene, none of these did succeed due to the absense of a supporting theory. Notable in these attempts was the introduction in May 1963 by Arlie G. Belcher (editor of *"The Craftsman"* magazine and president of the Leather Craftsman Inc.) of *"The Braiding, Knotting and Tying with Lace contest"*; unfortunately it did not survive. In fact lacing and braiding projects were very few and far between in *"The Leather Craftsman"*, *"The Craftsman"* and *"Make It With Leather"*, successive names for the leading magazine of the time (1956—1986).

Braiding is not just a handcraft; to the contrary, it is very much more than just that. A great deal of practical theory lies behind it. This theory is as essential to a braiding artisan as a hammer is to a carpenter. Hence the applied braiding theory is nothing but an **essential tool** or piece of equipment in the braiding craft. It formulates what can and cannot be achieved, it formulates the way or ways in which a desired end-result can be obtained. It is therefore an indispenable tool in the designing of new braid-forms, and hence in new braiding applications.

Many older braiders will, at least initially, tend to shy away from the theoretical aspects which govern the braiding processes. But in doing so, they will remain confined to the old well trodden pathways, and will consequently not be able to achieve what someone else with less experience, but with an understanding of the 'theoretical' aspects, is able to achieve.

This publication will address all aspects in braiding, hence it will be of value to all braiders irrespective of their attained skill-level. It will therefore cater for all braiders, from the beginner to the most advanced craftsman. The inclusion of a question and answer 'column' is envisaged, which should be of great value to beginners especially. This quarterly publication will contain besides its articles of an instructive nature also brief discriptions of the contents of our other publications which treat various braiding aspects in much more greater detail. This will enable the reader to decide whether or not it is worthwhile for him or her to obtain a particular publication.

The price of this quarterly publication is kept as low as possible. It may be obtained from the address given on the inside of the front-cover. Each issue, published in A4 size, can be bought individually, or alternatively a prepaid subscription for any desired number of issues can be taken out. Each issue will be posted by economy airmail in a medium size envelope. This means that it will be folded in three. If however a subscriber

does not wish to receive each issue in the folded condition as indicated above, he or she can for an additional postal charge of NZ$1.00 per issue receive the publication in a large envelope suitable for A4 material. The front-cover carries no page-numbers while the numbering of the other pages will be continuous throughout the consecutive issues. It is therefore important to note that back-issues will be available at any time. In order to facilitate the filing of the consecutive issues, the pages will not be stapled.

The question and answer column mentioned earlier will commence as soon as material for this column has been received.

Braiding Instructions and their Presentation

Any form of instruction, which serves the purpose of conveying the necessary steps in the production of the item concerned, makes use of some appropriate terminology. Hence also in braiding we will have to make use of some terminology which the braider must have learned to understand before he or she can expect to be able to follow the braiding instructions given. Naturally, learning the meaning of the terminology used requires some effort, just as it took some effort before he or she was able to walk, to eat independently, to read and to write, just to name a few of the everyday tasks we take for granted to accomplish without (too much) effort.

Traditionally, braids were (and are) drawn in pictorial fashion, accompanied by some written instructions which try to convey the necessary sequence of steps along the road to its creation. Although this method works fine for simple and, in most cases, small braid-forms, it becomes soon unmanageable for braid-forms which are either more extensive or complex. The pictorial way in presenting braids has the great disadvantage of its inability to show all the essential details of a three-dimensional object, and consequently not only are many pictures required in the set of instructions, but more importantly, an overview of the overall, complete braid cannot be obtained and hence presented. This inability to obtain and present such a complete overview has at times led to incorrect construction steps, which were not obvious from the appearance of the finished braid; while at other times it has led to the erroneous belief of having discovered a new braid. Besides these serious shortcomings associated with the pictorial braid drawings, the most serious one is that pictorial drawings prevent us from generalizing a braid-form. This in turn makes it impossible for us to find the braids which form a set family, to describe the common properties of such a family of braids, and to formulate their general construction procedures.

In order to overcome the above indicated problems, we make use of what we call **grid-diagrams**. A grid-diagram is a 'stylized' 2-dimensional representation of the complete braid in the form of a development (similar to the way a sheetmetal worker develops a sheetmetal object). It is 'stylized' in order to keep the diagrams as simple as possible, hence to avoid unimportant and irrelevant beautifications. *It is of the utmost importance that grid-diagrams can be drawn quickly, because they only serve as a tool which aids us in the actual construction of the associated braid.* To facilitate the quick drawing of grid-diagrams, we make use of a suitable graph-paper. The only commercially available graph-paper which is suitable is the so called **isometric graph-paper**. It consists

essentially of a grid which is formed by three sets of parallel lines, one set of parallel lines make an angle of 30° with a horizontal line, another set of parallel lines make an angle of 150° with a horizontal line, and a further set of parallel lines (which pass through the intersection-points of the previous two sets) perpendicular to a horizontal line (hence at an angle of 90° to a horizontal line). A sample of such an isometric graph-paper is shown below in Fig. 1.

There is also available an isometric graph-paper which has in addition to the above mentioned three sets of lines, a fourth set of horizontal lines. It is however better to avoid that type of isometric graph-paper since is is too cluttered with lines and consequently makes the reading of the diagrams unnecessary difficult.

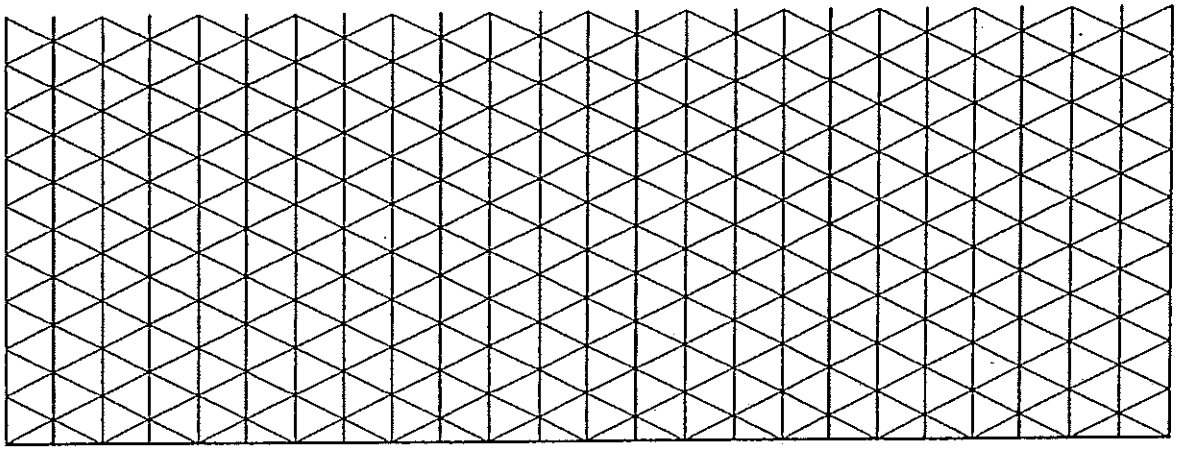


Fig. 1 — Isometric graph-paper.

We note that the slanting lines on this graph-paper intersect each other at an angle of 120°, whereas in actual braid-work the strings cross each other at 90°. Although this deforms the braid-work to some extent, it is of no significance since it does not affect any essential relationships within the braid. In fact most developments of braid-work will introduce deformations of some of the parts (irrespective of the grid-system) without affecting any of the essential relationships within the braid. Such deformations are not only often necessary, but also unavoidable, in order the present the braid-form clearly in a 2-dimensional lay-out.

Isometric graph-paper can be ordered from most stationary outlets.

In New Zealand it is available in two sizes:

(1). Foolscap size: 50 sheet pad, No. 0810 Isometric Grid, Gormack Graph Papers, Christchurch N.Z.

(2). A3 size: 50 sheet pad, No. L110X Isometric Grid, Gormack Graph Papers, Christchurch N.Z.

In order to assist other interested braiders, we will be grateful if braiders in other countries can supply us with the necessary details concerning the isometric graph-papers available in their country. We will then list these details in *"The Braider"*.

Every braid consists of two important parts:

(1). The **string-run**. This is the way the string is laid down, without regard to the interweaving required.

(2). The **coding**. This is the way in which the desired weave-pattern is achieved, as the string is laid down.

A braid is represented by a grid-diagram drawn on this isometric graph-paper. Its string-run follows the slanting lines of the isometric graph-paper; the coding is represented by small thick line-segments which indicate the crossing type between two strings at a crossing-point as shown below in Fig. 2.



Fig. 2 — The coding conventions.

This provides us with a simple and fast way to draw grid-diagrams which represent a braid and show all its essential properties. If desired, the codings may be coloured in various ways in order to clearly indicate the consecutive construction steps.

It is important to note that with this method of representing a braid, we are able to clearly show the **string-run** by omitting the **coding**. Then we can later superimpose the desired coding on this string-run. The separation and combination of string-run and coding are essential operations in representing braid-forms; they enable us to derive the general properties of families and related braid-forms.

Now we have given a rough outline of the way in which a braid-form is presented, let's have a look at some simple braids. We shall start with two very simple **Regular Flat Braids**; see Fig. 3.

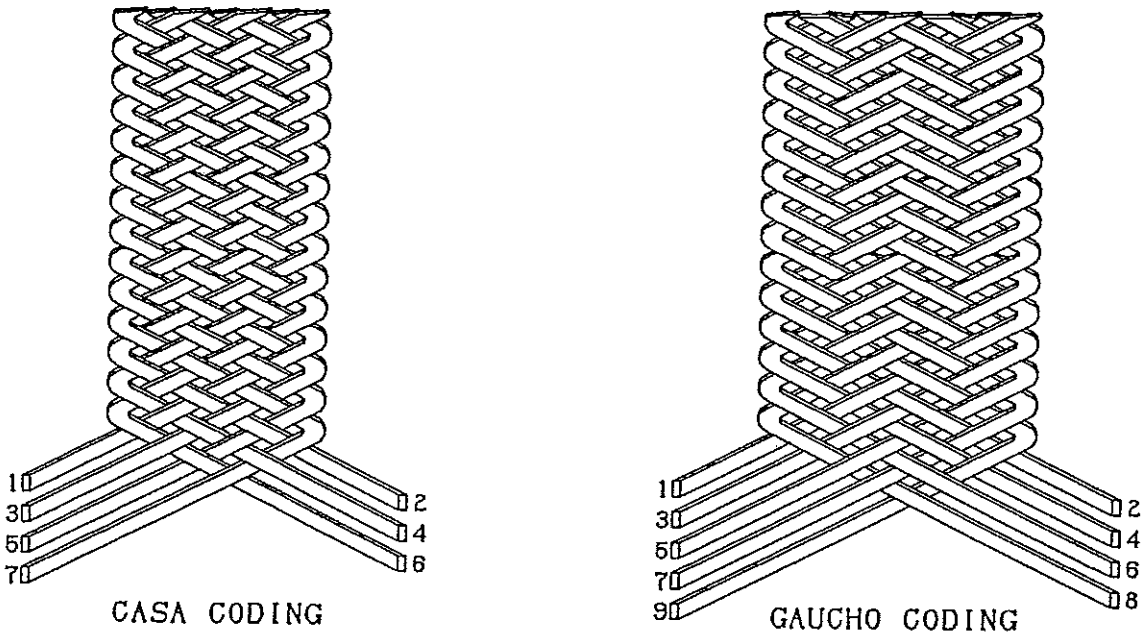


Fig. 3 — Two very simple Regular Flat Braids.

These pictorial drawings, although preferred by many braiders, have the serious disadvantage that they do not lend themselves for clearly showing the string-run; it is

not possible to separate the string-run and coding in pictorial drawings. At first sight, this may seem to be of little, if any, importance as far as such flat braids are concerned. However, the string-runs of such flat braids play a fundamental role in the properties of Cylindrical Braids (we shall discuss Cylindrical Braids in the next issue).

The disadvantages associated with the pictorial representation of Regular Flat Braids can readily be overcome by using their grid-diagrams instead. The upper-row central diagram in Fig. 4 shows the complete grid-diagram (*string-run* and *coding*) of the Casa-coded Regular Flat Braid in Fig. 3.

The lower-row of diagrams in Fig. 4 show the *string-run* diagram of the braid (this string-run is also shown in the top-right pictorial drawing).

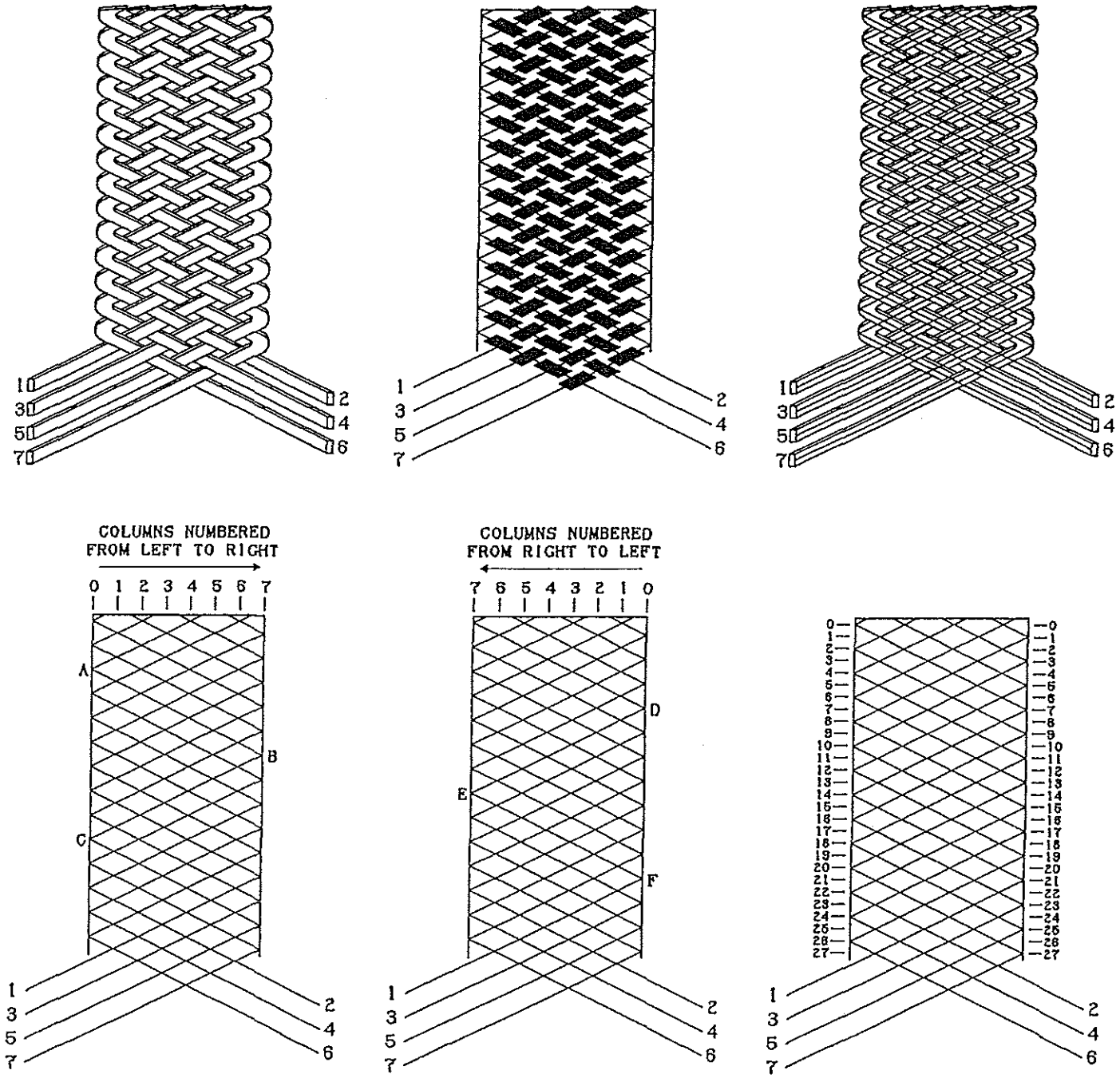


Fig. 4 — The Casa Regular Flat Braid.

The **Casa-coding** is an alternating over/under coding, which we called in our earlier publications a **Turk's Head coding**.

Through the bights-points and crossing-points of the string-run we can draw vertical lines, called **columns**. These *columns* are numbered from left to right (whereby the left-most column receives the number '0') and from right to left (whereby the right-most column receives the number '0').

The string-run consists of two sets of parallel **half-cycles** which run between the two bight-boundaries; one set consists of *half-cycles* which run from upper-left to lower-right (for example A $\longrightarrow$ B), and the other set consists of *half-cycles* which run from upper-right to lower-left (for example B $\longrightarrow$ C). Any two consecutive half-cycles (for example A $\longrightarrow$ B $\longrightarrow$ C, or D $\longrightarrow$ E $\longrightarrow$ F, make up a **cycle**.

> If the string-run diagram of a flat braid contains two sets of bights only, a left-hand set **all** on one single column and a right-hand set **all** on one single column, while its string-run consists of two sets of half-cycles of the types described above, then the flat braid is called a **Regular Flat Braid**.

We can also draw horizontal lines, called **rows**, through the bights-points and crossing-points of the string-run; in the lower right-most string-run diagram in Fig. 4 these *rows* are numbered downwards.

> If in a braid every single crossings-column has a constant coding (different crossings-columns may have different constant codings), then the braid is **Column-coded**. If in a braid every single crossings-row has a constant coding (different crossings-rows may have different constant codings), then the braid is **Row-coded**.

There are of course many other coding arrangements possible besides *column-coding* and *row-coding*.

Note that a Casa-coding is both a Column-coding as well as a Row-coding; it is the lowest limit of these coding forms.

The upper-row central diagram in Fig. 5 shows the complete grid-diagram (*string-run* and *coding*) of the Gaucho-coded Regular Flat Braid in Fig. 3.

The lower-row of diagrams in Fig. 5 show the *string-run* diagram of the braid (this string-run is also shown in the top-right pictorial drawing).

In the string-run diagrams we have again indicated the column numbering and the row numbering.

It is important not to confuse the *Gaucho-coding* with the somewhat similar looking, but in fact quite different **Herringbone-coding**, as has been done in at least one other publication (a *Herringbone-coded* Regular Flat Braid is shown by the left-most pictorial drawing in Fig. 6).

The end column-number in a *Gaucho-coded* Regular Flat Braid has always to be **odd**, whereas in a *Herringbone-coded* Regular Flat Braid it can be **odd** *or* **even**.

A Gaucho Regular Flat Braid has an **even** number of sets of adjacent intersection-columns. In a set, the coding of all the adjacent intersection-columns (a minimum of two) are identical; this coding consitutes the coding of the set. Adjacent sets have an over/under alternating coding.

Hence the Gaucho Regular Flat Braid shown has four sets of adjacent intersection-columns with two adjacent intersection-columns in each set. Because of the two adjacent intersection-columns in each set, the Gaucho-coding is a **2-pass** type.

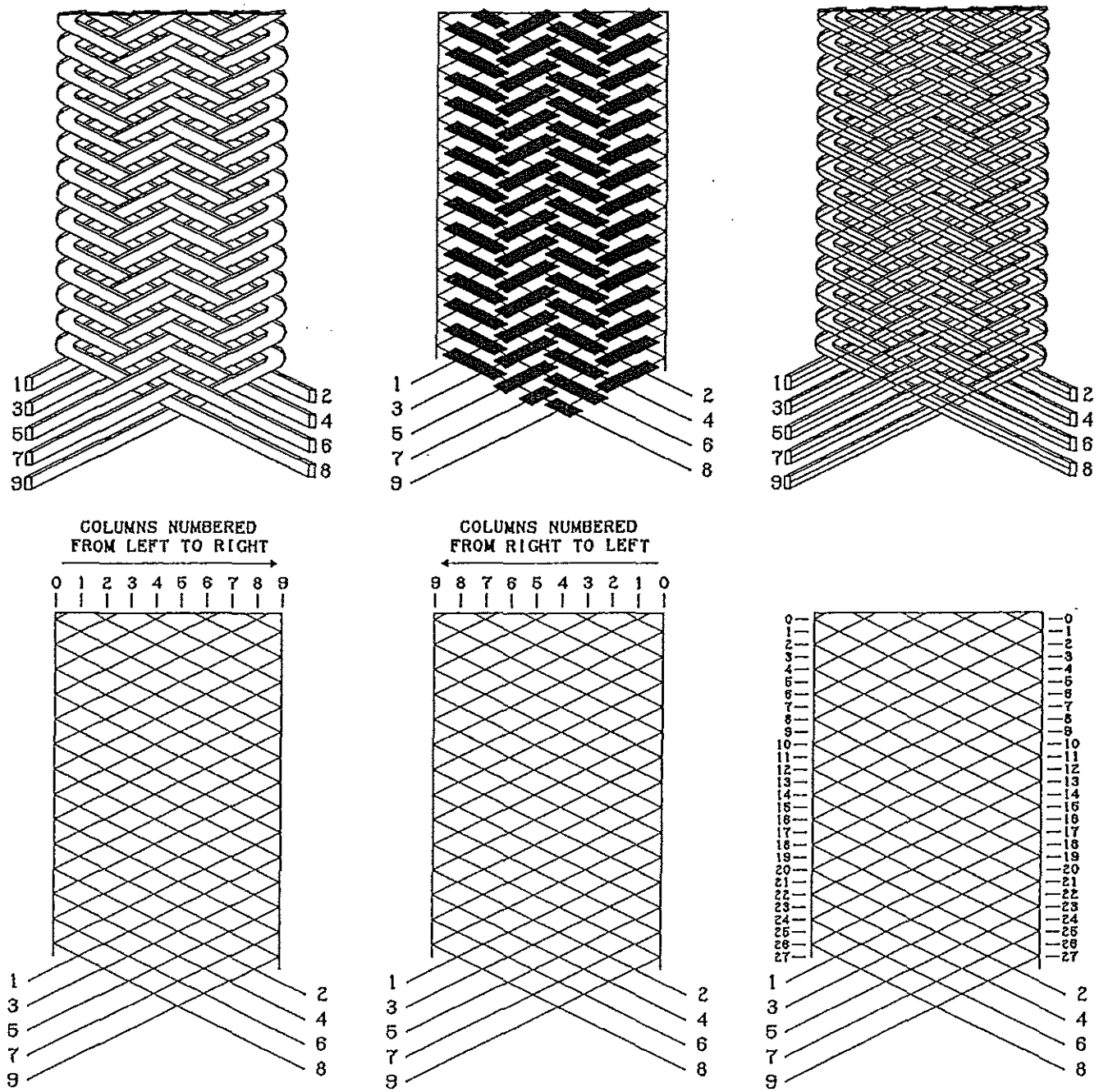


Fig. 5 — The Gaucho Regular Flat Braid.

Note that the Gaucho-coding is a **Column-coding** (compare the *Gaucho-coding* with the *Herringbone-coding*, which is a **Row-coding**).

Since the pictorial drawings of *Column-coded* Regular Flat Braids give a good indication of how their construction should take place, some braiders may initially prefer their pictorial drawings above their grid-diagrams. Nevertheless the drawing of these pictorial drawings takes too much time to make them a suitable medium for designing

even such simple braids. For *Row-coded* Regular Flat Braids, the pictorial drawings have in generally to be modified in order to readily show their construction. This modification becomes essential for the more "complicated" Row-coded Flat Braids. So let's have a look at two examples. In Fig. 6 the left-hand pictorial drawing shows a Herringbone Regular Flat Braid, and the right-hand pictorial drawing shows a Standard Herringbone Pineapple Flat Braid (both are of a 2-pass variety).

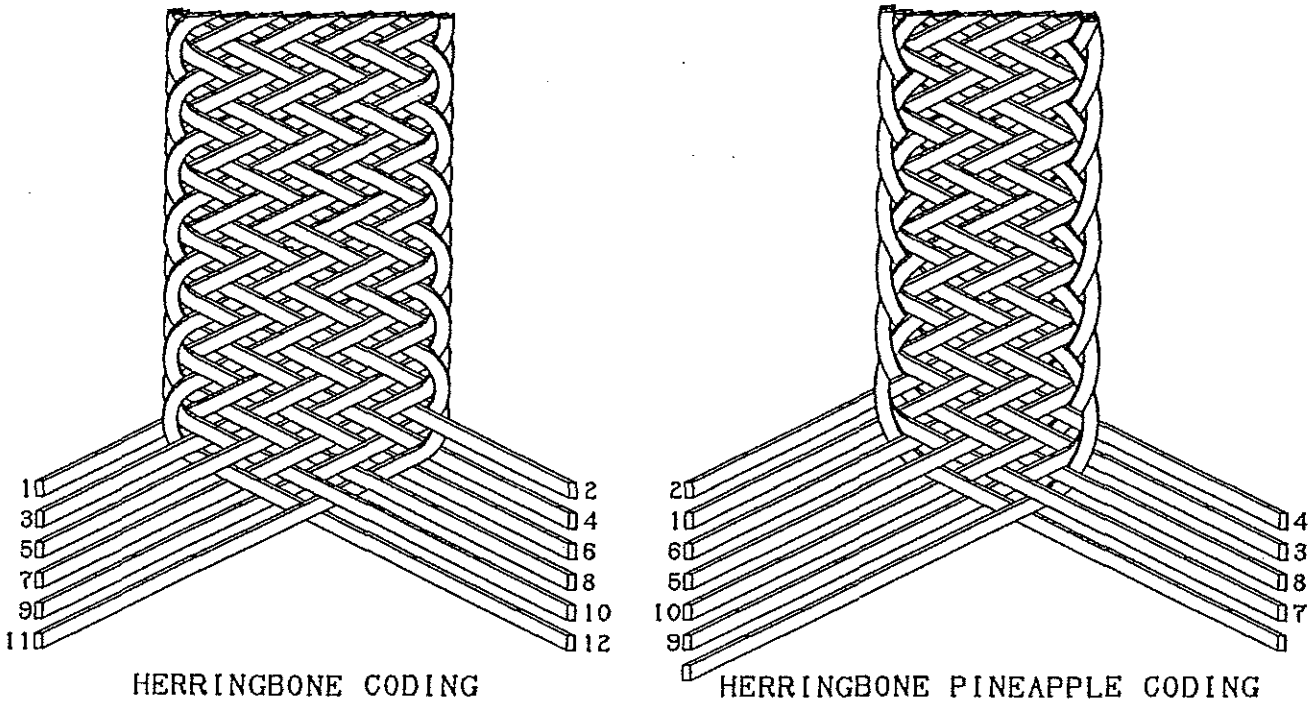


Fig. 6 — Two Row-coded Flat Braids.

Especially the pictorial drawing of the Standard Herrinbone Pineapple Flat Braid does not readily show its construction, but also the pictorial drawing of the Herringbone Regular Flat Braid is of insufficient clarity, despite that it is one of the simplest row-coded flat braids.

In order to make in a pictorial drawing their construction clear, we must separate the strings at the bight-boundaries. This has been done in Fig. 7. It is important to stress here that the pictorial drawings in Fig. 7 do not portray the finished appearance of the braids.

It should be realized that portraying the final appearance of a braid is only of aesthetic value and in most cases jeopardizes the clarity of construction details. Since our interest lies in the construction of braids, it is therefore advisable to limit such pictorial drawings to beautification purposes of the text, and to the familiarization process with grid-diagrams.

We have already mentioned that a braid consist of two essential parts:

(1) the string-run, and
(2) the coding.

Especially the string-run of a braid is of great importance, since it enables us to formulate essential relationships within the braid. These relationships in turn generalize the braid concerned, and enable us to design more complex braids of the same nature.

The pictorial drawings of the two row-coded flat braids in Fig. 6 do not clearly show the string-run of these braids, and even the modified pictorial drawings in Fig. 7 are too cluttered with irrelevant details to be of much value either as far as the string-runs of these braids are concerned.

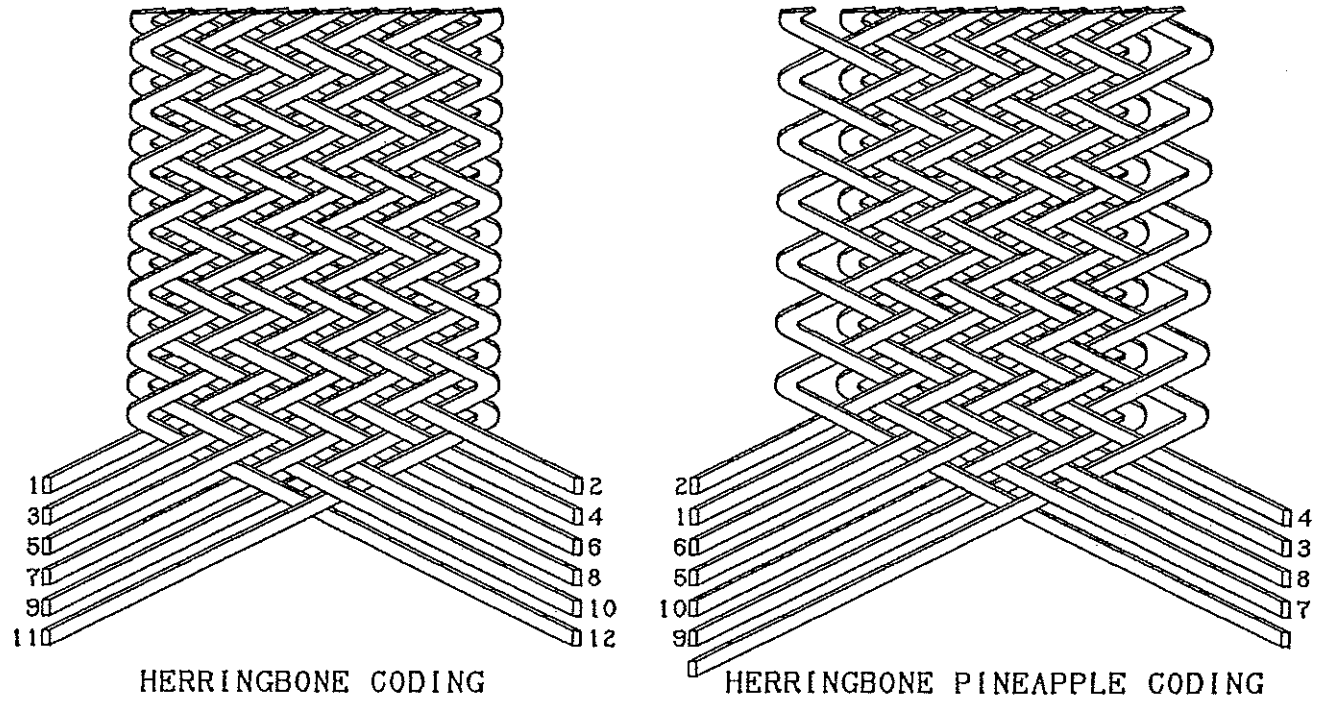


Fig. 7 — The modified pictorial drawings of the two Row-coded Flat Braids.

For string-run purposes, the string should be replaced with a single line in order to keep the string-run diagrams as simple and as clear as possible; all beautifications and irrelevant details should be dispensed with. Furthermore we should be able to superimpose the coding on the string-run in order to show the complete braid. This has been accomplished in the two lower diagrams of Figs. 8 & 9.

In Fig. 8 we have presented in the lower row of diagrams, at left the complete grid-diagram (*string-run* and *coding*), and at right the string-run diagram of the Herringbone Regular Flat Braid in Fig. 6. In order to compare these diagrams with the modified pictorial drawing of the Herringbone Regular Flat Braid in Fig. 7, we have repeated this diagram in the upper row, left-hand side.

After the reader has familiarized him or herself with the coding conventions it will be evident that the bottom row of diagrams convey the construction of this braid much more clearly. Besides this, these diagrams can be drawn very quickly on isometric graph-paper by even those braiders devoid of any artistic ability.

In Fig. 9 we have presented in the lower row of diagrams, at left the complete grid-diagram (*string-run* and *coding*), and at right the string-run diagram of the Standard Herringbone Pineapple Flat Braid in Fig. 6. Again in order to compare these diagrams with the modified pictorial drawing of the Standard Herringbone Pineapple Flat Braid in Fig. 7, we have repeated this diagram in the upper row, left-hand side.

Again it will be evident that the bottom row of diagrams convey the construction of this braid much more clearly. It will also readily be seen that the drawing of these

diagrams on isometric graph-paper remains just as easy as the drawing of grid-diagrams and string-run diagrams of simple Column-coded Regular Flat Braids on such graph-paper. This ease with which everyone can draw grid-diagrams is a most important aspect. It will enable every braider to design new braiding applications, which even the most talented artist will not be able to do with his or her pictorial drawings.

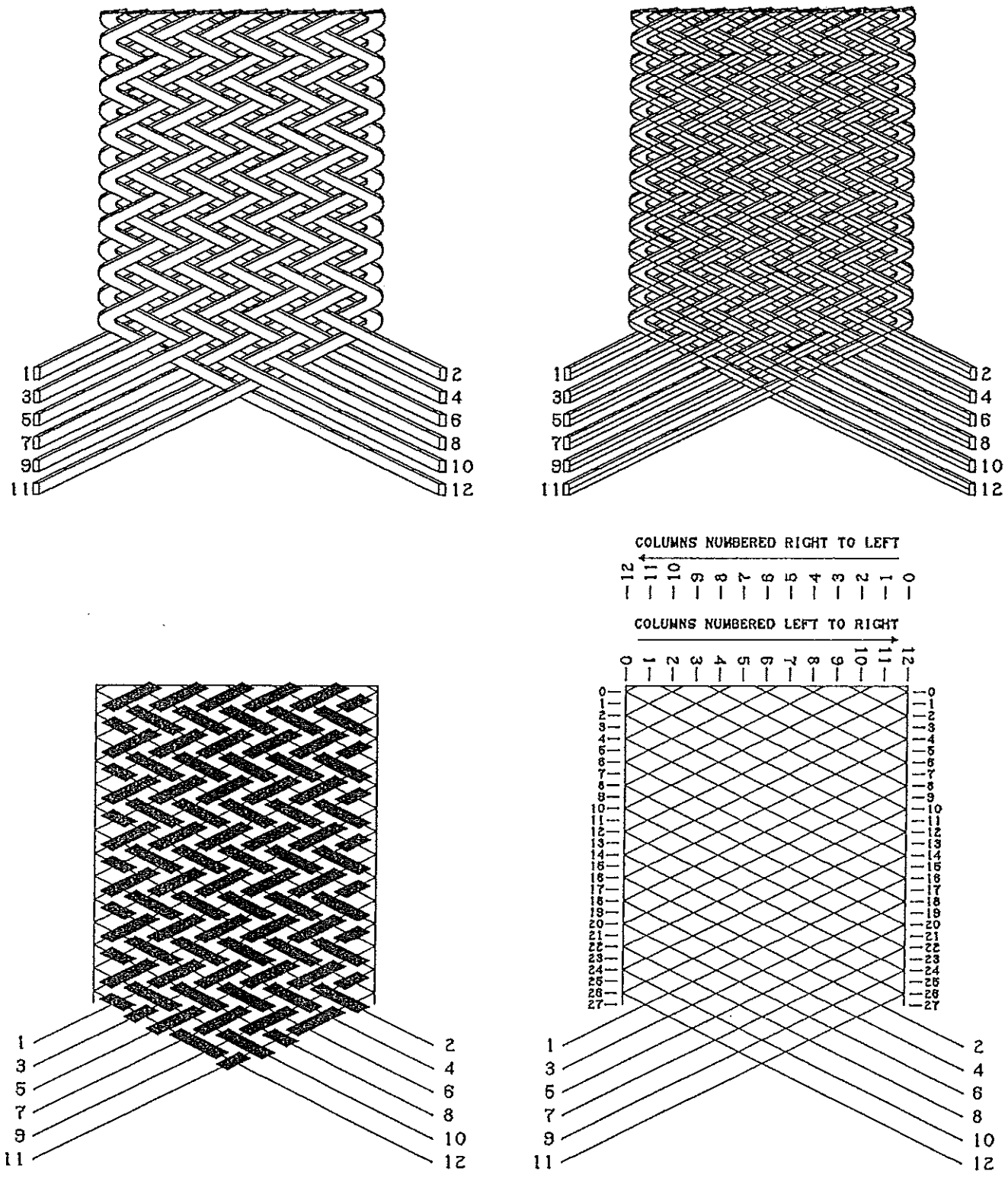


Fig. 8 — The Herringbone Regular Flat Braid.

We mentioned earlier that the minimum number of passes in a Gaucho-coding is two. The same applies to a uniform Herringbone coding. However, it will be evident that a Casa-coded Regular Flat Braid with an even number of intersection-columns can be regarded as being the lowest limit of a Gaucho Regular Flat Braid, as well as being the lowest limit of a Herringbone Regular Flat Braid.

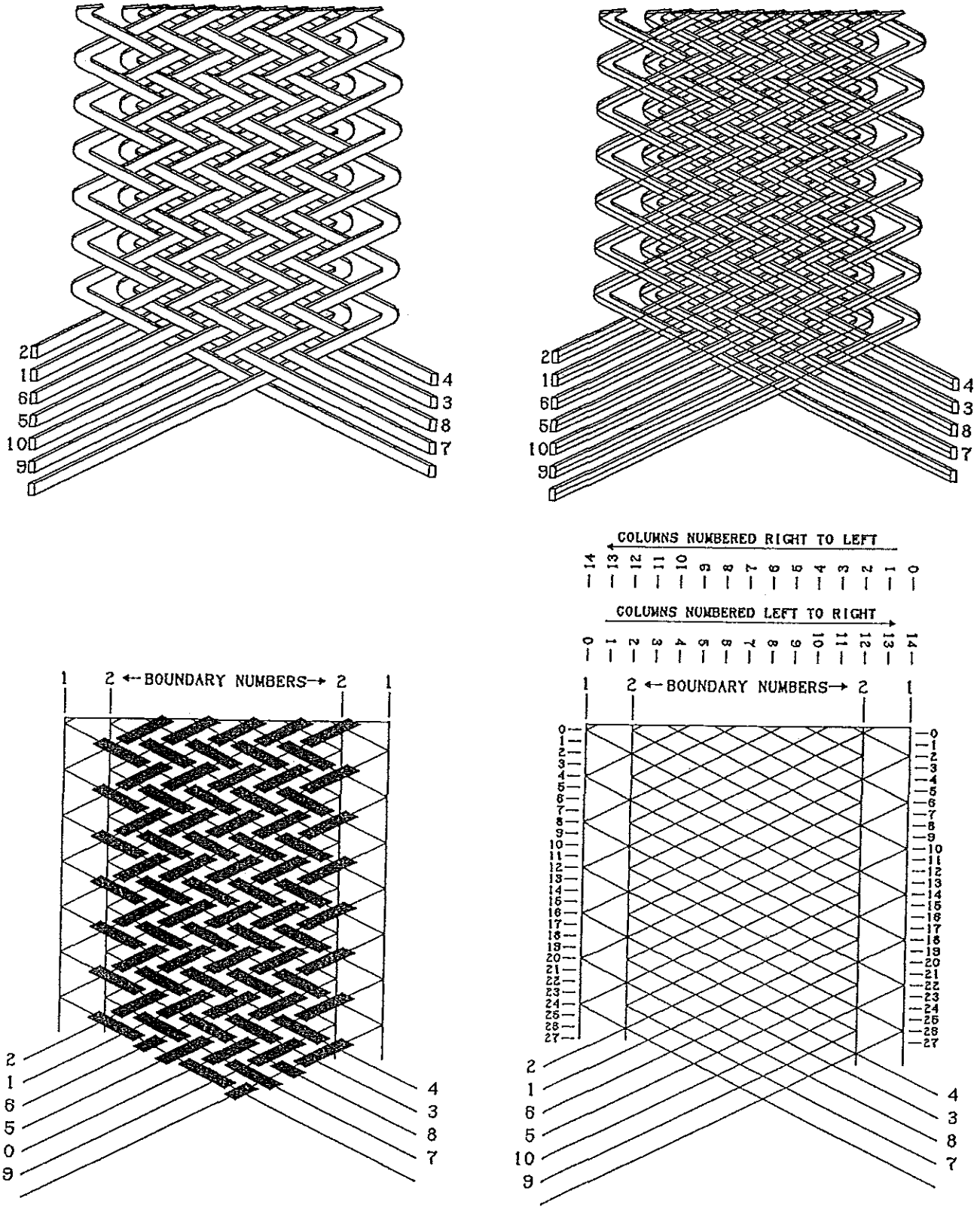


Fig. 9 — The Standard Herringbone Pineapple Flat Braid.

It will readily be seen from the string-run diagram of the Standard Herringbone Pineapple Flat Braid that this braid is an interbraid of two Regular Flat Braids; one with its string-run between the bight-boundaries 1—1, and another with its string-run between the bight-boundaries 2—2. The one which runs between the bight-boundaries 1—1 has six intersection-columns, and the one which runs between the bight-boundaries 2—2 has four intersection-columns. From the coding we readily deduce that both these flat braids have a Casa-coding.

In this article we haven't discussed the purpose of numbering the columns and the rows; this will be done in a future issue. Let it in the meantime suffice to say that it is associated with the braiding instructions used for the construction of Cylindrical Braids. Flat braids form the foundation on which cylindrical braids are based, and with these braids we shall continue our discussion in the next issue.

An Introduction to Flat Braids — Pamphlet No. 5

This Pamphlet in A4 format (295x210mm.), soft cover, stapled and taped spine, contains 36 pages and more than 68 illustrations.

It discusses Flat Braids in more detail, and hence can be recommended for further study.

Four pages are devoted on Turk's Head (Casa) Regular Flat Braids; four colour patterns with their starts in these braids are discussed and shown.

Four pages are devoted to the Secret Plait. The main emphasis is on the construction procedures of a "perfect" loop in these braids (such as a hand-loop at one end of a dog-leash). The correct preparation of the transition between the blank sections and the braided section is detailed for the 5-lead and 7-lead braids.

Seven pages are devoted to the Herringbone Regular Flat Braid. The proper starting procedures for this braid are fully outlined. Other, less desirable and the not recommended starting procedures are shown. Some colour patterns are discussed.

Nineteen pages are devoted to the Standard Pineapple Flat Braids. The construction of a two-colour hat-band in this braid, together with its integrated Turk's Head (Casa) end-knots and two-colour integrated Standard Herringbone Pineapple Knots as keepers, is fully outlined.

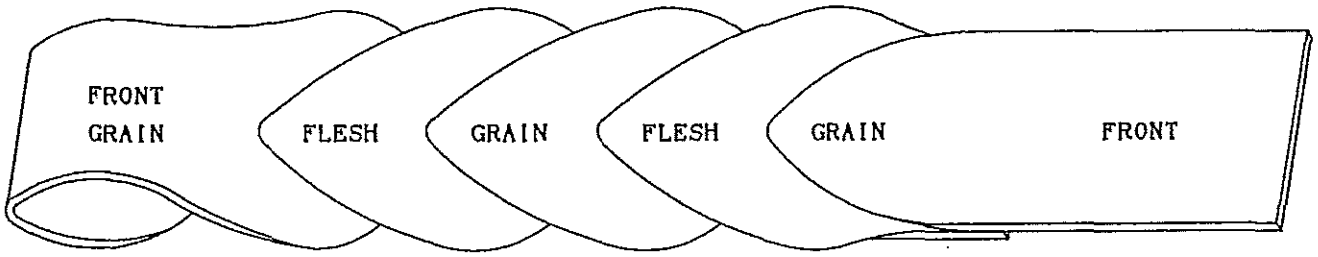
The Pamphlet is mainly concerned with applications, and hence it will be found to be very useful by all braiders irrespective of their attained skill-level. The beginner will be able to learn the correct braiding procedures from the start, and hence will establish a sound foundation on which to build further.

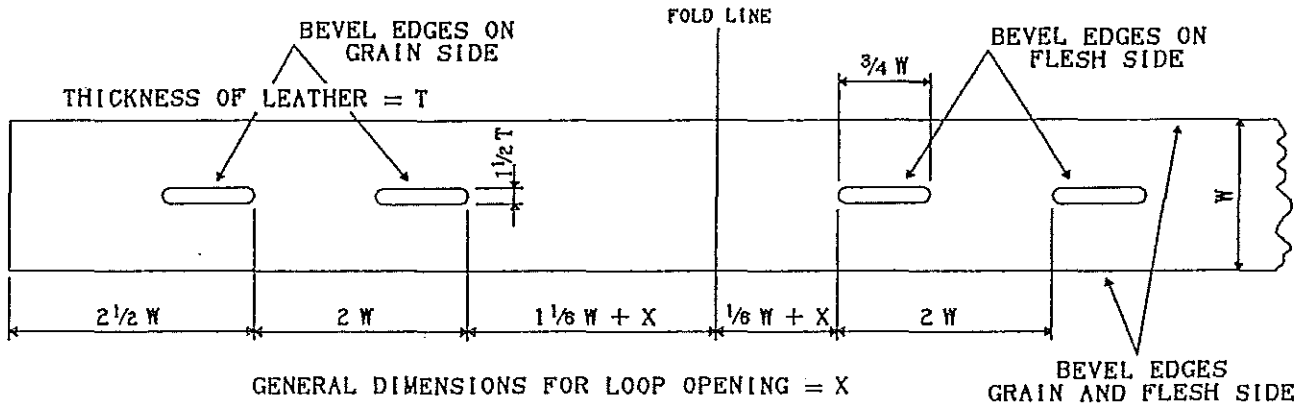
It is recommended that the braids discussed be constructed, especially all the starting procedures outlined. This will give the braider a better understanding of the way in which desirable starts should be designed and how less desirable and poor starts can be recognised at the design stage before braiding commences.

For ordering information and price calculation see the publication, price, weight, and postal charge tables.

Slit Braids

Slit Braids are discussed in many different publications. Although there are two different ways in constructing these braids, the most commonly discussed form is as in Bruce Grant's books; see for example the *"Encyclopedia of Rawhide and Leather Braiding"* pgs. 50, 51, 68, 69. In this construction form the edges of the leather are visible from the front-side of the finished braid. However, a much superior construction-form is the one in which the edges of the leather are only visible from the back of the braid. Not only is this construction-form neater and hence more pleasing from the front, it is also more suitable for the various applications wherein this braid can be employed. Hence we shall limit our discussion to this construction-form only.





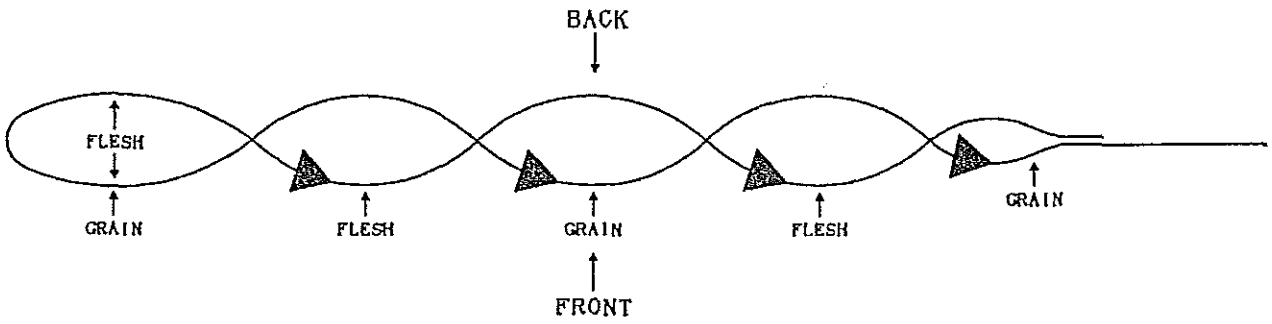


Fig. 10 — The recommended Slit Braid construction and its general dimensions.

In the preparation phase of the leather blank, the generally described process is cutting slits with a knife or chisel. This is not a method that can be recommended since

it obviously leaves sharp ends to the slits. Instead of slits, one should use slots with half circular ends.

No literature appears to give, in their preparation instructions for the leather blank, any information concerning suitable general dimensions. In Fig. 10, such suitable general dimensions are indicated, together with some other relevant leather-blank preparation details.

In Fig. 11 below, two applications of the Slit Braid are shown; the top one depicts a brow-band for a bridle, and the bottom one depicts a belt. Close-ups of the important details are shown in Figs. 12 and 13 respectively.

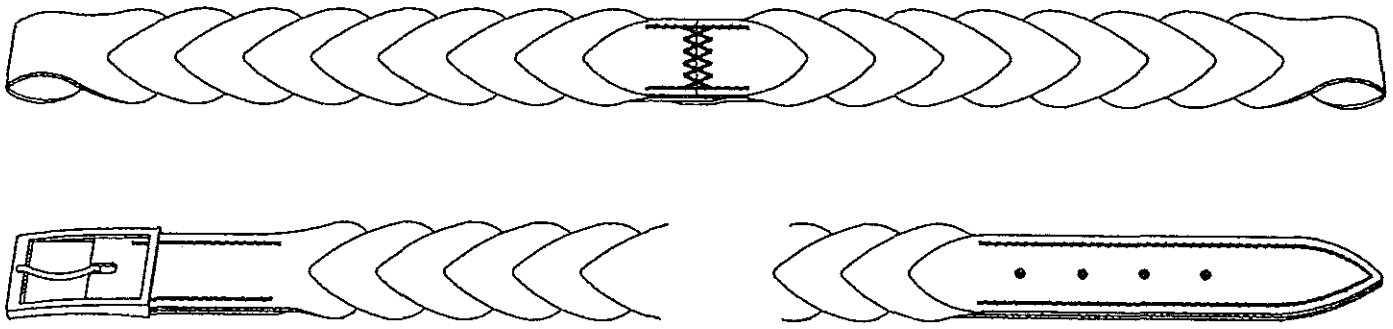


Fig. 11 — Two Slit Braid applications.

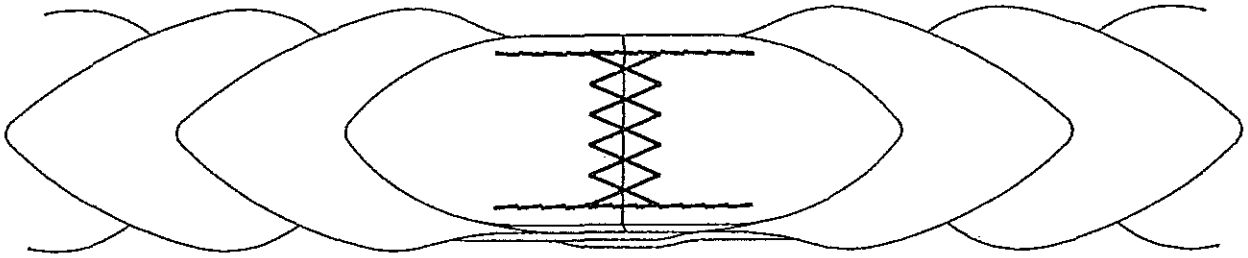


Fig. 12 — Detail of the joint between the two Slit Braids in the brow-band application.

In the brow-band application we need two identical lengths of Slit Braids. Each Slit Braid should preferably finish with the grain-side on the front. The back-parts of each braid have their ends skived, and overlap each other; the overlapped parts are then glued together. The front-parts are butted with a small bevel and glued to the assembled back-parts. This overall assembly is then sewn together as shown with the locked saddle-stitch (do not use the ordinary saddle-stitch, but always use the locked saddle-stitch, or double locked saddle-stitch). Keep the stitch-lines 3mm from the edges, and use a 5mm stitch-pitch. Decorate the butted front-parts with a cross-stitch, executed in the form of a locked saddle-stitch. For the sewing-thread use a black or brown braided polyester thread.

The central part between the slit braids may also be covered with a contrasting piece of thin leather. The stitching should then be slightly recessed in order to obtain a smooth surface for the thin leather covering piece, which should be glued to the central part. The cross-stitching should of course be dispensed with in this case. This finish is

especially good when at the end of each slit braid the flesh-side is on the front.

In the belt application we use two leather blanks. This is done for two reasons. By placing the grain-side of one leather blank against the flesh-side of the other leather blank we ensure that only the grain-side of the leather is visible at the front. Secondly, we don't require very long lengths of leather blanks, which may be difficult to obtain.

The sewn parts are first skived if necessary, and glued together. The sewing is done as described above.

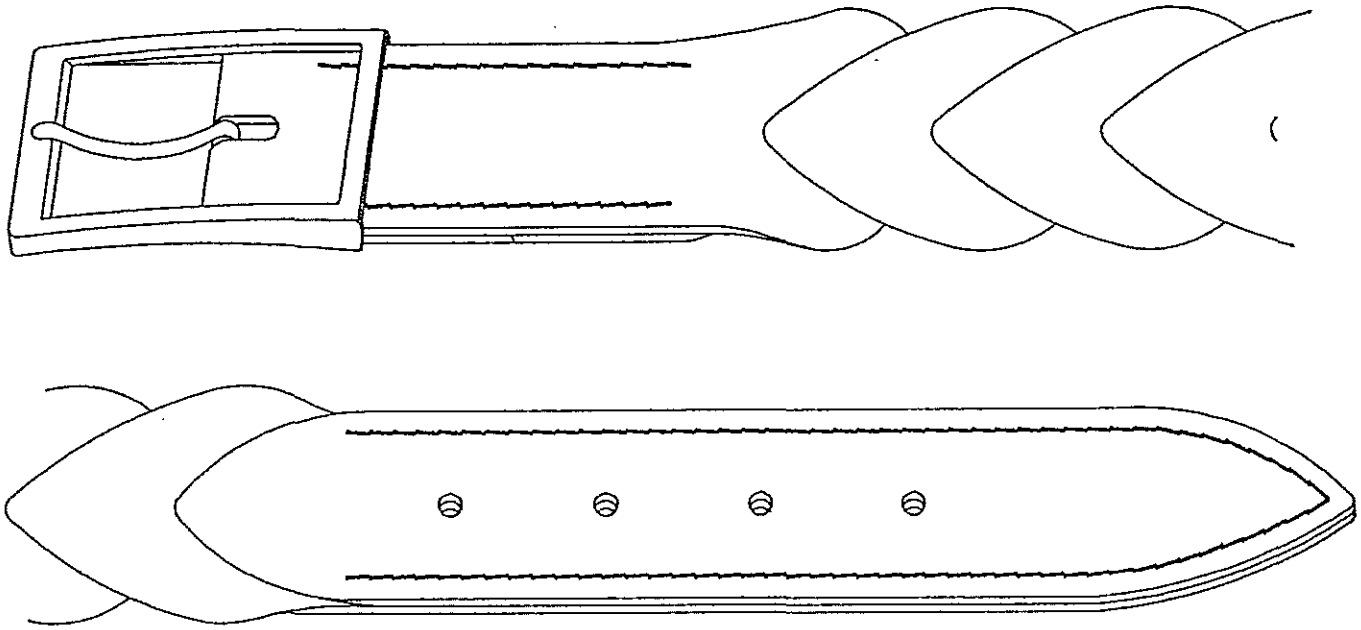


Fig. 13 — Details of the belt construction.

These belts are very comfortable to wear, and are generally preferred above other belt-types. We like to stress that two leather blanks should be used for ensuring that only the grain-side of the leather is shown on the front. In at least one publication this "only grain-side on the front" is achieved by giving one long leather blank half a twist on the back close to the buckle, and hammering this half-twist "flat". This method is an extremely poor one and should never be used by any self-respecting leather-worker.

Furthering the Braiding Craft

In order to further the braiding craft it is of some importance that braiders can get in contact with each other. To facilitate in this we shall, at regular intervals, list the subscribers (together with their full address) to this publication.

Since there are many application fields in the braiding arena, we would very much appreciate if each subscriber to this publication would be so kind to let us know the application field(s) of his or her special interest, in order for this to be included with the above mentioned list. Furthermore, we hope that subscribers will assist us in keeping the list up to date.

Western Tack Tips

This is a 129 pages practical manual by Tom Hall. Its format is 280x215mm., spiral bound with soft cover. It gives the measurements needed to make plain and braided horse gear. Naturally, these measurements should be seen to act as a guide since in some instances they may have to be varied according to the size and breed of the horse for which the item is intended.

The instructions, which include detailed braiding and knot-tying procedures, are given in pictorial form, which are without exception very clear and hence easy to follow by even the most inexperienced braider.

Furthermore some algorithm-tables (formulae) for tying Casa-coded Knots are given.

This book may be obtained from:

S-T Leather Company
P.O.Box 78188
ST. LOUIS, MO 63178
U.S.A.

for US$19.95.

It may also be obtained from any of *"The Leather Factory"* stores for US$24.95.

Reviews

Under this heading we shall review publications by authors which are **not** associated with this quarterly publication.

Since proper reviews of braiding literature appear to be as scarce as hen's teeth, it is important that braiders can somewhere find a review which will critically examine a publication. We shall indicate the braiding level for which the publication is in our opinion suitable. Some publications have little if any value for any level, but are nevertheless always of some use even if it is for showing the braider how it should not be done; such publications will be indicated accordingly and are mainly of value to the more advanced intructor only.

As mentioned above, proper reviews appear to be non existent. We often encounter reviews in braiding Journals which consist of little pieces of the published material. Such reviews are obviously done by people who have either a very limited braiding knowledge, are too lazy to review a publication properly, or who desire to either place the publication in a favourable or unfavourable light depending of the type and size of the little pieces selected. Such reviewers should, if they can't curb their desire in reviewing a publication, limit themselves in printing the *"Introduction"* or *"Foreword"* of a publication only.

A proper review, however, is of great value to those braiders who may contemplate obtaining a copy of the publication concerned, since cheap publications are non existent today and nobody is keen to waste a fairly large sum of money on a for him or her useless publication.

We shall start our reviews with the books by Bruce Grant.

Leather Braiding ISBN 0-87033-039-X 173 pages 215x140mm., soft cover, sewn and glued spine.

Author: Bruce Grant.

Publisher: Cornell Maritime Press, Inc.; Centreville; Maryland; U.S.A.

This was the first book of any significane as far as braiding is concerned in the English language. It was the first attempt to provide some general knowledge on rawhide and leather braiding.

The book starts off with six pages of historical aspects concerning leather braiding, followed by a brief description and illustration of leather-braiding tools and a brief description and illustration of thong cutting.

The procedures for braiding some *flat braids, round braids* and *cylindrical braids* (placed under the heading 'Spanish knots and Turk's Heads') are fully described and illustrated by means of pictorial drawings. These drawings are very clear and hence easy to follow. The book further contains sections on *edge-lacing, buckle covering, braided applique*, and a small section on some simple *buttons*. All these processes are clearly described and profusely illustrated by means of clear pictorial drawings.

Although this book was first published some 45 years ago in the spring of 1950, it still can be considered to be a good book. It is a pity that it has not been up-dated, especially with respect to relationships between different braid-forms, and with respect to braiding procedures for cylindrical braids. The braiding instructions for the braiding of cylindrical braids are generally not the best, and are the cause that some braiders find them difficult to follow. Also some of the procedures in the edge-lacing section can be considerably improved.

Although the book is quite suitable for beginners, there are for them much more suitable publications available today.

How to Make Cowboy Horse Gear ISBN 0-87033-034-9 186 pages 215x140mm., soft cover, sewn and glued spine.

Authors: Bruce Grant and Lee M. Rice.

Publisher: Cornell Maritime Press, Inc.; Centreville; Maryland; U.S.A.

This book, first published in 1953 and enlarged in 1956, outlines applications for the braids described in *"Leather Braiding"*, the book reviewed above. It contains a good section of over six pages on some historical aspects, mainly concerned with rawhide braiding. This is followed by a section of eleven pages on making and working rawhide. The making of all kind of cowboy gear is clearly described and profusely illustrated with excellent pictorial drawings. Together with Tom Hall's *"Western Tack Tips"* it should form an excellent source for braiding projects.

The book ends with a 43 pages chapter on how to make a western saddle by Lee M. Rice.

The book is suitable for those who have thoroughly mastered the braiding procedures outlined in *"Leather Braiding"* in order to achieve the proper results.

Encyclopedia of Rawhide and Leather Braiding ISBN 0-87033-161-2 528 pages 230x155mm., hard bound cover.

Author: Bruce Grant.

Publisher: Cornell Maritime Press, Inc.; Centreville; Maryland; U.S.A.

This publication contains besides all the material of the previous two books, more than 200 pages of further braiding material. Most of the pictorial drawings additional

to the ones in the previous two books are of a much lesser quality, although they may be considered to be sufficient.

This book was first published in 1972, and does neither appear to have been updated nor corrected where errors occured, hence the same comments as under *"Leather Braiding"* above, apply here. We should take into consideration that this publication is more than twenty years old, in fact some parts are 45 years old. Although there are now publications with considerably better pictorial drawings, and hence braiding instructions, it is still the publication which contains in one volume the greatest amount of braiding knowledge. It is therefore a very important publication for any braider.

Its suitability level is as indicated under *"Leather Braiding"* above.

The above books by Bruce Grant are most likely the best known books on braiding. They were undoubtedly the first which tried to overcome the trade-secret syndrome of which braiding especially has suffered for hundreds of years. Although great strides have been made in the second half of the twentieth century, there are still people who chronically suffer from that disease. Most of these people did however at some time obtain their knowledge from someone else, be it by direct personal contact, books, or both. After obtaining the necessary knowledge they became selfish and greedy, forgetting that knowledge is one of the things which cannot be patented and should be available to anyone who appreciates its value. Hence the braiding publications by Bruce Grant are not only of great value for the braiding knowledge they contain, but are also of great historical significance.

Publications available to Braiders only

Enclosed with this first issue of *"The Braider"* are six pages of information concerning publications which may be obtained by braiders only from the address shown. These publications are not supplied to general resellers, such as bookshops and the like, and hence are not generally available. From the information on these six pages, a braider who wants to order one or more of the listed publications can calculate the cost involved.

It will be noticed that there is a relatively cheap postal rate for weights up to 0.20kg. As much use as possible should be made of the cheapest postal rates in order to save on costs. Hence in many cases multiple postings work out cheaper than one single posting for orders of more than one publication. Ensure however that each posting falls within a specified weight limit. Future publications and price changes will be announced in *"The Braider"*.

THE BRAIDER

No.2 MAY 1995.

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A quarterly publication for the braiding artisan

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Copies may be obtained from:

A.G. Schaake,
21 Sundown Cresc.,
Hamilton,
New Zealand.

REGULAR FLAT BRAIDS
AND THEIR
STARTS FOR BELTS

In the first issue of *"The Braider"* the reader was introduced to some Flat Braids and their grid-diagrams. Since many readers will not yet be familiar with grid-diagrams and their use in the actual braiding process, we shall now braid some of those Flat Braids. We shall limit our discussion here to Casa-coded and Herringbone-coded Regular Flat Braids, and introduce the reader to some colour patterns that can be obtained in those braids. We shall furthermore pay special attention to the way in which a Regular Flat Braid should be started in order to achieve a "start" which is not only aesthetically pleasing, but is form-fast as well.

For most braids the actual length of braid obtained is about 2/3 of the string-length, hence the length of each string has to be $1\frac{1}{2}$ times the intended braid-length. When the flat braid is intended for a belt, we start the braiding process with the half-way position of the thongs, at the billet-tip. Thus the length of a thong has to be twice a string-length, or 3 times the intended braid-length; with the exception of one thong when braiding an odd number of strings.

Example 1 : In this Example we shall braid a seven-string Casa-coded Regular Flat Braid which has four dark coloured strings and three light coloured strings. In order to obtain these strings, cut two dark coloured thongs and one light coloured thong, each twice the length of a string, or 3 times the length of the intended flat braid. Furthermore cut one light coloured thong the length of a string, or $1\frac{1}{2}$ times the length of the intended flat braid. The grid-diagram of this seven-string Casa-coded Regular Flat Braid is shown in Fig. 14.

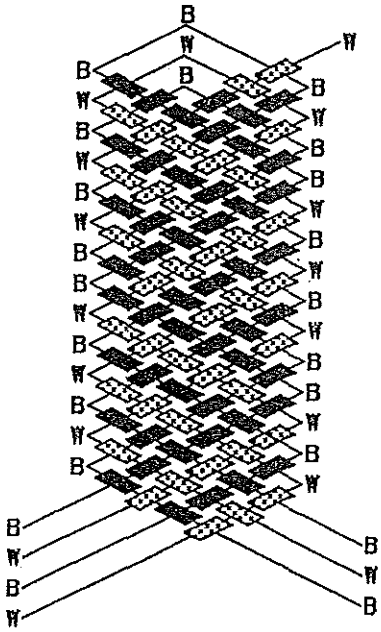


Fig. 14 — Grid-diagram of the 7-string Casa-coded Regular Flat Braid of Example 1.

The first four drawings in Fig. 15 show how to place and hold the thongs and how braiding begins.

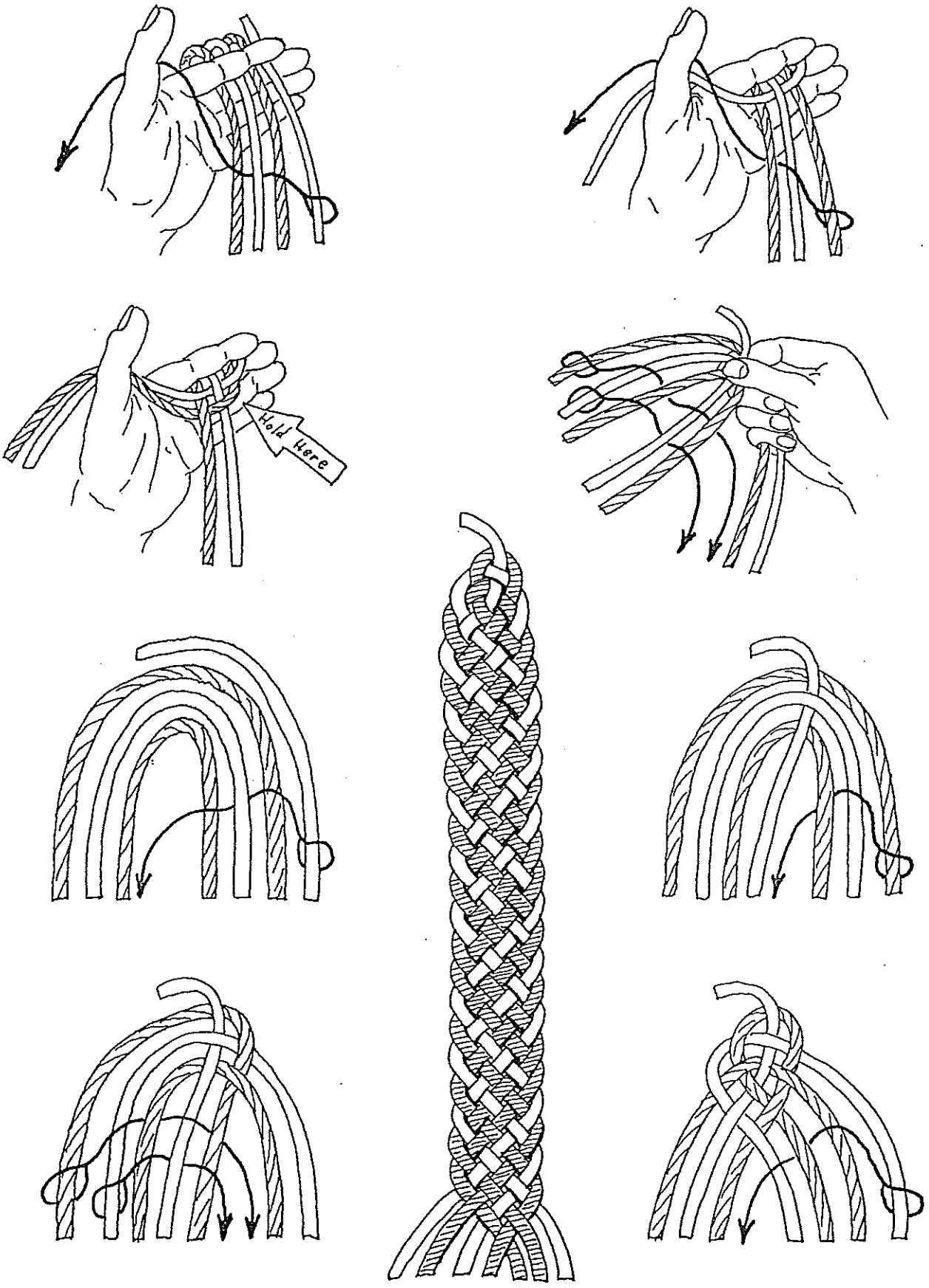


Fig. 15 — Pictorial drawings associated with the braiding of the 7-string Casa-coded Regular Flat Braid of Example 1.

The colour-pattern obtained in this braid is more noticeable in the actual braid and its grid-diagram than that it is in the pictorial drawing.

The tip (where the braiding-process begins) of Regular Flat braids which have an odd number of strings leaves much to be desired, since there will always be one string which has to be back-braided. This causes a certain "weakness" in the tip-construction. On the other hand, these braids do enable us to obtain a symmetrical braid-pattern (note that *braid-pattern* should not be confused with *colour-pattern*).

Example 2 : The braid in this Example is also a seven-string Casa-coded Regular Flat Braid, but with a different arrangement of the coloured strings. This arrangement gives a very pleasing colour-pattern, although the tip-construction is of a lesser quality.

We have depicted three different tip-constructions (starts). They are all of a comparable quality when braided with the correct tightness, and hence it is very much a personal preference which of these tip-constructions is used. The tip-construction in Fig. 16 gives the most pointed tip, the tip-construction in Fig. 17 gives a little more rounded tip, and the tip-construction in Fig. 18 gives the most rounded tip.

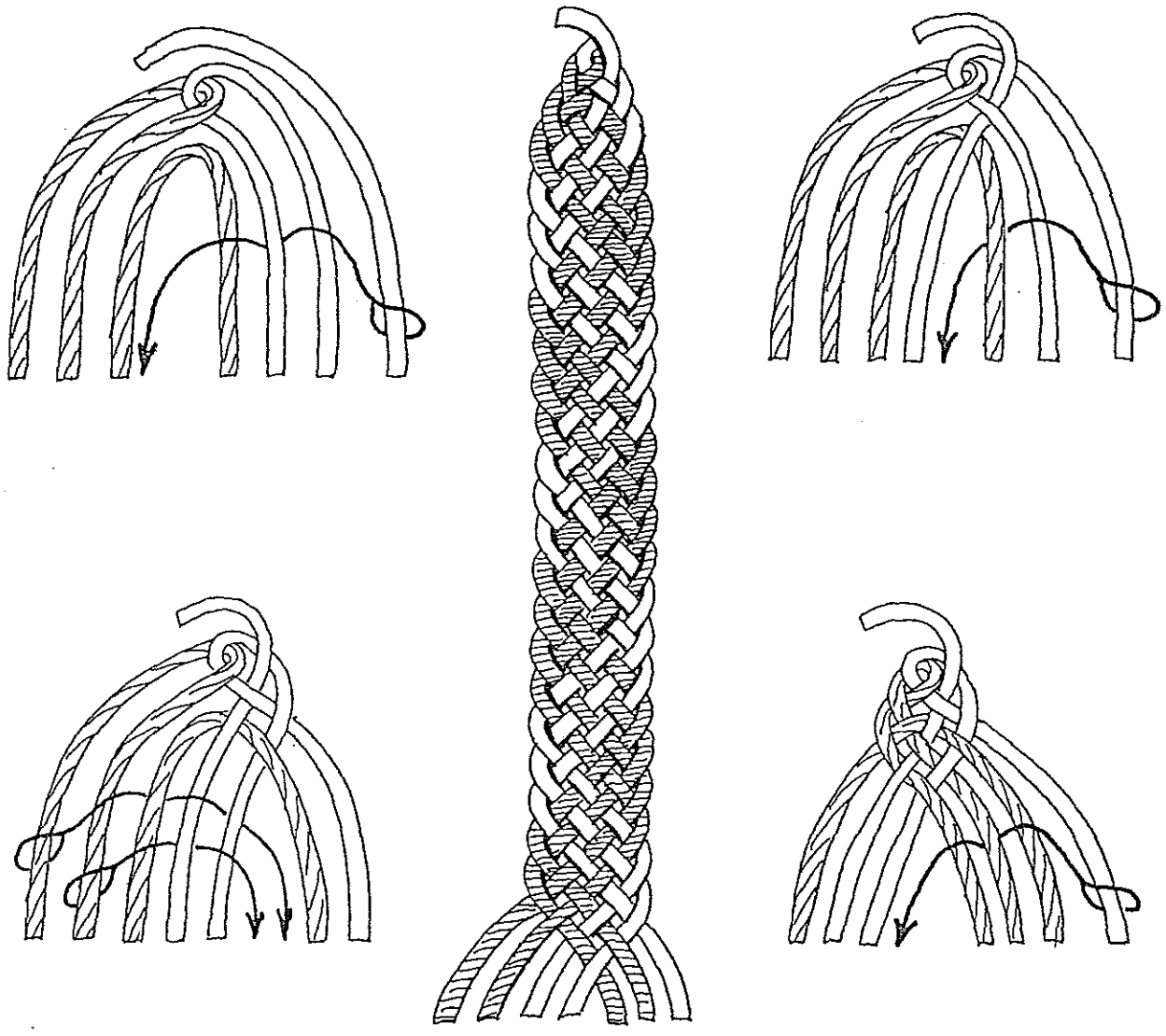


Fig. 16 — Pictorial drawings associated with the braiding of the 7-string Casa-coded Regular Flat Braid of Example 2.

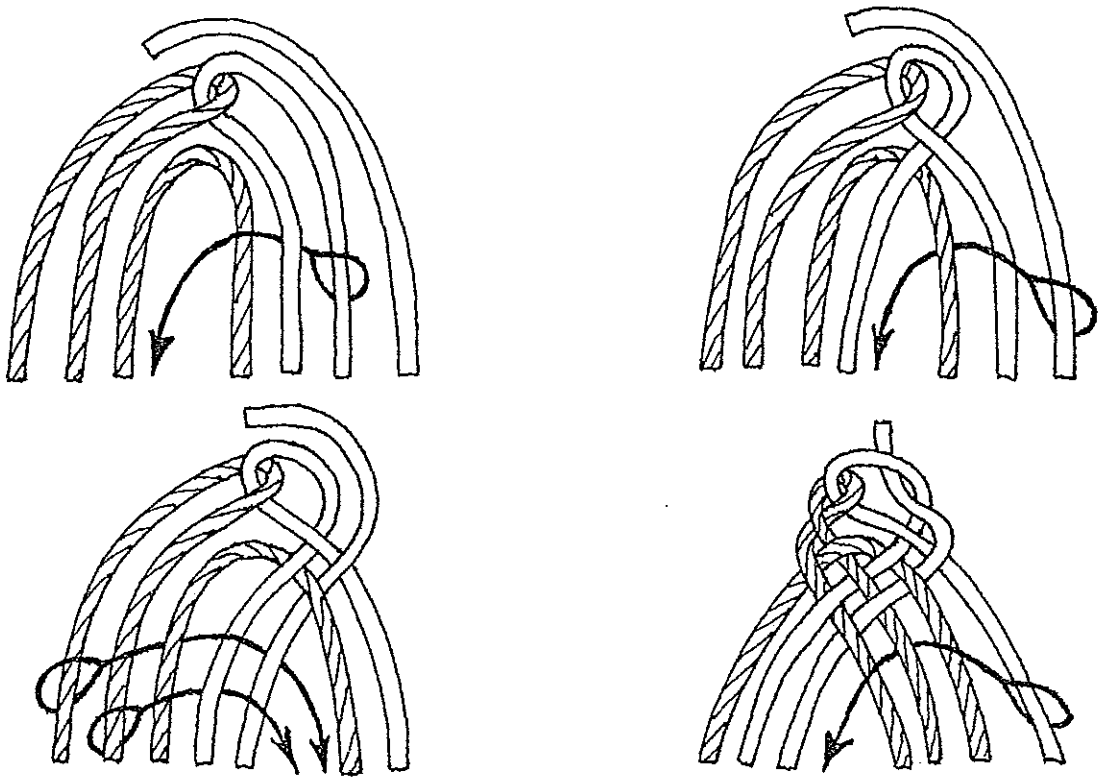


Fig. 17 — Alternative tip-construction to Fig. 16.

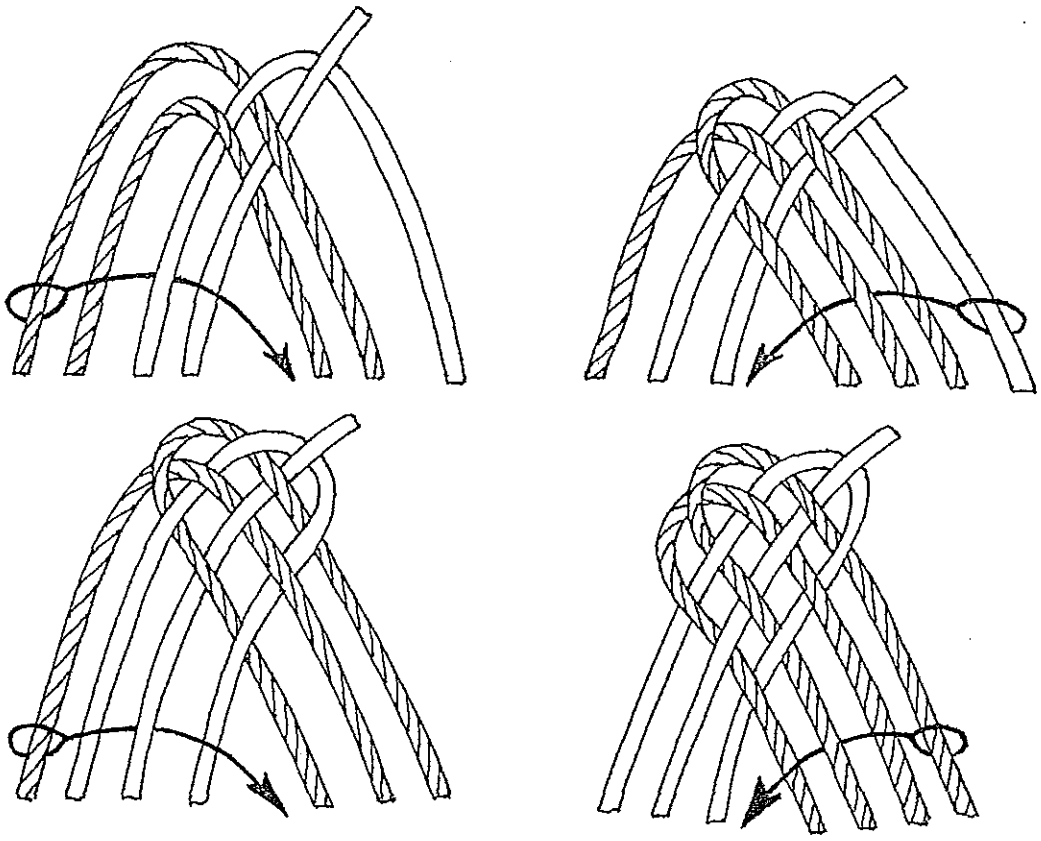


Fig. 18 — Alternative tip-construction to Figs. 16 and 17.

In Fig. 19 are presented the respective grid-diagrams associated with the braid constructions in Figs. 16, 17, 18. The upper-left grid-diagram is associated with Fig. 16, the upper-right grid-diagram is associated with Fig. 17, and the lower grid-diagram is associated with Fig. 18.

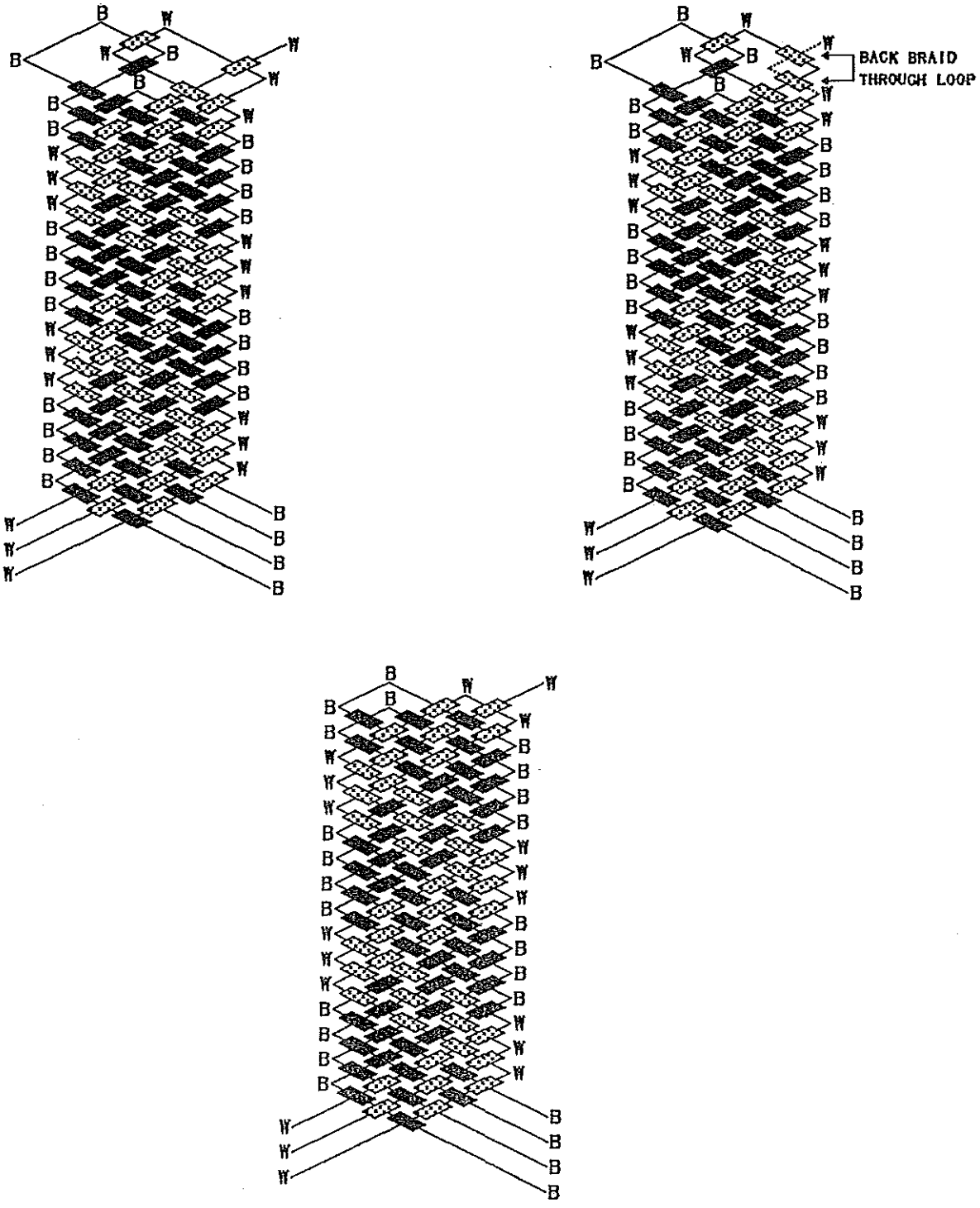


Fig. 19 — Grid-diagrams associated with the braiding of the 7-string Casa-coded Regular Flat Braid of Example 2.

Example 3: Here we have an Example of an eight-string Casa-coded Regular Flat Braid. We require a total of four thongs, each twice the length of a string or three times the length of the intended braid.

Two thongs are of a dark colour and two thongs are of a light colour. They are arranged in the same order as for the braid in Example 1. The braiding-process is depicted in Fig. 20. The pictorial drawing clearly shows the alternating dark and light striped colour-pattern of this braid. The grid-diagram associated with this braid is presented in Fig. 21.

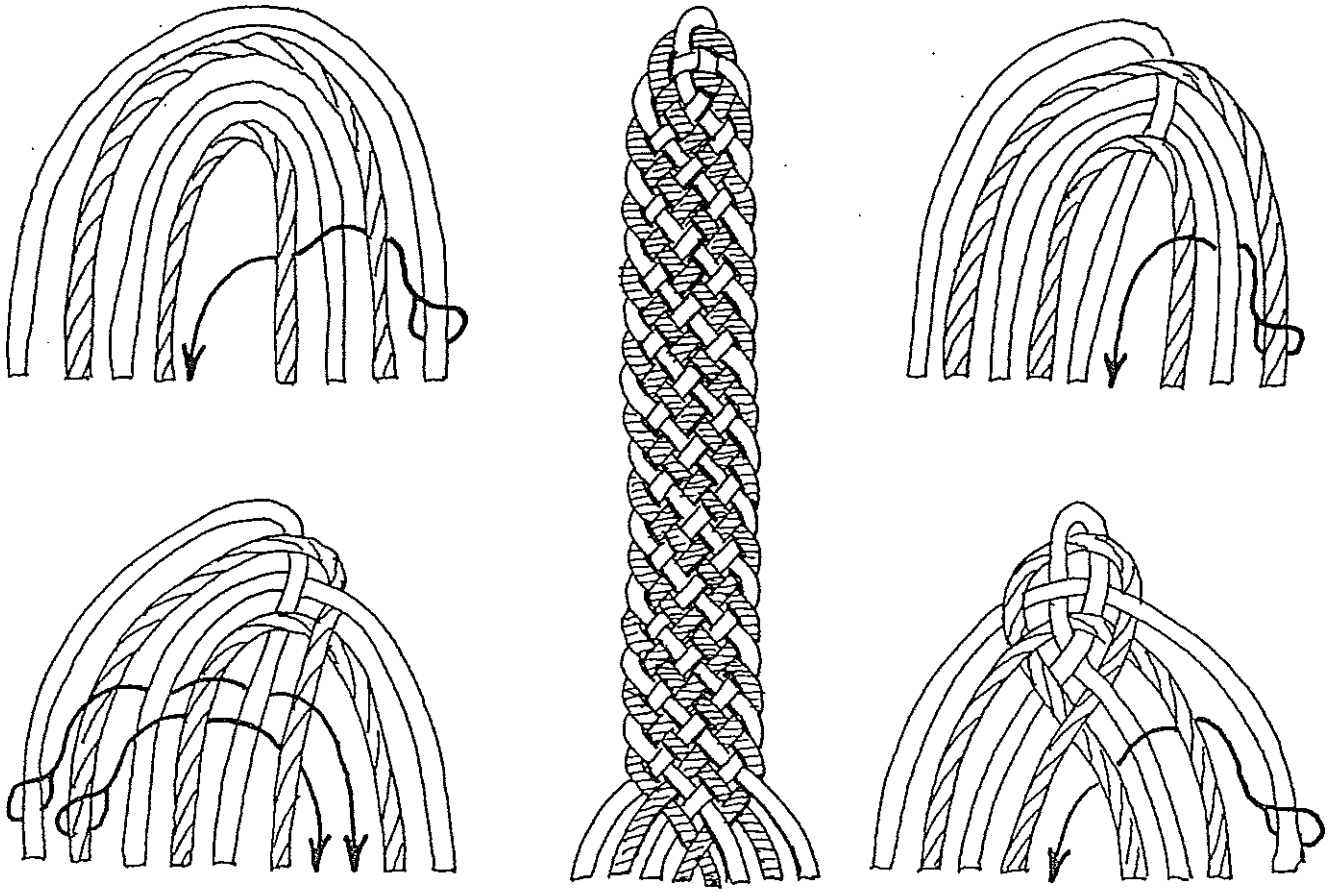


Fig. 20 — Pictorial drawings associated with the braiding of the 8-string Casa-coded Regular Flat Braid of Example 3.

The tip-construction in a Regular Flat Braid with an *even* number of strings can always be of a quality which ranges from very good to excellent. The "disadvantage" of an even number of strands is that both bight-edges are not the same in a Casa-coded Regular Flat Braid. If one has an over-coding, then the other must have an under-coding.

We can of course change the coding so that the codings of both bight-edges are identical. It is however very important that the very tip, whatever the coding of the Regular Flat Braid may be, has a Casa-coding in order to ensure that the tip is form-fast. To ensure that it is of a good quality the tip should furthermore always be tightly braided. Often far too little attention is given to these very important tip-construction aspects with the result that otherwise good braid-work can only be awarded a low quality value. In the next Example we shall discuss a tip-construction in some detail.

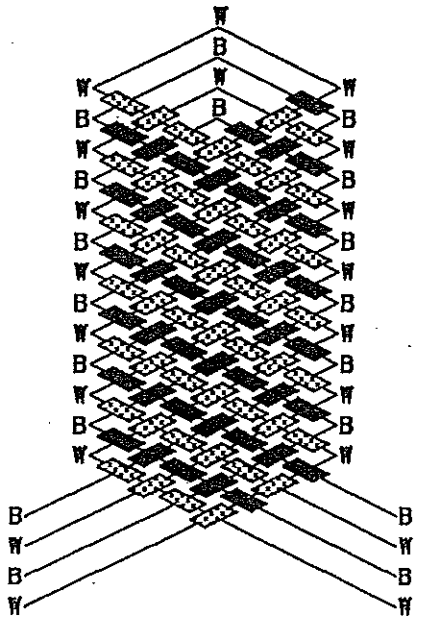


Fig. 21 — Grid-diagram associated with the braiding of the 8-string Casa-coded Regular Flat Braid of Example 3.

Example 4 : In this Example we shall have a look at a 10-string Standard Herringbone-coded Regular Flat Braid. We shall use five light coloured thongs, each three times the length of the intended braid. Each thong we dye for half its length dark. Hence we finish up with five light coloured strings and five dark coloured strings. The braiding process is depicted in Fig. 22. Such Herringbone-coded Regular Flat Braids, with both edges of the same colour, have a number of strings that is equal to twice an odd positive integer. Their colour-pattern on the back is opposite to the one on the front. They are therefore good for belts since they can be used reversed.

There are a number of different tip constructions for this braid; they range from very poor to excellent. Hence a little more detailed discussion concerning the tip construction should be of value to the braider.

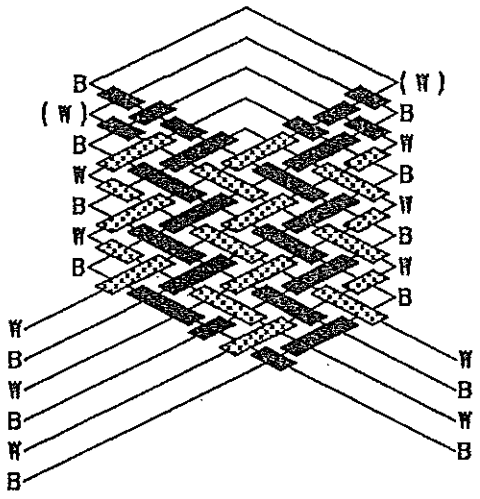


Fig. 22 — The best tip construction for the braid of Example 4.

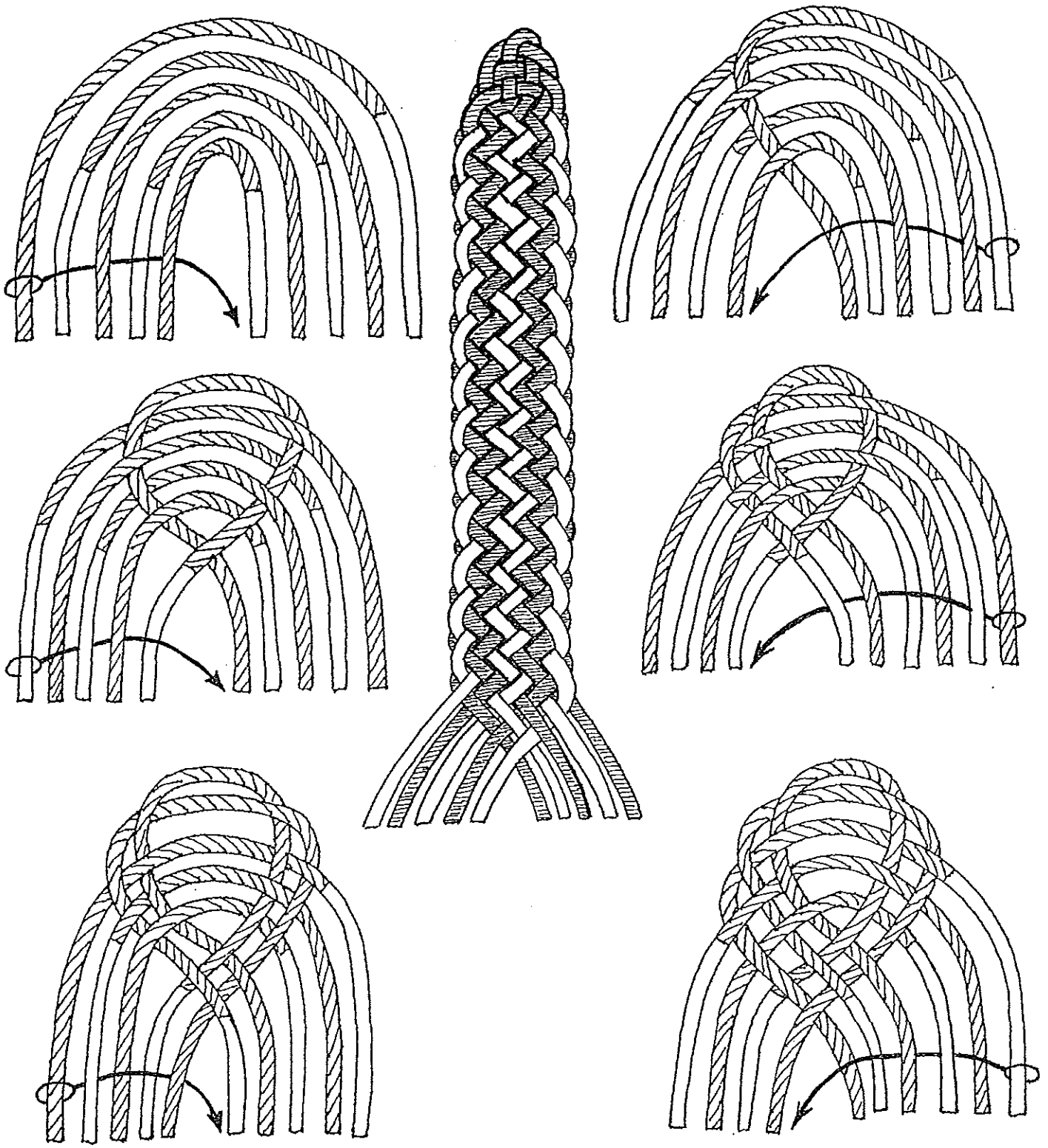


Fig. 23 — Pictorial drawings associated with the braiding of the 10-string Herringbone-coded Regular Flat Braid of Example 4.

Many braiders are inclined to continue with the Herringbone-coding to the end, or nearly the end, of the tip. Such a commonly encountered tip construction for a Herringbone-coded Regular Flat Braid is depicted in Fig. 24.

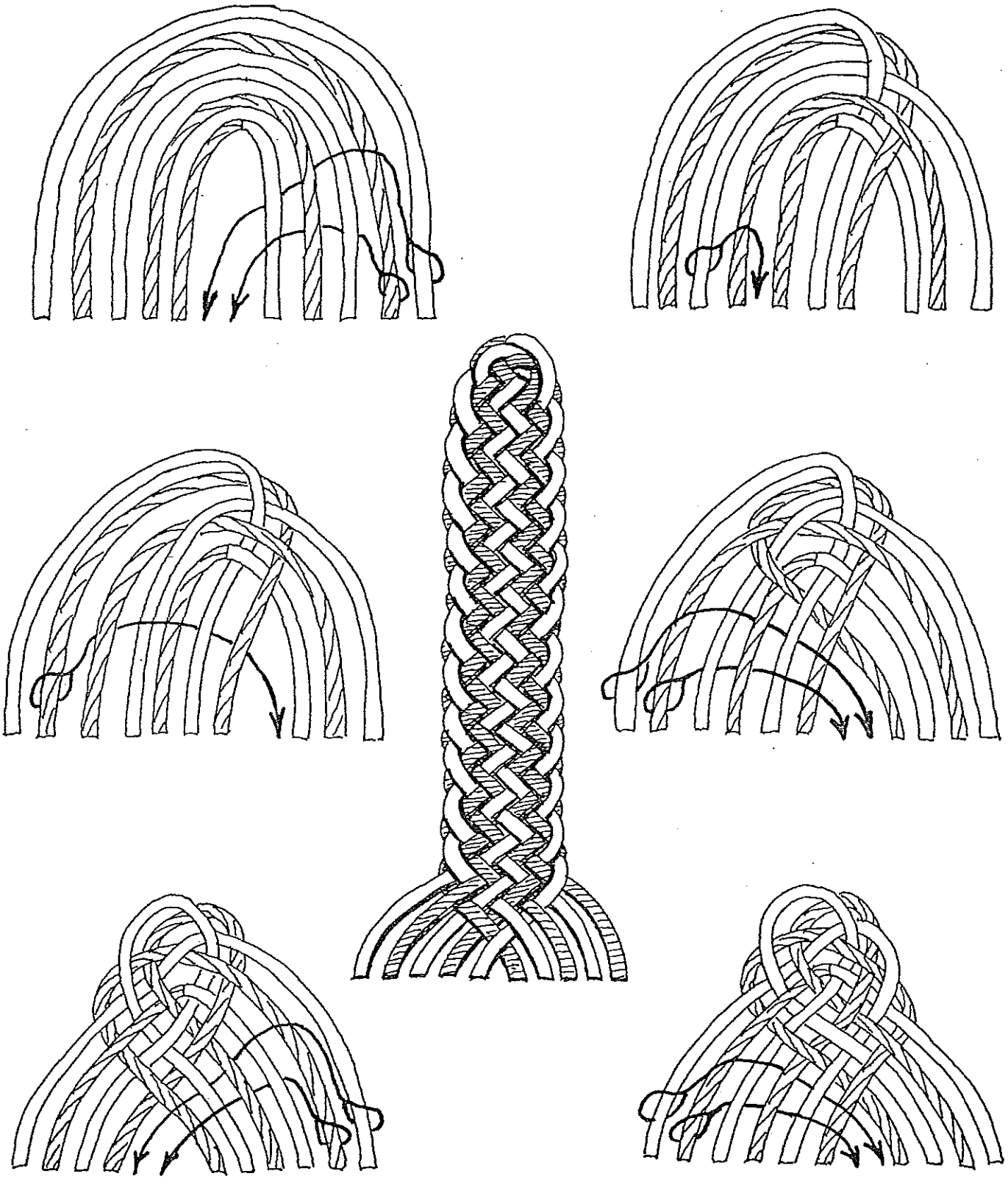


Fig. 24 — A not recommended but commonly encountered tip construction for the 10-string Herringbone-coded Regular Flat Braid.

This type of tip construction invariably results in a tip which is not form-fast. When such a tip is tightly braided, the strings will "roll over" at the end. **We cannot stress enough that the coding of a tip should always be a Casa-coding, and that a tip should always be braided tightly.** In order not to disturb the colour pattern of the Regular Flat Braid, it is therefore advisable to have the tip in one colour, hence to dye the tip if necessary. However, an one colour tip can often be achieved by moving the thongs along a little.

The grid-diagrams of Fig. 25 both represent the tip construction of Fig. 24. These diagrams clearly show that this construction is not form-fast.

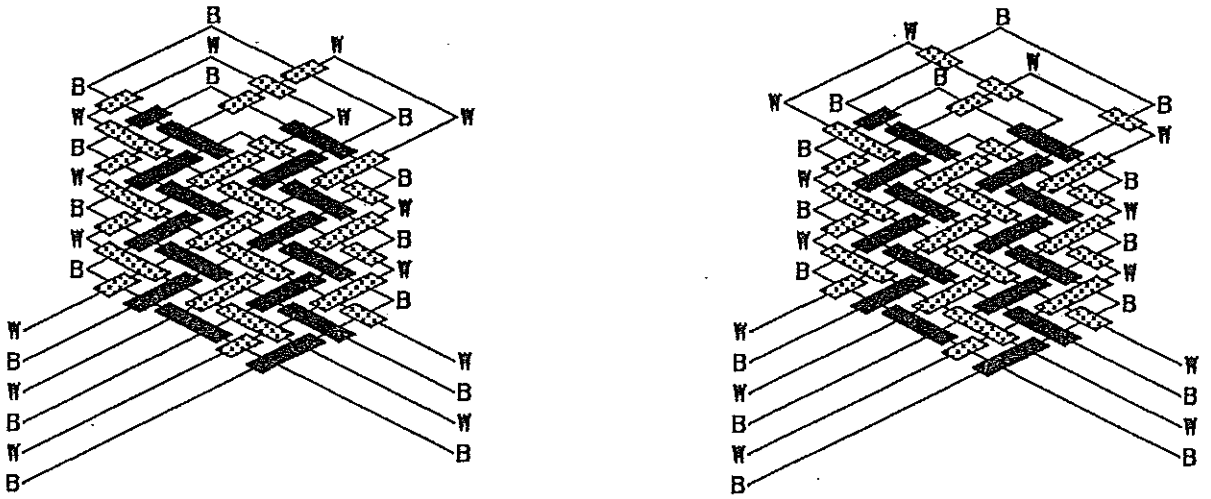


Fig. 25 — Grid-diagrams representing the tip construction of Fig. 24.

The tip construction of Fig. 24 requires only one thong which is for half the length light-coloured and for half the length dark coloured. However, the same applies for the much better tip construction of Fig. 26.

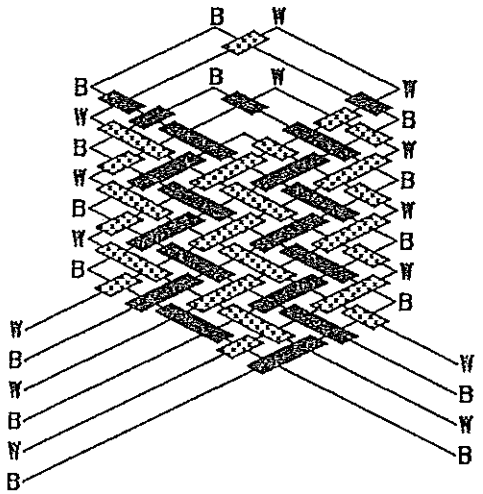


Fig. 26 — Grid-diagram of an acceptable tip construction.

In Figs. 27 and 28 we have shown Casa-coded tip constructions for a ten-string Herringbone-coded Regular Flat Braid which can be classed as good and very good respectively.

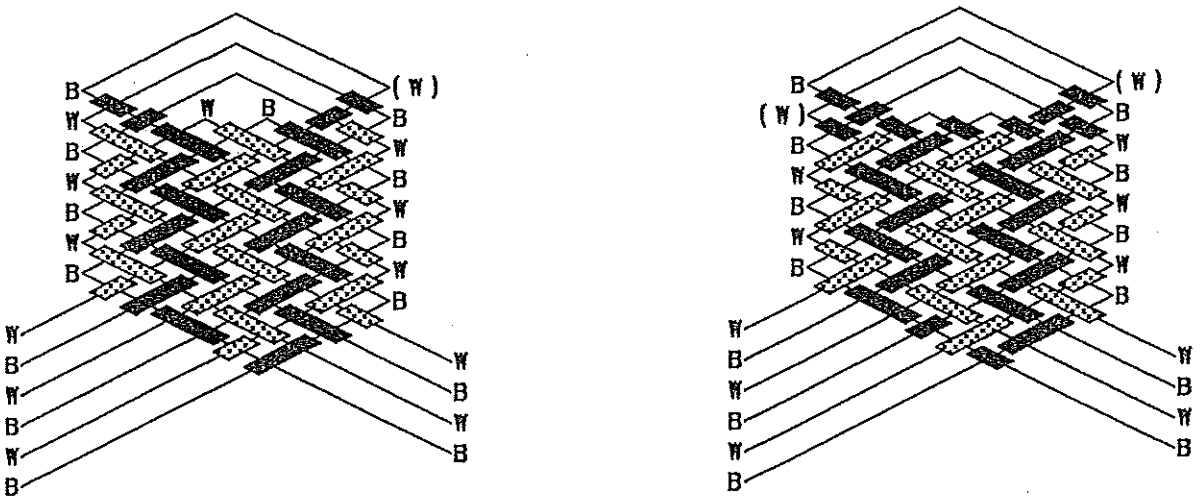


Fig. 27 — Grid-diagrams of good tip constructions.

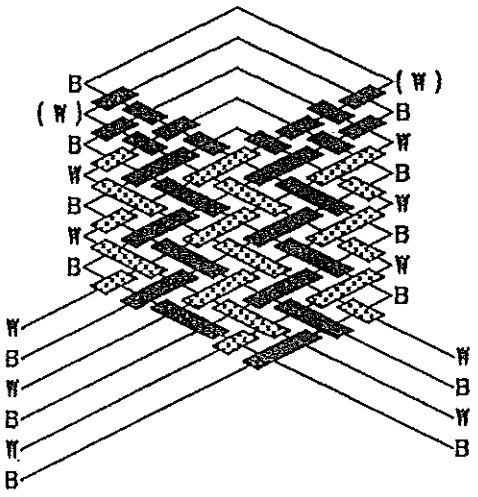


Fig. 28 — Grid-diagram of a very good tip construction.

Braiding Instructions and their Presentation

In the previous issue we have seen a few examples of Flat Braids and how they are depicted in grid-diagrams, drawn on an *isometric grid*. The reason for using such a grid is that it is the only suitable grid which is commercially available as graph-paper, which in turn allows the braider to design a braid quickly without the requirement of having to possess some artistic ability. We mentioned towards the end of the previous article that Flat Braids form the foundation on which Cylindrical Braids are based and that in this issue we would continue with discussing them. Since there are many classes of Flat Braids, so are there many classes of Cylindrical Braids.

The most simple class of Flat Braids is the class of the *Regular Flat Braids* (see pg. 6). It should be noted that in the definition of Regular Flat Braids, no mention is made of any *coding* form. Hence it defines the string-run only. Similarly other flat braid classes are defined by their string-run only. For example, the flat braid class to which the *Standard Herringbone Pineapple Flat Braid* discussed in the previous issue belongs, is the class of the *Regular-Nested Flat Braids.*

Since the *Regular Flat Braids* form the most simple class of *Flat Braids*, so form the *Regular Cylindrical Braids* the most simple class of *Cylindrical Braids.* The *Regular Cylindrical Braids* have essentially the shape of a hollow cylinder with two parallel bight-boundaries which form its ends. If we cut such a Regular Cylindrical Braid along a cylinder generating line, we obtain a length of Regular Flat Braid. If we now reverse this process, hence take a length of Regular Flat Braid and after wrapping it around a cylinder join the ends in such a way that there are no irregularities, then we have produced a Regular Cylindrical Braid.

If we take the length of Regular Flat Braid such that its number of bights along one of the edges (of course both edges have the same number of bights) is coprime† with the number of its strings, then and only then can the resulting Regular Cylindrical Braid be made from one single string. Such 1-string Regular Cylindrical Braids we call **Regular Knots.** If the length of Regular Flat Braid is such that the number of bights along one of the edges and the number of its strings have a common divisor, then the resulting Regular Cylindrical Braid requires more than one string in its construction. In fact the number of strings required is equal to the greatest common divisor (g.c.d.) of these two numbers. Such Regular Cylindrical Braids are called **Semi-Regular Knots.**

It is important to note that in this classification the *coding* plays no role; it is concerned with the string-run only.

We have already seen that a **length** of Regular Flat Braid is fully defined by the number of bights along **one** of its two parallel bight-boundaries. Hence with this number of bights we indicate the length of a piece of Regular Flat Braid. It will be obvious that we need to indicate the number of strings in the Regular Flat braid by some other parameter, which has an identical meaning in both the Regular Flat Braid and the Regular Cylindrical Braid. We note that the numbering of the columns in the string-run of the Regular Flat Braid always results in the highest column-number being equal to the number of strings in the Regular Flat Braid concerned, and that this number is equal to the number of equal **parts** in which a half-cycle is divided by opposing half-cycles. This number of *parts* in which a half-cycle is divided, does not change with whatever we do with a length of Regular Flat Braid, and hence is a much more suitable parameter than its number of strings to which it is equal. Thus:

The string-run of a length of Regular Flat Braid is fully defined by its number of parts and its number of bights.

We shall restrict our further discussion to the **Regular Knots**, hence the 1-string Regular Cylindrical Braids.

Let's look at an example, for which we shall use a length of the Regular Flat Braid depicted in Fig. 29. This is a Regular Flat Braid with seven parts. The piece of length we have taken has five bights as depicted in the uppermost right pictorial drawing in Fig. 29. Note that the numbers 7 and 5 are coprime.

† the two numbers are coprime when they are not divisible by the same integer other than 1.

We wrap this length around a cylindrical object and join the identical lettered thong-ends. Since the string-run is: top 'A' = bottom 'A' ⟶ top 'B' = bottom 'B' ⟶ top 'C' = bottom'C' ⟶ top 'D' = bottom'D' ⟶ top 'E' = bottom'E' ⟶ top 'F' = bottom'F' ⟶ top 'G' = bottom'G' ⟶ top 'A', this results in a Regular Cylindrical braid which can be made from one length of thong, hence the result is a Regular Knot.

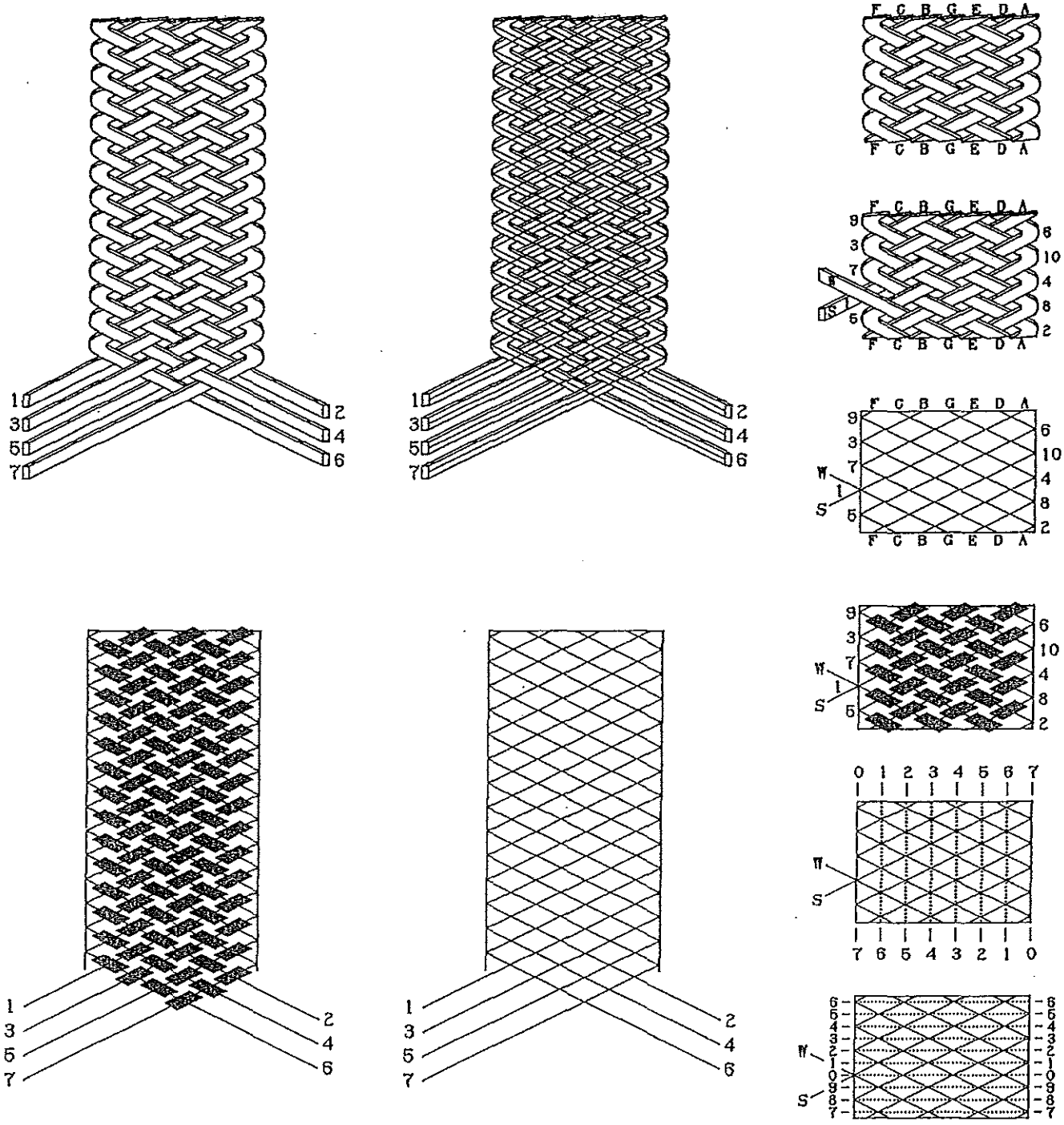


Fig. 29 — A 7-string (7-parts) Casa-coded Regular Flat Braid and a 7-parts/5-bights Casa-coded Regular Knot.

This uppermost right pictorial drawing in Fig. 29 is redrawn in the next lower right-

hand pictorial drawing in which the ends of the thong are shown. Furthermore, the half-cycles are numbered, whereby half-cycle 1 starts with the Standing-end 'S' and half-cycle 10 ends with the Working-end 'W'.

The string-run of this Regular Knot is shown by the right-hand diagram, third from the top.

The next lower diagram shows this string-run diagram with the superimposed Casa-coding, and hence it depicts the complete grid-diagram of a 7-parts/5-bights Casa-coded Regular Knot.

The next lower diagram shows the string-run diagram with the intersection columns dotted. For the half-cycles which run from Left to Right the columns are numbered from Left to Right, starting with '0' for the leftmost column (the left-hand bight-boundary) and ending with '7' (equal to the number of parts) for the rightmost column (the right-hand bight-boundary). For the half-cycles which run from Right to Left the columns are numbered from Right to Left, starting with '0' for the rightmost column (the right-hand bight-boundary) and ending with '7' (equal to the number of parts) for the leftmost column (the left-hand bight-boundary).

The next lower diagram shows the string-run diagram with the intersection rows (crossings-rows) dotted. The intersection row which passes through the bight where the Standing-end begins and the working-end ends, receives the number '0'. The rows are numbered upwards consecutively, and hence the last row (the row immediately below row '0') receives the number '9' ($= 2b - 1 = 2 \times 5 - 1$), hence we have ten rows ($= 2b$).

Note that in these diagrams we have deliberately placed the cylinder generating cutting-line in such a position that it does not coincide with an intersection-row, but instead runs halfway between two adjacent intersection-rows.

To draw the grid-diagram of a Regular Knot on a piece of isometric graph-paper, we follow the following procedure (we shall use as example a 7-parts/5-bights Casa-coded Regular Knot):

On a piece of isometric graph-paper (see Fig. 30) we choose a vertical line on which we shall later place the left bight-boundary of the knot. From this line we draw to the right a horizontal line (the cylinder generating line used in cutting the knot, and which passes halfway (in vertical sense) between adjacent rows of intersection points. The length of this line, measured in columns, is equal to the number of parts of the knot.

There are two possible ways in which to draw this line as shown in Fig. 31 (the upper horizontal lines). Which of these two ways we adopt is immaterial. This same cutting-line must of course also be drawn b bights below the top lines as shown in Fig. 31.

Then we connect the left-hand starting-point of each upper horizontal line with the left-hand starting-point of its corresponding lower horizontal line (the left-hand bight-boundary of the knot), and we connect the right-hand end-point of each upper horizontal line with the right-hand end-point of its corresponding lower horizontal line (the right-hand bight-boundary of the knot). The result is shown in Fig. 32.

Next we draw lines over the isometric grid-lines within these "knot-frames" and choose a bight-point where we position the Standing-end of the string (see Fig. 33). This gives us the string-run diagram of the knot.

Note that the left-hand and right-hand string-run diagrams in Fig. 33 are equivalent, and hence it is immaterial which one we choose for representing the string-run of the knot.

In Fig. 34 we have superimposed on these string-run diagrams an identical Casa-coding, and hence the grid-diagrams in Fig. 34 are equivalent. It is therefore immaterial which one of these grid-diagrams we choose to represent the knot.

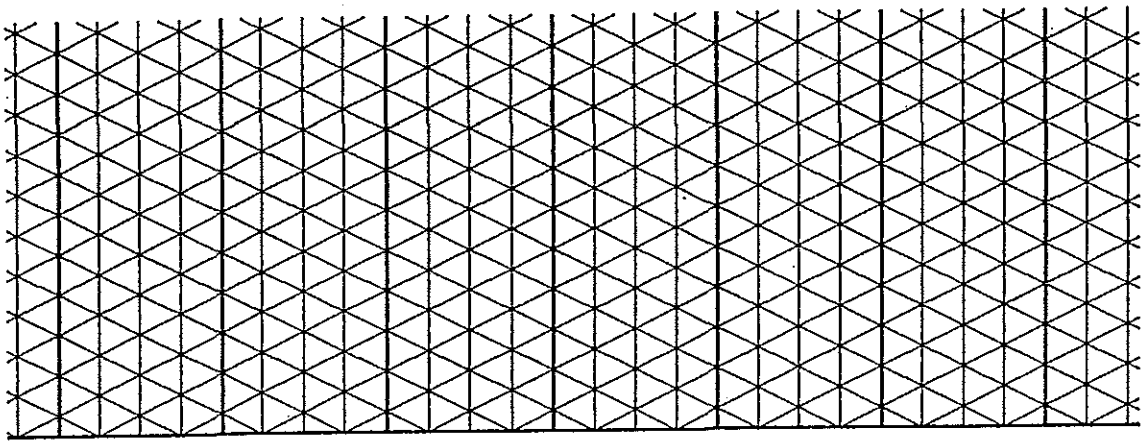


Fig. 30 — Isometric graph-paper.

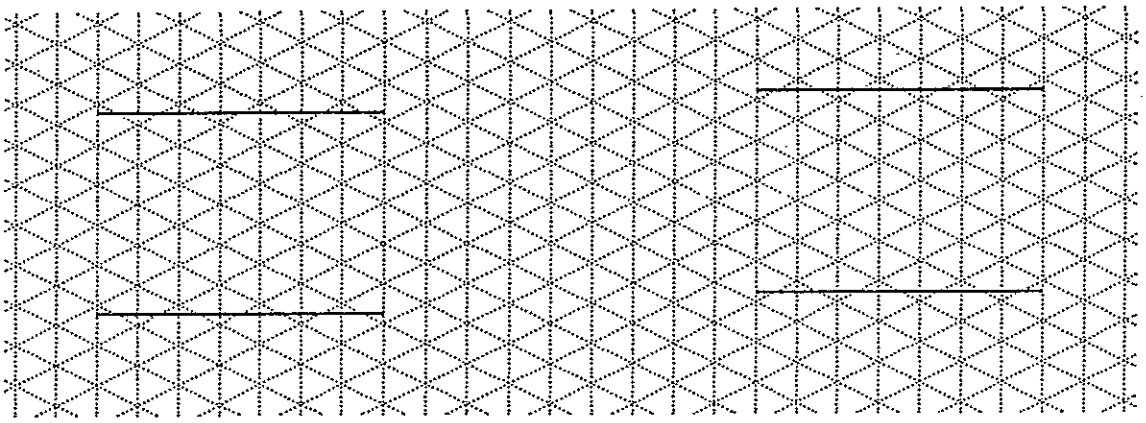


Fig. 31 — The two ways in drawing the cylinder-generating cutting-line.

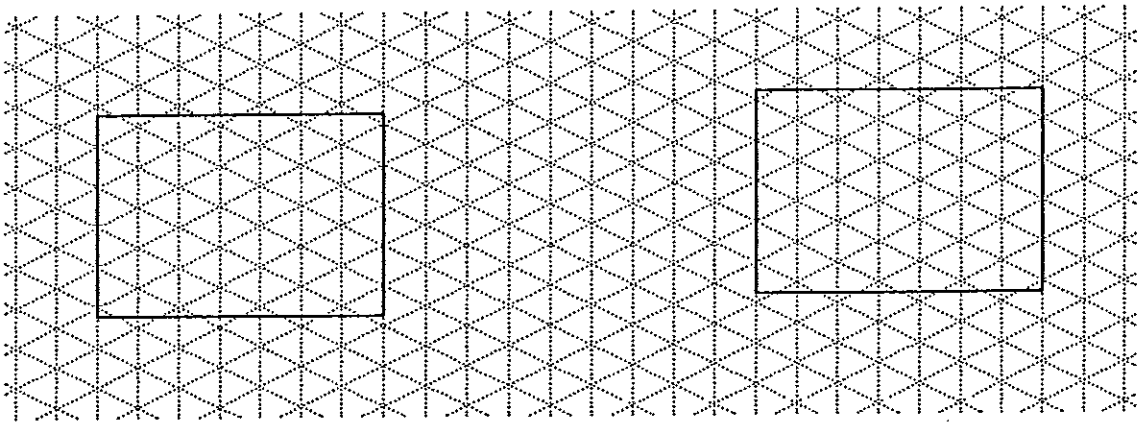


Fig. 32 — The two ways in drawing the "knot-frame".

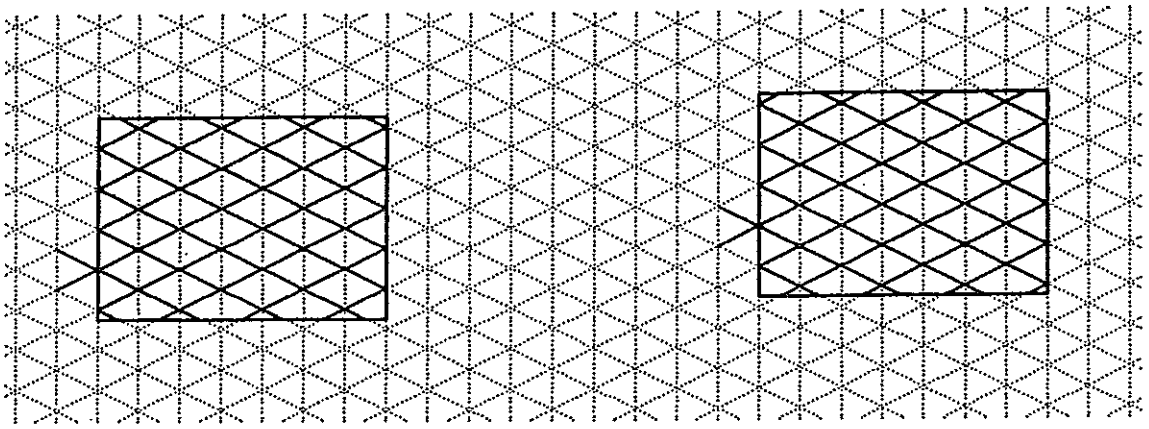


Fig. 33 — The two ways in drawing the string-run diagram.

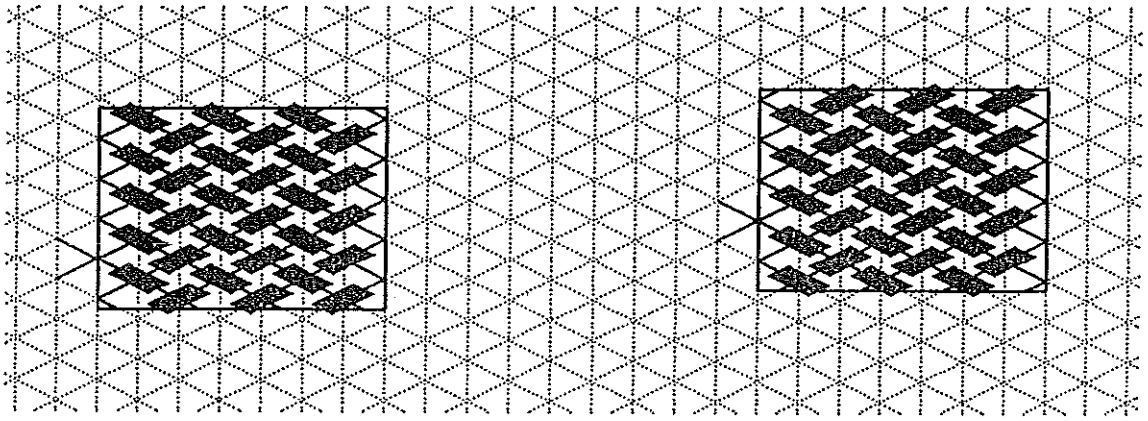


Fig. 34 — The two ways in drawing the grid-diagram.

Any Cylindrical Braid can be cut along a cylinder generating line, then rolled out in a flat plane, thus giving a length of Flat Braid. This length of Flat Braid can then readily be represented by a grid-diagram, which in turn represents the Cylindrical Braid in question. By using isometric graph-paper (which is commercially available), just any braider is thus able to draw quickly the grid-diagram of any Cylindrical Braid, irrespective of its coding and the number of strings used in its construction. One of the great advantages of using grid-diagrams is that we can see the whole braid, not just a part of it as is the case in a three-dimensional (pictorial) drawing. Since grid-diagrams show us the whole braid in a stylized form without distracting beautifications, we are able to see immediately any string-run and/or coding inconsistencies which may be present in a given braid. We are also able to design quickly different coding forms, and with some experience we will be able to distinguish between the more and the less aesthetically pleasing ones, before we braid the knots in question.

We can also use grid-diagrams in the derivation of the required braiding sequences. Furthermore we can depict the consecutive braiding sequences accurately and hence clearly in an associated set of grid-diagrams; each consecutive braiding sequence can be shown in relation to previous sequences in such a way that it can readily be distinguished

from them. An example of this is given in Fig. 35 in which the construction process of a 7-parts/5-bights Casa-coded Regular Knot (uppermost left-hand grid-diagram) is depicted. Since this knot has five bights (=b), and hence ten half-cycles (=$2b$), it may be constructed by braiding the consecutive ten half-cycles. This is shown in the remaining ten diagrams of Fig. 35.

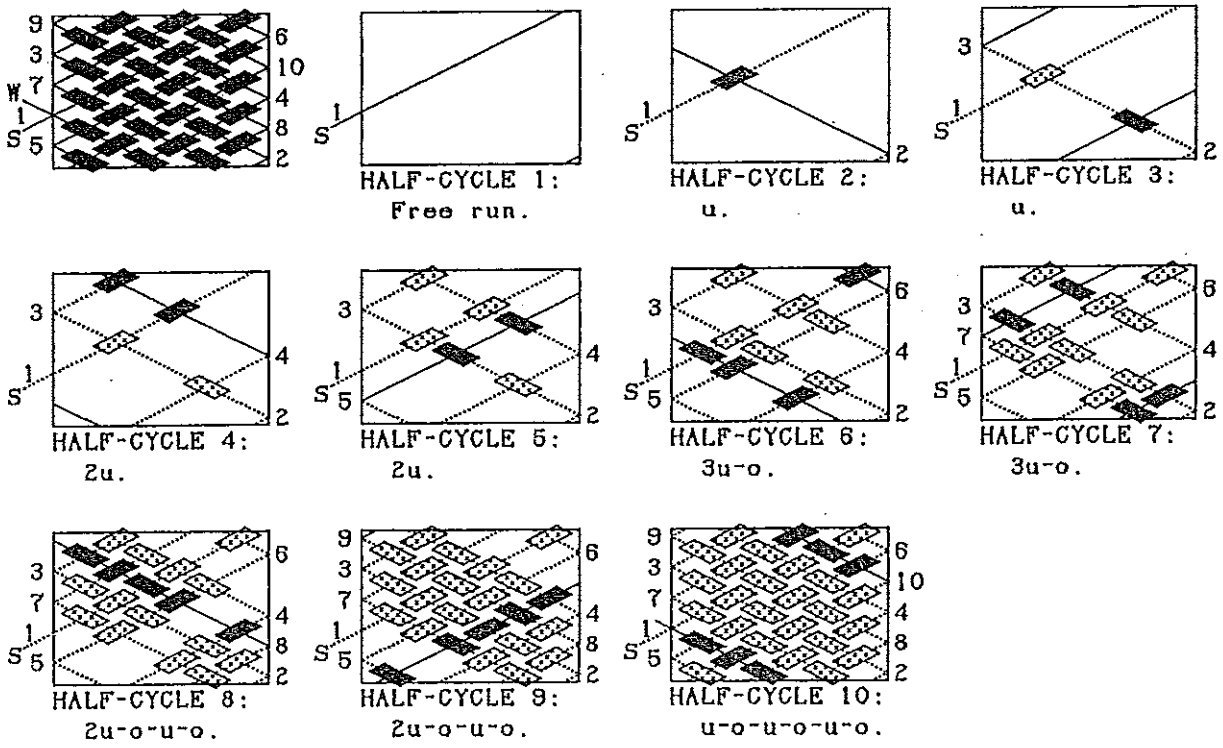


Fig. 35 — The braiding of the 7-parts/5-bights Casa-coded Regular Knot by means of its ten consecutive half-cycles.

Another very important property of a grid-diagram is that it contains the string-run of the braid depicted, and that by removing the coding we are left with this string-run in a clear uncluttered way. The string-run contains many specific properties of the class of braids to which the braid in question belongs. We have already seen an example of this above in the case of Regular Knots, where the number of parts (p) and the number of bights (b) must be coprime (this property is independent of the coding). In future issues we shall encounter several other important properties associated with these knots, properties which will aid us in the actual braiding of these knots. But before diving into these aspects, we shall in the next issue first show how a grid-diagram readily exposes undesirable construction methods, and how it aids us in correcting these.

Some New Publications for Braiders

We have published, since the previous issue of *"The Braider"*, three further Pamphlets which, as is the case with all our other publications, are only made available

to braiders. Please add the details concerning these new publications to the lists and cost-tables which were a supplement to issue No. 1. The details are as follows:

Supplement to Pamphlet No. 4 — *The Regular Knot Tree and Enlargement Processes* — **Casa-coded Regular Knots.** A.G.Schaake and T.Hall.

This Pamphlet in A4 format (295 × 210mm.), soft cover, stapled and taped spine, contains 31 pages and numerous illustrations of grid-diagrams and their associated pictorial drawings.

Price: NZ$ 4.00; Weight: 0.11 kg.

With this Supplement it is the first time in braiding history that the braiding of Casa-coded Regular Knots, by means of enlargements, is fully treated. It is only fair to say that this braiding process is very poorly, and hence incompletely, understood by braiders. Contrary to the statements in the available literature which deals with this subject, just any Regular Knot can be readily braided with a Casa-coding via Enlargement steps. It is in fact the easiest and most convenient method for braiding these knots, since we do not require half-cycle algorithms or algorithm-diagrams. All we have to know are a few rules, which are very easy to remember. Any braider who is still able to divide 3 by 2 and work out the remainder, will encounter no difficulties. Many fully illustrated Examples are presented.

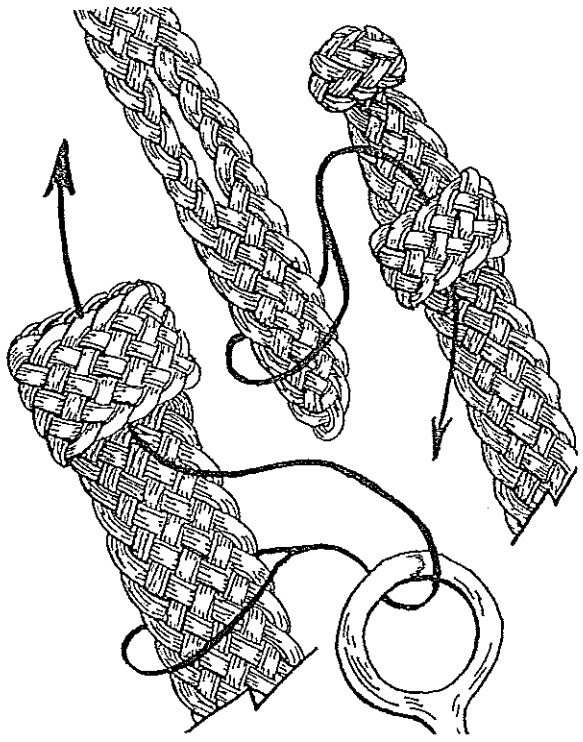
It is in our opinion one of those publications nobody who wants to call him or herself a braider can do without.

Pamphlet No. 10 Pt. 1 — *Special Braid Forms Pt. 1* — **End-keepers and Mid-keepers.** A.G.Schaake, T.Hall and J.C.Turner.

This Pamphlet in A4 format (295 × 210mm.), soft cover, stapled and taped spine, contains 176 pages and numerous illustrations of grid-diagrams and their associated pictorial drawings.

Price: NZ$ 13.00; Weight: 0.49 kg.

Rather than trying to describe what End-keepers and Mid-keepers are, the following illustration should make it much clearer than a hundred words.



An End-keeper is shown in the leftmost drawing and a Mid-keeper in the rightmost drawing. Obviously a Mid-keeper may be placed anywhere along the Flat Braid, not necessarily near the end.

In this Pamphlet the braiding of Regular End-keepers and Regular Mid-keepers is fully discussed with the aid of many Examples. Several concepts which are new to the braiding-world and are essential for the further development of braiding, have been introduced. The first 116 pages (bar pg. 1) deal with the Regular End-keepers, while the remaining pages (bar pg. 176) deal with the Regular Mid-keepers.

The material is suited for the more advanced braider, and in fact for many of them it may require some serious effort to become fully acquainted with the subject. However for those braiders whose desire it is to become braiding artisans, the processes dealt with in this Pamphlet are essential since the End-keepers and Mid-keepers discussed, form an integral part of the overall braid. This is of course essential for good quality braidwork, where their commonly seen "bush" counterparts, made from separate strings, have no place.

Also here with this Pamphlet, any braider who still can count, divide 3 by 2 and work the remainder out, will be able to master the few simple calculation procedures which are involved easily.

Pamphlet No. 13 — The Braiding of Long Regular Knots with Maximum Number of Free-Run Half-cyles. A.G.Schaake and T.Hall.

This Pamphlet in A4 format (295 × 210mm.), soft cover, stapled and taped spine, contains 50 pages and illustrations of the 3-row modular algorithm-diagrams and grid-diagrams associated with the knots discussed.

Price: NZ$ 5.50; Weight: 0.17 kg.

Although the braiding of Long Knots from one string is quite time consuming, there are many applications where for various reasons Long Knots braided from one string are preferable to those braided from several strings. In order to keep braiding-time and hence labour-cost down, we often use in such circumstances Long Knots which possess the **maximum** number of '**free-run**' half-cycles. Such knots are of necessity **Column-coded** and possess specific **weaving-patterns**. The number of different knots for a given number of bights depends on this number of bights. When the number of bights is *even*, the *weaving-patterns* are symmetric relative to the length-wise centres of the knots. When we require different *weaving-patterns* for the two bight-edges, then the number of bights must be *odd*.

Since specific applications often require specific weaving-pattern, it is important to have available a type of data-base from which the braider can readily select the appropriate Long Knot with its half-cycle algorithms. This Pamphlet not only fully discusses the necessary procedures to generate such a data-base, but supplies such a data-base for Long Knots with 2 to 12 bights.

Please Note

Pamphlet No. 10 Pt.2 — The Braiding of CFC Braids, and **Pamphlet No. 11 — Braiding Application – Bridle and Reins** have not yet been published. Their publication will in due course be announced in *"The Braider"*.

Reviews

In this issue we shall review some of the braiding booklets by Ron Edwards. The contents of the following individual booklets on braiding:

(1). Stockmen's Plaited Belts; format: 246 × 162mm.; 28 pages; ISBN 0 909901 28.
(2). The Secret Plait; format: 245 × 162mm.; 20 pages; ISBN 0 909901 29 5.
(3). Whipmaking Book One; format: 246 × 164mm.; 20 pages; ISBN 0 909901 32 5.
(4). Whipmaking Book Two; format: 248 × 160mm.; 32 pages; ISBN 0 909901 42 2.
(5). Leather Lacing Manual; format: 248 × 163mm.; 16 pages; ISBN 0 909901 36 8.
(6). Plaits, Round & Flat; format: 240 × 161mm.; 20 pages; ISBN 0 909901 37 6.
(7). Braided Belts; format: 244 × 161mm.; 20 pages; ISBN 0 909901 41 4.
(8). Bushmen's Belt Pouches; format: 243 × 162mm.; 24 pages; ISBN 0 909901 46 5.

are also to be found in his book entitled *Bush Leatherwork*. This last mentioned book contains furthermore the contents of a further four booklets which are of little or no value to braiders; they are:

Homemade Leatherworking Tools; Counterlining Stock Saddles;
Saddle Repairs; Rolling a Thread.

The booklets have a soft cover and a stapled spine.

Apart from the irregular format, and in some booklets the too narrow margins, it are first class publications for the beginner. It should, however, be kept in mind that we are dealing here with "bush" braidwork, hence there are instances where the braiding procedures described do not result in good quality braidwork (braidwork of a high standard). The braiding procedures are clearly explained with an abundance of excellent pictorial drawings.

Booklet (1) deals with Flat Belts which have a Casa-coding. Discussed are the Regular Flat Braids with an *even* number of parts (also some colour patterns for these braids are dealt with), a Crocodile Ridge Braid, two Raised Edge Braids, lace cutting, the twin ring belt, and the buckle belt.

Booklet (2) describes the braiding procedure for the Secret Braid in an excellent manner (definitely the best on this subject). It further describes with excellent drawings the Gulf Knot and the Trinity Knot.

Booklet (3) describes the construction procedures for basic stockwhips.

Booklet (4) deals with the kangaroo hide stockwhips.

Booklet (5) describes, with excellent drawings, various edge lacing procedures. The procedures described are the same as those which can be found in many other publications. Hence some do need updating.

Booklet (6) describes besides the braiding of some round and flat braids, the construction process of a bullwhip.

Booklet (7) deals mainly with a few examples of appliqué braidwork. It also describes the construction process of a snakewhip.

Booklet (8) contains only thirteen pages which are devoted to braidwork, where the braiding of various pouches is described; such as a braided knife pouch, a braided knife pouch fixed to a braided belt, and a braided pouch.

The main merit of these publications lies in the abundance of excellent drawings and hence they are eminently suited for the beginner. However, in many aspects they fall a very long way short of forming a standard reference or part thereof.

THE BRAIDER

No.3 AUGUST 1995.

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A quarterly publication
for
the braiding artisan

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This publication is available to braiding artisans only.

Copies may be obtained from:

A.G. Schaake,
21 Sundown Cresc.,
Hamilton,
New Zealand.

THE BRAIDER'S NOTEBOOK

When braiders begin to feel a little bit more comfortable with grid-diagrams they soon get the urge to design initially various knots, soon to be followed by all manner of braid-forms. From their created grid-diagrams they then read the braiding algorithms which will enable them to actually braid their creations. At least for some time this actual braiding of these created braid-forms is essential as otherwise the braider will never properly **comprehend** what the grid-diagram portrays, hence a theoretician will never become a braider, let alone a braiding artisan. The braider will derive from the process: visualizing a braid-form $\longrightarrow$ creating its grid-diagram $\longrightarrow$ braiding the braid-form in accordance with this grid-diagram $\longrightarrow$ comparing the obtained braid-form with the envisaged braid-form, a wealth of knowledge which he or she will be able to apply to further designs. Eventually the braider will be able to accurately predict the practical result of what the grid-diagram depicts. Such a process **should** ultimately develop the braider into a braiding artisan. However, if the braider is not careful, and get carried away with the new possibilities which have just opened up for his or her creative mind, then the braider will not reach this ultimate goal. Often in their desire to create braid-forms which are new to them, and have to fulfil an envisaged application, they forget or neglect some of the most important braiding aspects. Of course anyone who can read and comprehend a grid-diagram, and hence can see in his or her mind the braid-form so depicted, should have no trouble in designing literally hundreds of different braid-forms which may be used in practical applications. In fact it will be child's play, and only in the eye of the novice will this appear to be expertise and craftmanship. Of course nothing could be further from reality.

We have seen in the issues 1 and 2 that a braid-form consists of two independent parts:

(1). the string-run,
(2). the coding.

Although from the string-run we cannot obtain the braiding algorithm and hence cannot braid the braid-form, it is nevertheless initially the most important item. It will tell the braider how many strings are required for the particular braid. If that number is not what is wanted, then the braider will have to adjust his or her design, so that the required number of strings is equal to the number wanted. Only after the condition: *number of strings required = number of strings wanted* has been fulfilled, can the braider devote his energies on designing a coding for the string-run just obtained. It would, however, be a stroke of luck if the size of the obtained braid-form fits the intended application. Hence the braider will normally have to adjust the size of the braid-form once again. Hence even the design of a suitable string-run becomes a process of trial and error. To complicate matters further, the braider might not be able to superimpose on this string-run a suitable or desirable coding (other than a Casa-coding which can always be superimposed), with the result of having to find other suitable string-runs on which, hopefully, a suitable or desirable coding can be superimposed.

It is therefore essential to possess the necessary tools, in the form of relationships between the various parameters involved, which will allow the braider to form an overview of what is and isn't possible as far as the string-run is concerned. Only then can he or she dispense with the trial and error method of finding a suitable string-run. It should be stressed here that a trial and error braider is **not** a braiding artisan and is most unlikely ever to become one unless he or she outgrows the trial and error habit.

But even when the braider has derived all the necessary relationships between the parameters involved for the braid-form in question, it may well be that for practical purposes it is much more convenient to assemble these relationships in a series of general string-run diagrams from which the various valid solutions may be read. In fact this will be the case for most of the more involved braid-forms.

Hence we see that generally a considerable amount of work has to be done to even obtain that essential overview of what is and isn't possible as far as the string-run is concerned. It should be stressed that what isn't possible is just as, if not more, important than that what is possible.

Let's say that the braider has solved the string-run problems, which is the **major** step in developing new braid-forms. However, the dangers of going astray are far from over. In fact their presence could even lead the more experienced braider who has mastered this important stage just as easy astray as they often tend to do the novice. The dangers of going astray revolve around two very important concepts, which are far too often overlooked. They are:

(1). proportions,
(2). balance and symmetry.

It is of the utmost importance that the braider pays due attention to these two concepts. Unfortunately many braiders fail to do so, with the result that their braidwork, which may have been very neatly executed, has to be downgraded to that of a low quality (standard). However, as easy as it is to neglect these two aspects, just as easy is it to take them into account, hence there is no excuse for their neglect.

First of all the aspect of **proportions** should be taken into account with each design. The general rule that *small components* should be associated with other *small components*, and that *large components* should be associated with other *large components*, will no doubt be more than obvious. Nevertheless, how often don't we see this most obvious rule violated; no doubt in many cases is this associated with the trial and error braider, who is not in the possession of the necessary tools (in the form of relationships between the various parameters involved) which will provide him or her with the means to form an overview of what is and isn't possible as far as the string-run is concerned. It is sometimes also associated with braiders, who blindly follow a given design, change and/or add some components, without critically assessing the new overall design.

Then, the aspect of balance and symmetry should receive its due attention. In braidwork we encounter two different forms under this aspect:

(1). balance and symmetry related to weaving-pattern,
(2). balance and symmetry related to colour-pattern.

We have grouped balance and symmetry together, since both exert their influence often together. When a weaving-pattern is symmetric, then it is also automatically balanced. For example a Gaucho-coding or a Headhunter's-coding (see Fig. 36).

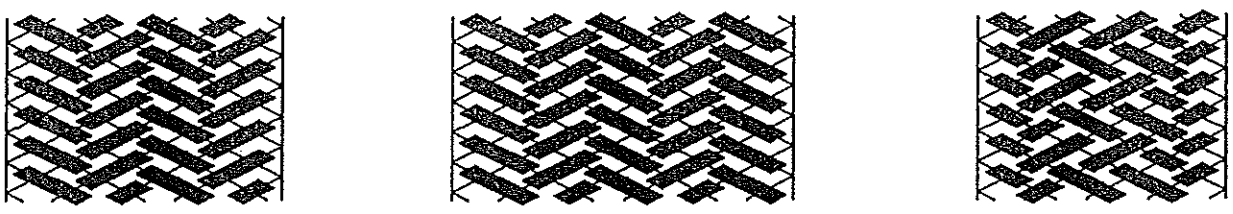


Fig. 36 — Balance and Symmetry in weaving-patterns.

The rightmost grid-diagram in Fig. 36 shows a coding where *balance* and *symmetry* work together to give a good coding design.

In Fig. 37 we have depicted two Examples of lopsided codings. They obviously create in most cases lopsided weaving-patterns, and hence are very seldom of any use, especially when the geometric shape of the braid is symmetric.

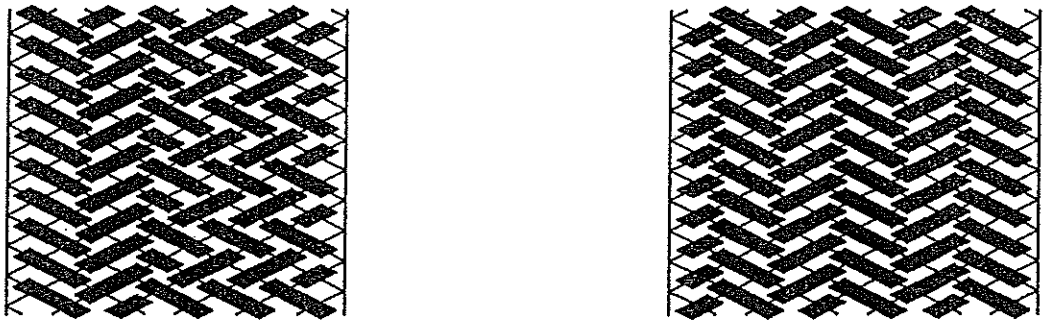


Fig. 37 — Examples of Lopsided coding arrangements.

Although there are many braiders who don't seem to have much eye for balance and symmetry, it is also true that there are many who possess an automatic eye for it. Those who don't have an eye for balance and symmetry should do their utmost best to cultivate one, since its presence or absence will either make or break a design.

Balance and symmetry as related to colour-pattern is often vastly different to balance and symmetry as related to weaving-pattern. A good illustration of this may be found in flat belts. Balance and symmetry related to the weaving-pattern demands that the weaving-pattern near the upper and lower bight-edges must be symmetric, while the the rest of weaving-pattern must at least be balanced. Balance and symmetry related to the colour-pattern does not necessarily demand symmetry near the upper and lower bight-edges; here balance demands that if there is no colour-symmetry near these two bight-edges, then the braid near the upper bight-edge must be of a lighter colour than the braid near the lower bight-edge.

Those braiders who have difficulty with the aspects of balance and symmetry, or those who have not given those aspects their due attention, should braid several sample pieces of various flat braids to begin with. They should use different coding designs, and different colour designs and compare the aesthetic appearance of their creations very critically. Very soon the aspects of balance and symmetry should not present problems anylonger.

This Quarterly Publication and its Purpose

No doubt, different braiders will have different expectations of this publication with regards to the type of articles and their contents. The purpose of this publication is to assist those braiders whose desire it is to acquire a thorough understanding of our new braiding theory with the aim of applying it to their braidwork. It is not meant to be a substitute for our other publications, to the contrary, it is meant to complement these publications. Braiders will often encounter parts in those publications, which they might find difficult to understand. In this quarterly publication we shall then try to

present those parts in a different way, or in some greater detail. Hence when there are queries, let's hear about them. Naturally, querries not directly associated with our other publications are just as welcome, and will receive their due attention.

In order to present the material in an orderly fashion, which is of course essential if it will be of real use to the reader, it is important that we don't get carried away with little titbits, which are not yet supported by a more thorough treatment in one or more of our other publications. It is very important that a braider gets a full and thorough understanding of a particular braid-form, rather than to excite him or her with a view of a fabulous looking dish, without being able to taste it properly. It is understandable, that those who have been exposed to a very tiny and incomplete glimpse of a braid-form new to them, get excited and like to let others partake in the view. Not only will this have little purpose, it harbours the danger of jumping the gun. However, all those exciting braidforms of which there are literally hundreds with practical applicability, will in due course get their deserved attention, hence the braider who asked to show in this publication a braided keeper assembly for a ranger style belt (which was by the way only one of the braid-forms of this category), will not be disappointed, provided he has the patience to wait a few years.

As we have said in the opening article of this issue, a braider has to be presented with the full set of tools which will enable him or her to form a complete overview what is and isn't possible with a particular braid-form. Finding a string-run for a set number of strings by means of a trial and error method is only of very little value.

In due course, the diversity of articles will increase, but initially the articles are meant to make the braider fully conversant with the basic tools of trade. When we have learned to walk, a little running will be good at times, but not before we can walk properly without effort though. To ascend the braiding-ladder, each step will require effort and patience. All good things come slowly!!!

Braiding Instructions and their Presentation

Grid-diagrams enable us to obtain a complete overview of braids, an overview which is impossible to obtain with pictorial drawings. From this overview we can readily distinguish good designs from bad ones before we start with the actual braiding process. Many bad designs can then at the design stage readily be modified into good designs. This will avoid the waste of time and materials in braiding a braid-form which, when completed, is not satisfactory or of a poor standard. Unfortunately many braiders use at times sub-standard braid-forms. This is in most cases due to their inability to recognise such braid-forms. Sub-standard braid-forms may be divided into two main categories: (1). Those with merit. These can be divided into two sub-categories:

(1a). Bush braid-forms. These braid-forms are generally discovered in an experimental way. Their assets lie in the ease with which their braiding-process can be remembered and executed. Their importance is restricted to bush braidwork. However, these bush braid-forms can readily be modified into braid-forms of a high standard. A bush braid-form often carries the name of its discoverer, however we shall attach this name to the modified form of a high standard.

(1b). Flawed braid-forms. These braid-forms are generally also discovered in an experimental way and contain one or more faulty construction sequences which don't exhibit themselves during a casual examination of the completed braid. These faults can readily be rectified in order to obtain braid-forms of a high standard. A flawed braid-form often carries the name of its discoverer, however we shall attach this name to the modified form of a high standard.

(2). Those without any merit.

Mongrel braid-forms. These braid-forms are generally created by inexperienced braiders, braiders with a very limited amount of braiding-knowledge, or by braiders who neglect (or have no eye for) the design aspects of *balance and symmetry.* It are generally those asymmetric braid-forms for which no justifiable application exists.

In this issue we shall look at two types of **mongrel braid-forms** and at a **bush braid-form.**

In the previous issue we looked briefly at the 10-parts Herringbone-coded Regular Flat Braid. We showed a few of the several possible tip-constructions. Among those, we showed an often encountered but not recommended form (Figs. 24 & 25). The grid-diagrams 1 and 2 of Fig. 38 are those of Fig. 25; they depict two of **several** different grid-diagrams depicting this particular construction-form; a **mongrel braid-form.**

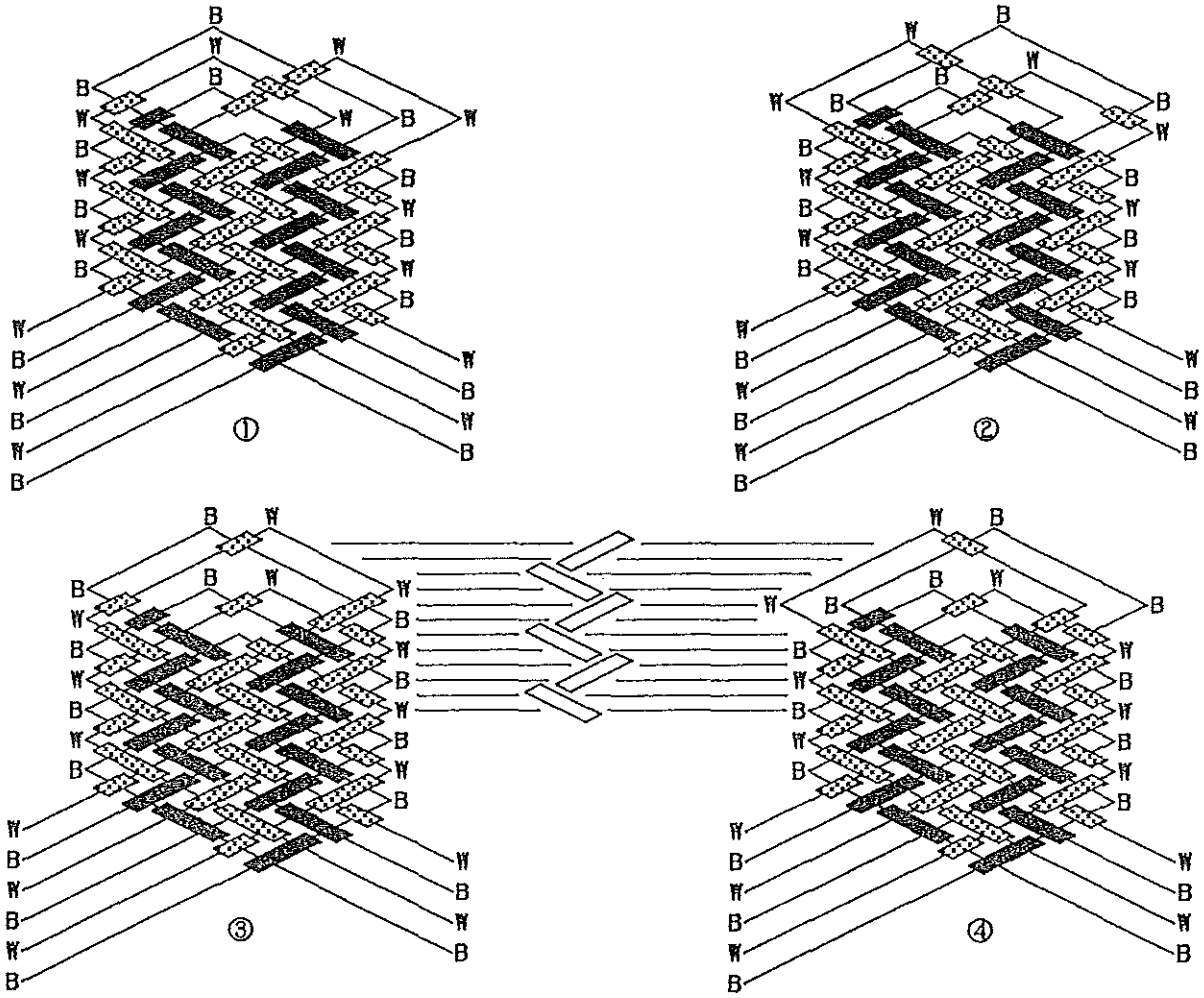


Fig. 38 — Some grid-diagrams depicting the tip-construction of Fig. 24.

We did mention that many braiders are inclined to continue with the Herringbone-

coding to the end of the tip, and that this will always lead to a tip that is not form-fast. Hence let's have a little closer look at this construction-form.

When a particular tip-construction can be depicted by different grid-diagrams, it indicates that this tip-construction is not form-fast.

We can readily rearrange the string-run of grid-diagram 1 without affecting the weaving-pattern, and obtain grid-diagram 3. This grid-diagram clearly shows that the Herringbone coding has been continued to the very end of the tip (the Herringbone-coding has been depicted between the diagrams 3 and 4, together with the associated intersection-rows).

The braider should ensure that the string-run of the Herringbone-coded Regular Flat Braid is maintained at the parallel bight-edges. This has not been done in diagram 4 (which can readily be obtained from diagram 2), and consequently its coding may falsely suggest that the Herringbone-coding has not been continued to the very end of the tip.

Although these grid-diagrams give an unsatisfactory tip-construction, the diagrams 3 and 4 have a symmetrical string-run.

Since a good tip-construction requires a symmetrical string-run, it may well be that such a **good** (and hence **form-fast**) braid-form is obtainable with a small change in the coding. We shall show this to be the case for the string-run of diagram 3.

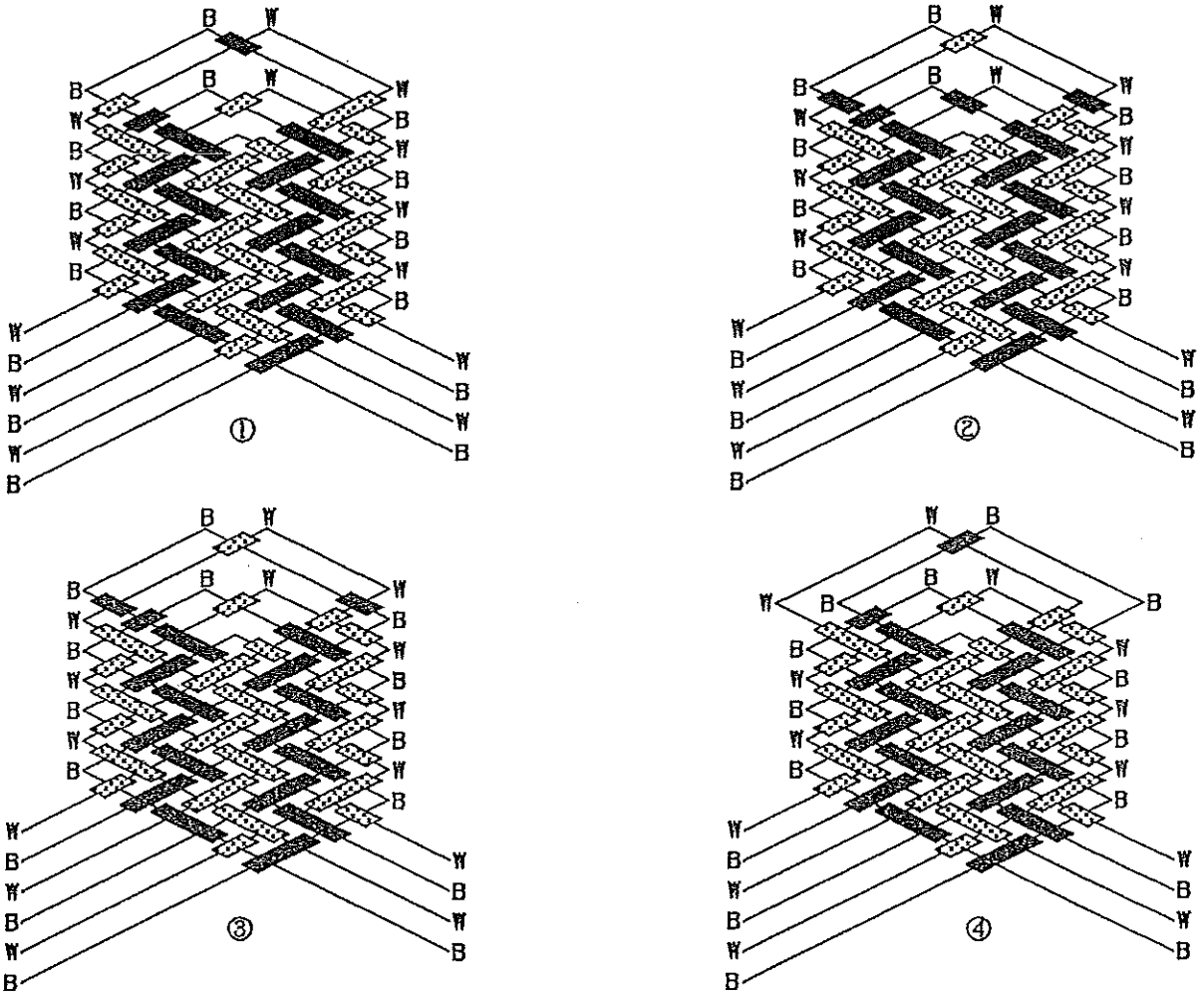


Fig. 39 — The grid-diagrams 3 and 4 of Fig. 38 with modified coding.

In Fig. 39 we have depicted three grid-diagrams (1, 2 and 3) which have the string-run of diagram 3 in Fig. 38, and one grid-diagram which has the string-run of diagram 4 in Fig. 38.

By changing in diagram 3 of Fig. 38 the coding of the uppermost crossing, we obtain the form-fast and good tip-construction depicted in grid-diagram 1 of Fig. 39.

Each of the grid-diagrams in Fig. 39 have a string-run with the tip-form:

Double Apex — Double Apex — Single Apex.

A tip-form which contains *Double Apex — Double Apex* never gives a satisfactory tip-construction when the *Double Apex — Double Apex* is surrounded by a Casa-coding. This is the reason why the tip-construction depicted by diagram 2 in Fig. 39 is only of an acceptable nature, and the tip-constructions depicted by diagrams 3 in Fig. 39 is of an unacceptable nature.

Generally, Nested Edge Bights in Regular Flat Braids do not give a satisfactory tip-construction, since they tend to make the string 'roll over' at the edges of the tip. For this reason the tip-construction depicted by diagram 4 in Fig. 39 has to be regarded as unsatisfactory, notwithstanding that also here the coding of the uppermost crossing in diagram 4 of Fig. 38 has been changed in order to make the tip form-fast.

It cannot be stressed enough, that a form-fast tip requires a Casa-coding, or at least a Casa-type coding, at the end of the tip.

It is important that the braider actually braid these various tip-constructions and compares the obtained results. Furthermore, by doing this with various Flat Braids, he or she will accumulate a valuable amount of knowledge which can later be used in the designs of further tip-constructions. It is very important **not** to use in these exercises braided thonging made from synthetics or natural fibres. These materials, due to their flexibility, often give totally false impressions. Only rawhide, leather, or fabric-reinforced plastic (such as "Dura Flex" lace) should be used. The braider should at all times ensure that the tip is braided as tight as possible.

The next type of **mongrel braid-form** is much easier to avoid, and is normally only used by inexperienced braiders. A typical example are the knots which are purported to be Gaucho Knots based on Casa Knots. For the alleged 2-pass variety, they come in two forms: those braided from two strings and those braided from a single string. Let's first illustrate the two string variety of these mongrel knots by means of their grid-diagrams (see Figs. 40).

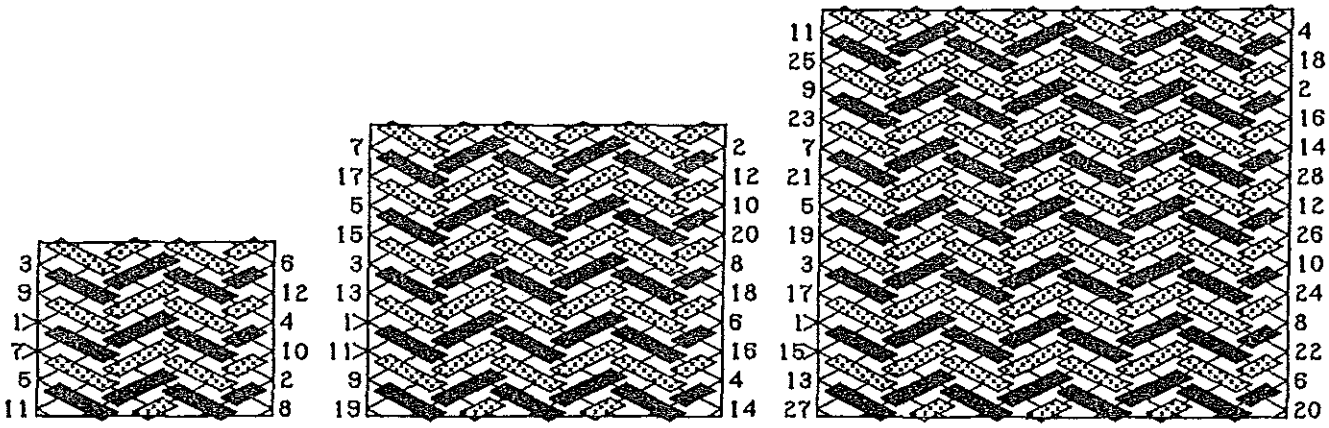


Fig. 40 — Grid-diagrams of the two string knots.

These knots are Column-coded Semi-Regular Knots. The leftmost knot in Fig. 40 has 8-parts and 6-bights, the second knot from the left has 12-parts and 10-bights, and the rightmost knot has 16-parts and 14-bights. For each of these knots the g.c.d. (greatest common divisor) of their number of parts and bights is equal to 2. Consequently since these knots are Regular Cylindrical Braids, each one of them requires two strings in its construction. It will immediately be seen from the respective grid-diagrams that these knots do **not** possess a 2-pass Gaucho-coding.

Since in a Regular Cylindrical Braid the number of parts is always 1 more than the number of crossing-points on a half-cycle, it is impossible to give these knots (to superimpose on their string runs) a Gaucho-coding, which always requires an **even** number of crossing-points on a half-cycle, and hence always requires an **odd** number of parts (recall from issue No.2 that in a Regular Cylindrical Braid the number of parts is always 1 more than the number of intersection-columns).

It will readily be seen that each of the knots in Fig. 40 is an interbraid of two Casa knots, both having an identical number of parts and an identical number of bights. In the leftmost knot each of the two interbraided Casa knots has 4-parts and 3-bights, in the second knot from the left each of the two interbraided Casa knots has 6-parts and 5-bights, and in the rightmost knot each of the two interbraided Casa knots has 8-parts and 7-bights. Each two interbraided Casa knots have an identical coding (note that this does not mean that they have identical strings; in our case the strings have a different colour).

These knots are mongrel knots because they lack symmetry and balance, the coding adjacent to one bight-boundary is of the "1-pass" type and adjacent to the other bight-boundary it is of a "2-pass" type, with the result that when the "1-pass" bight-edge is of the correct tightness, the "2-pass" bight-edge is too loose. Sometimes it is claimed that their merit lies in that they are easy to braid, but such a claim only testifies the very limited braiding-knowledge of the claimant, since all column-coded knots may be braided with the same ease.

Some braiders might wonder if these knots could possibly have some merit due to the colour-pattern which can be created. Also here the answer is no, since the same colour-pattern can be created with properly balanced and symmetric knots, which are just as easy to braid. Two examples are depicted by the grid-diagrams in Fig. 41.

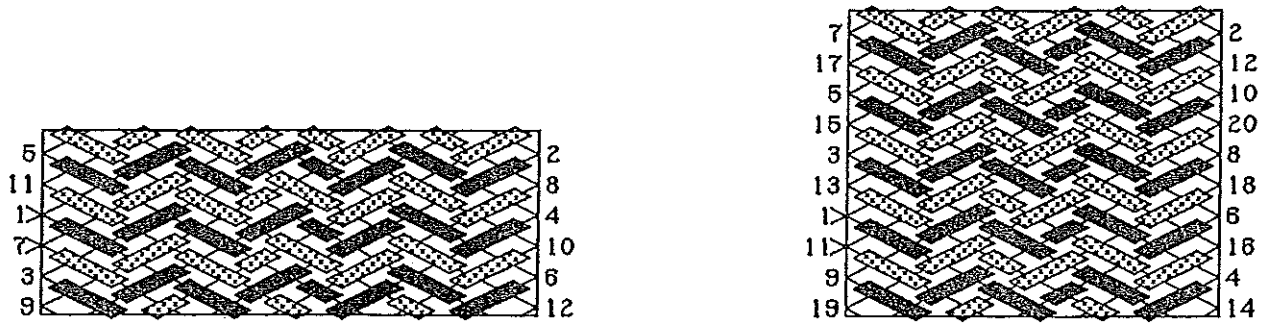


Fig. 41 — Examples of two string knots which give the same colour-pattern.

Although in the grid-diagrams of Figs. 40 and 41 the Standing-ends of the two strings are placed in adjacent bight-points, in practice we should space them at regular intervals around the bight-boundary. The reason why we have not done this is not only to relate the grid-diagrams in Fig. 40 to those in Fig. 44, but also to depict these knots in the

form they are shown in the literature. The spacing of the Standing-ends does not affect their braiding-algorithms.

Before discussing the single string varieties of these purported Gaucho knots, we shall first list the braiding-algorithms for the knots depicted in Figs. 40 and 41.

The leftmost knot in Fig. 40; 8-parts/6-bights:

half-cycle:

1. L ⟶ R: Free Run.
2. R ⟶ L: o.
3. L ⟶ R: u.
4. R ⟶ L: $u - o$.
5. L ⟶ R: $o - u$.
6. R ⟶ L: $o - u - o$.

---

7. L ⟶ R: $u - o - u - o$.
8. R ⟶ L: $u - o - u - 2o$.
9. L ⟶ R: $u - o - 2u - o$.
10. R ⟶ L: $u - o - 2u - 2o$.
11. L ⟶ R: $u - 2o - 2u - o$.
12. R ⟶ L: $u - 2o - 2u - 2o$.

Note that we immediately observe from the last half-cycle algorithm that this knot does not possess a Gaucho-coding, since for such a coding a last half-cycle algorithm must read: $2u - 2o - 2u - 2o$.

The central knot in Fig. 40; 12-parts/10-bights:

half-cycle:

1. L ⟶ R: Free Run.
2. R ⟶ L: o.
3. L ⟶ R: u.
4. R ⟶ L: $u - o$.
5. L ⟶ R: $o - u$.
6. R ⟶ L: $o - u - o$.
7. L ⟶ R: $u - o - u$.
8. R ⟶ L: $u - o - u - o$.
9. L ⟶ R: $o - u - o - u$.
10. R ⟶ L: $o - u - o - u - o$.

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11. L ⟶ R: $u - o - u - o - u - o$.
12. R ⟶ L: $u - o - u - o - u - 2o$.
13. L ⟶ R: $u - o - u - o - 2u - o$.
14. R ⟶ L: $u - o - u - o - 2u - 2o$.
15. L ⟶ R: $u - o - u - 2o - 2u - o$.
16. R ⟶ L: $u - o - u - 2o - 2u - 2o$.
17. L ⟶ R: $u - o - 2u - 2o - 2u - o$.
18. R ⟶ L: $u - o - 2u - 2o - 2u - 2o$.
19. L ⟶ R: $u - 2o - 2u - 2o - 2u - o$.
20. R ⟶ L: $u - 2o - 2u - 2o - 2u - 2o$.

Note that we immediately observe from the last half-cycle algorithm that this knot does not possess a Gaucho-coding, since for such a coding a last half-cycle algorithm must read: $2u - 2o - 2u - 2o - 2u - 2o$.

The rightmost knot in Fig. 40; 16-parts/14-bights:

half-cycle:

1. L ⟶ R: Free Run.
2. R ⟶ L: o.
3. L ⟶ R: u.
4. R ⟶ L: $u - o$.
5. L ⟶ R: $o - u$.
6. R ⟶ L: $o - u - o$.
7. L ⟶ R: $u - o - u$.
8. R ⟶ L: $u - o - u - o$.
9. L ⟶ R: $o - u - o - u$.
10. R ⟶ L: $o - u - o - u - o$.
11. L ⟶ R: $u - o - u - o - u$.
12. R ⟶ L: $u - o - u - o - u - o$.
13. L ⟶ R: $o - u - o - u - o - u$.
14. R ⟶ L: $o - u - o - u - o - u - o$.

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15. L ⟶ R: $u - o - u - o - u - o - u - o$.
16. R ⟶ L: $u - o - u - o - u - o - u - 2o$.
17. L ⟶ R: $u - o - u - o - u - o - 2u - o$.
18. R ⟶ L: $u - o - u - o - u - o - 2u - 2o$.
19. L ⟶ R: $u - o - u - o - u - 2o - 2u - o$.
20. R ⟶ L: $u - o - u - o - u - 2o - 2u - 2o$.
21. L ⟶ R: $u - o - u - o - 2u - 2o - 2u - o$.
22. R ⟶ L: $u - o - u - o - 2u - 2o - 2u - 2o$.
23. L ⟶ R: $u - o - u - 2o - 2u - 2o - 2u - o$.
24. R ⟶ L: $u - o - u - 2o - 2u - 2o - 2u - 2o$.
25. L ⟶ R: $u - o - 2u - 2o - 2u - 2o - 2u - o$.
26. R ⟶ L: $u - o - 2u - 2o - 2u - 2o - 2u - 2o$.
27. L ⟶ R: $u - 2o - 2u - 2o - 2u - 2o - 2u - o$.
28. R ⟶ L: $u - 2o - 2u - 2o - 2u - 2o - 2u - 2o$.

Note that we immediately observe from the last half-cycle algorithm that this knot does not possess a Gaucho-coding, since for such a coding a last half-cycle algorithm must read: $2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o$.

The left-hand knot in Fig. 41; 16-parts/6-bights:

half-cycle:

1. L ⟶ R: Free Run.
2. R ⟶ L: $2u$.
3. L ⟶ R: $2u$.
4. R ⟶ L: $4u - o$.
5. L ⟶ R: $2u - o - u - o$.
6. R ⟶ L: $u - o - 2u - o - u - o$.

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7. L ⟶ R: $u - o - u - o - u - o - u - o$.
8. R ⟶ L: $u - o - 2u - o - u - o - 2u - o$.
9. L ⟶ R: $u - o - 2u - o - u - o - 2u - o$.
10. R ⟶ L: $2u - o - 2u - 2o - u - o - 2u - 2o$.
11. L ⟶ R: $2u - o - 2u - o - 2u - o - 2u - 2o$.
12. R ⟶ L: $2u - 2o - 2u - 2o - u - 2o - 2u - 2o$.

The right-hand knot in Fig. 41; 12-parts/10-bights:

half-cycle:

1. L ⟶ R: Free Run.
2. R ⟶ L: o.
3. L ⟶ R: o.
4. R ⟶ L: $u - o$.
5. L ⟶ R: $u - o$.
6. R ⟶ L: $o - u - o$.
7. L ⟶ R: $2u - o$.
8. R ⟶ L: $2o - u - o$.
9. L ⟶ R: $o - 2u - o$.
10. R ⟶ L: $u - 2o - u - o$.

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11. L ⟶ R: $u - o - u - o - u - o$.
12. R ⟶ L: $u - o - u - o - u - 2o$.
13. L ⟶ R: $u - o - u - o - u - 2o$.
14. R ⟶ L: $u - o - u - o - 2u - 2o$.
15. L ⟶ R: $u - o - u - o - 2u - 2o$.
16. R ⟶ L: $u - o - 2u - o - 2u - 2o$.
17. L ⟶ R: $u - o - u - 2o - 2u - 2o$.
18. R ⟶ L: $u - 2o - 2u - o - 2u - 2o$.
19. L ⟶ R: $u - 2o - u - 2o - 2u - 2o$.
20. R ⟶ L: $2u - 2o - 2u - o - 2u - 2o$.

We shall now look at the one-string form of these two-string mongrel knots depicted in Fig. 40. The one-string forms are imitation knots of these two-string Regular Cylindrical Braids, since they are **not** Regular Cylindrical Braids, but only imitate them.

In order to imitate a multi-string Regular Cylindrical Braid by a one-string braid-form, the coding of the imitating string-run will always have to differ from the coding of the multi-string Regular Cylindrical Braid, and hence their respective braiding algorithms can never be identical.

This important universal rule is frequently violated by many braiders, with the result of a clearly noticeable irregularity near the bight-point where braiding commences.

First we shall show how the string-runs of the knots in Fig. 40 will have to be changed in order to obtain the string-runs of their one-string imitations.

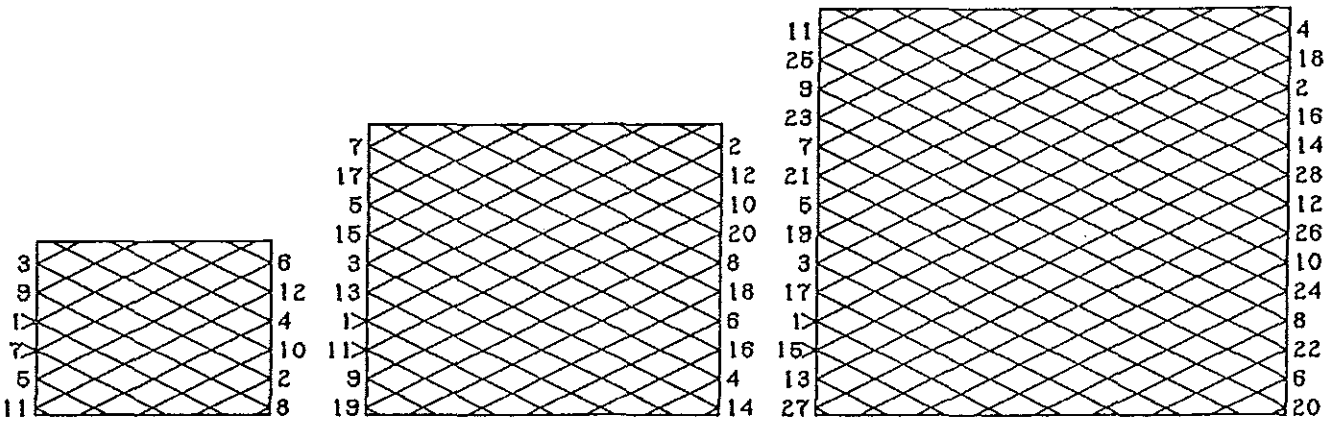


Fig. 42 — The string-runs of the knots in Fig. 40.

In Fig. 42 are depicted the string-runs of the knots in Fig. 40, and in Fig. 43 the string-runs of their one-string imitations.

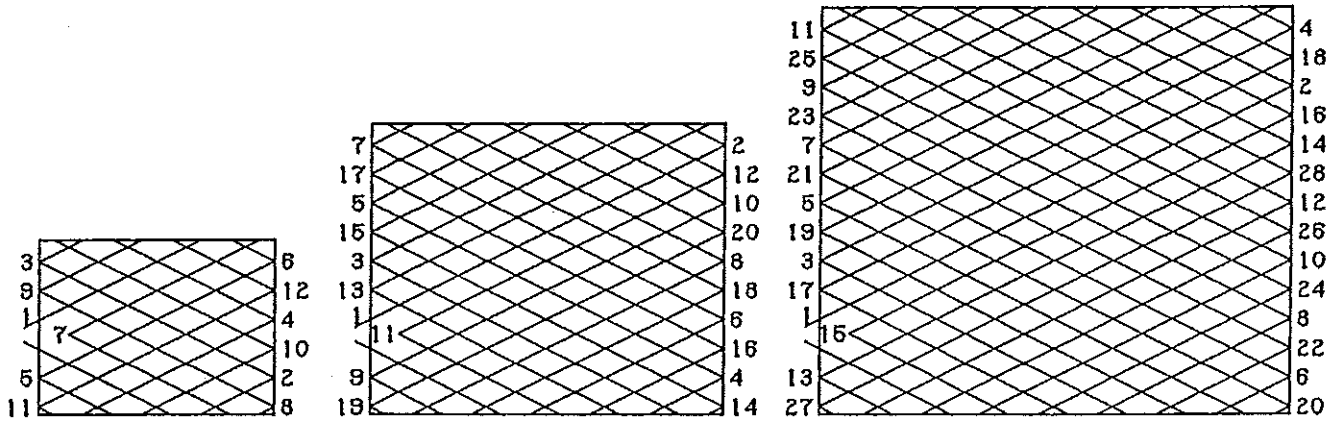


Fig. 43 — The string-runs of the one-string knots which imitate the knots in Fig. 40.

The grid-diagrams in Fig. 44 show the correct coding superimposed on the string-runs in Fig. 43.

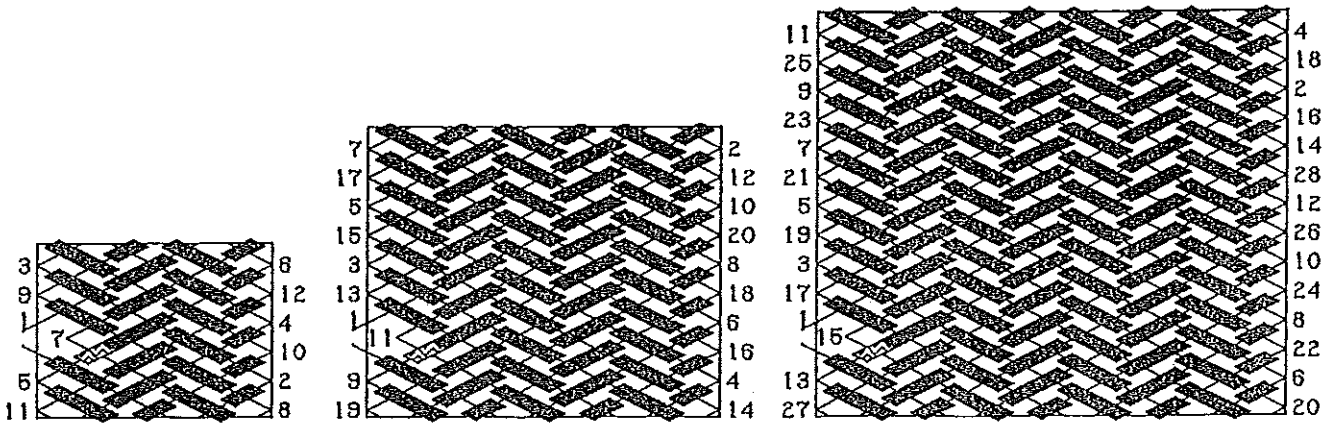


Fig. 44 — The grid-diagrams of the one-string knots which imitate the knots in Fig. 40.

It is very important to note the coding change at the last crossing of the respective half-cycles 6, 10, and 14. This change in coding is necessary in order to work away the string-ends properly, and hence to form the associated bights correctly. In Fig. 45 we have shown this process in detail.

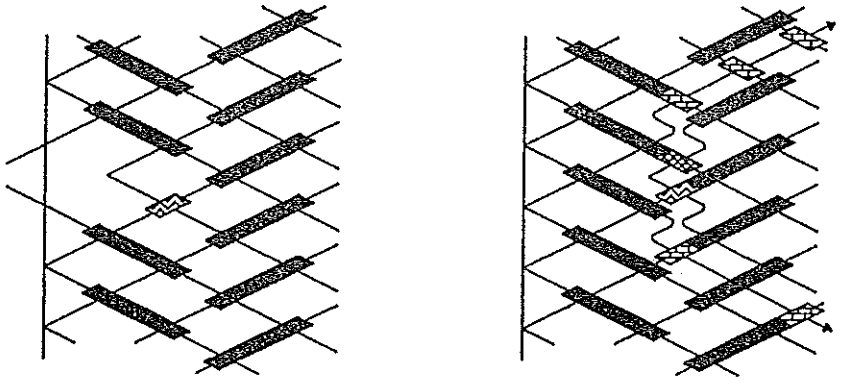


Fig. 45 — Working the string-ends away.

Below we have tabulated the braiding-algorithms for these one-string imitation knots. Observe that in the table of the 8-parts/6-bights one-string imitation knot the half-cycle algorithms for the half-cycles 6 and 7 are different to the corresponding half-cycles of the two-string knot. Similarly are the half-cycle algorithms for the half-cycles 10 and 11 of the 12-parts/10-bights one-string imitation knot different to the corresponding half-cycles of the two-string version, and so are the half-cycle algorithms for the half-cycles 14 and 15 of the 16-parts/14-bights one-string imitation knot different to the corresponding half-cycles of its two-string version.

The leftmost knot in Fig. 44; 8-parts/6-bights:

half-cycle:

1. L ⟶ R: Free Run.
2. R ⟶ L: o.
3. L ⟶ R: u.
4. R ⟶ L: $u - o$.
5. L ⟶ R: $o - u$.
6. R ⟶ L: $o - 2u$.
7. L ⟶ R: $o - u - o$.
8. R ⟶ L: $u - o - u - 2o$.
9. L ⟶ R: $u - o - 2u - o$.
10. R ⟶ L: $u - o - 2u - 2o$.
11. L ⟶ R: $u - 2o - 2u - o$.
12. R ⟶ L: $u - 2o - 2u - 2o$.

Note that we immediately observe from the last half-cycle algorithm that this knot does not possess a Gaucho-coding, since for such a coding a last half-cycle algorithm must read: $2u - 2o - 2u - 2o$.

The central knot in Fig. 44; 12-parts/10-bights:

half-cycle:

1. L ⟶ R: Free Run.
2. R ⟶ L: o.
3. L ⟶ R: u.
4. R ⟶ L: $u - o$.
5. L ⟶ R: $o - u$.
6. R ⟶ L: $o - u - o$.
7. L ⟶ R: $u - o - u$.
8. R ⟶ L: $u - o - u - o$.
9. L ⟶ R: $o - u - o - u$.
10. R ⟶ L: $o - u - o - 2u$.
11. L ⟶ R: $o - u - o - u - o$.
12. R ⟶ L: $u - o - u - o - u - 2o$.
13. L ⟶ R: $u - o - u - o - 2u - o$.
14. R ⟶ L: $u - o - u - o - 2u - 2o$.
15. L ⟶ R: $u - o - u - 2o - 2u - o$.
16. R ⟶ L: $u - o - u - 2o - 2u - 2o$.
17. L ⟶ R: $u - o - 2u - 2o - 2u - o$.
18. R ⟶ L: $u - o - 2u - 2o - 2u - 2o$.
19. L ⟶ R: $u - 2o - 2u - 2o - 2u - o$.
20. R ⟶ L: $u - 2o - 2u - 2o - 2u - 2o$.

Note that we immediately observe from the last half-cycle algorithm that this knot does not possess a Gaucho-coding, since for such a coding a last half-cycle algorithm must read: $2u - 2o - 2u - 2o - 2u - 2o$.

The rightmost knot in Fig. 44; 16-parts/14-bights:

half-cycle:

1. L ⟶ R: Free Run.
2. R ⟶ L: o.
3. L ⟶ R: u.
4. R ⟶ L: $u - o$.
5. L ⟶ R: $o - u$.
6. R ⟶ L: $o - u - o$.
7. L ⟶ R: $u - o - u$.
8. R ⟶ L: $u - o - u - o$.
9. L ⟶ R: $o - u - o - u$.
10. R ⟶ L: $o - u - o - u - o$.
11. L ⟶ R: $u - o - u - o - u$.
12. R ⟶ L: $u - o - u - o - u - o$.
13. L ⟶ R: $o - u - o - u - o - u$.
14. R ⟶ L: $o - u - o - u - o - 2u$.
15. L ⟶ R: $o - u - o - u - o - u - o$.
16. R ⟶ L: $u - o - u - o - u - o - u - 2o$.
17. L ⟶ R: $u - o - u - o - u - o - 2u - o$.
18. R ⟶ L: $u - o - u - o - u - o - 2u - 2o$.
19. L ⟶ R: $u - o - u - o - u - 2o - 2u - o$.
20. R ⟶ L: $u - o - u - o - u - 2o - 2u - 2o$.
21. L ⟶ R: $u - o - u - o - 2u - 2o - 2u - o$.
22. R ⟶ L: $u - o - u - o - 2u - 2o - 2u - 2o$.
23. L ⟶ R: $u - o - u - 2o - 2u - 2o - 2u - o$.
24. R ⟶ L: $u - o - u - 2o - 2u - 2o - 2u - 2o$.
25. L ⟶ R: $u - o - 2u - 2o - 2u - 2o - 2u - o$.
26. R ⟶ L: $u - o - 2u - 2o - 2u - 2o - 2u - 2o$.
27. L ⟶ R: $u - 2o - 2u - 2o - 2u - 2o - 2u - o$.
28. R ⟶ L: $u - 2o - 2u - 2o - 2u - 2o - 2u - 2o$.

Note that we immediately observe from the last half-cycle algorithm that this knot does not possess a Gaucho-coding, since for such a coding a last half-cycle algorithm must read: $2u - 2o - 2u - 2o - 2u - 2o - 2u - 2o$.

It is important to note that these one-string imitation knots are **not** based on Casa knots; this is also clearly evident from the algorithms for the respective half-cycles 6, 10, and 14.

Recently, Ron Edwards described in *The Australian Whipmaker (Number 38, April 1995)* these one and two string mongrel knots, with a series of nice pictorial drawings, as Gaucho knots. If he only would have taken a little effort in learning the use of grid-diagrams, he would not only have saved a great deal of wasted time, associated with the production of the various pictorial drawings, but would also have avoided the errors which are invariably made in the half-cycle algorithms for the one-string imitation varieties. Furthermore it would have prevented leading the novice up the garden path.

The reader may wonder if there are any Gaucho knots (Regular knots with a Gaucho-coding) which are based on Casa knots. The answer is yes (see Pamphlet No. 6). In due course we shall discuss some aspects of this in this quarterly, but for the time being it should suffice to present a short list of the 2-pass Gaucho knots which are based on Casa knots (in this list we have left the 2-pass Spanish Ring knots out, which may be regarded as 2-pass Gaucho knots with the minimum number of parts).

Casa knot parts/bights:		2-pass Gaucho knot parts/bights:	Casa knot parts/bights:		2-pass Gaucho knot parts/bights:
5/1	⟶	9/2	5/4	⟶	9/7
5/6	⟶	9/11	5/9	⟶	9/16
5/11	⟶	9/20	5/14	⟶	9/25
5/16	⟶	9/29	5/19	⟶	9/34
5/21	⟶	9/38	5/24	⟶	9/43
⋮	⟶	⋮	⋮	⟶	⋮
7/1	⟶	13/2	7/6	⟶	13/11
7/8	⟶	13/15	7/13	⟶	13/24
7/15	⟶	13/28	7/20	⟶	13/37
7/22	⟶	13/41	7/27	⟶	13/50
7/29	⟶	13/54	7/34	⟶	13/63
⋮	⟶	⋮	⋮	⟶	⋮
9/1	⟶	17/2	9/8	⟶	17/15
9/10	⟶	17/19	9/17	⟶	17/32
9/19	⟶	17/36	9/26	⟶	17/49
9/28	⟶	17/53	9/35	⟶	17/66
9/37	⟶	17/70	9/44	⟶	17/83
⋮	⟶	⋮	⋮	⟶	⋮

Note that each of these Casa knots has an **odd** number of parts, hence an **even** number of crossing-points on a half-cycle. Logically, those Casa knots may be regarded as 1-pass Gaucho knots.

Let's now have a look at an example of **bush braid-forms**. We have already mentioned that bush braid-forms serve a real purpose. In bush braidwork it is the aim to produce a serviceable article by means of quick and simple processes. The standard of braidwork plays little if any role, all that matters is that the braided article must fulfil the demands associated with its intended use. The environmental situation demands that the braiding-processes used must be easy to remember and excecute, hence it will be obvious that many of such processes leave much to be desired when the aspects of *quality standard* are also important. Hence a bush-braider is not a braiding artisan, nor does generally such a person have any desire to become one.

The bush-braids we are going to have a look at, we shall call **Hansen's bush knots** in order to distinguish them from their proper and correct counterparts, which we shall call **Hansen knots**.

Hansen's bush knots are based on Casa knots which have an **odd** number of parts. These bush knots are supposed to imitate Gaucho knots. In Fig. 46 we have depicted two examples of such knots by their string-runs and grid-diagrams. The upper two diagrams are of a 15-parts/6-bights Hansen's bush knot, and the lower two diagrams are of a 21-parts/12-bights Hansen's bush knot.

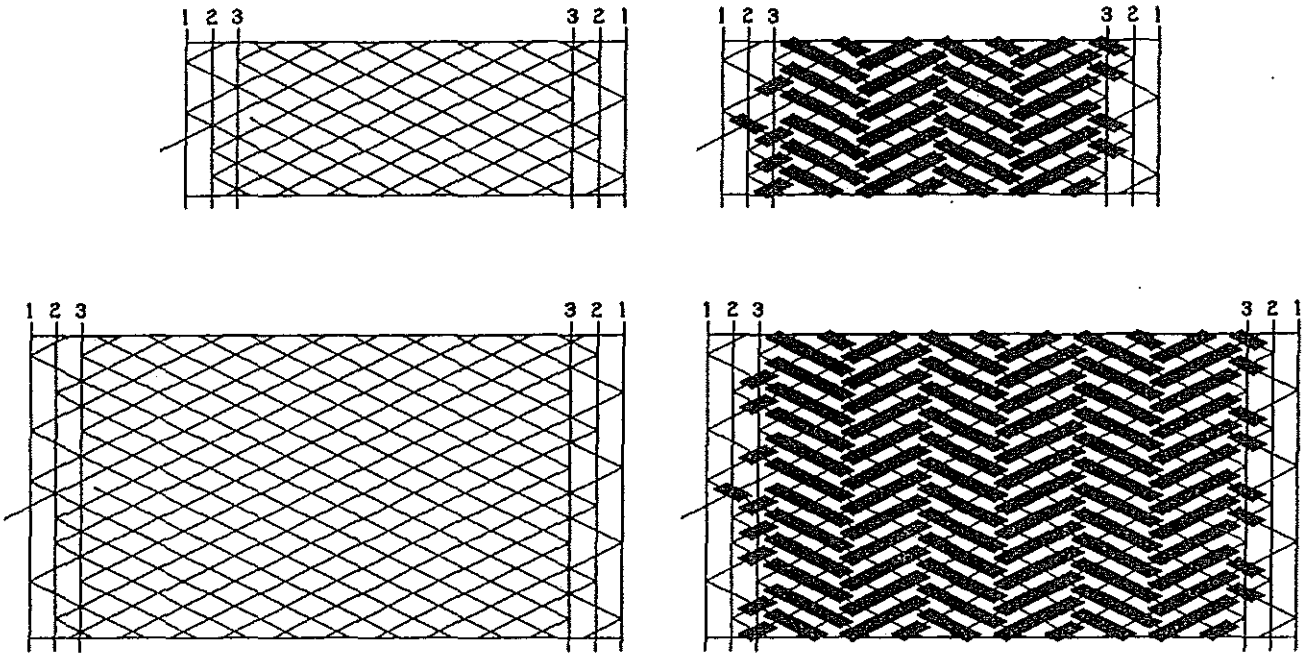


Fig. 46 — 15-parts/6-bights Hansen's bush knot,
21-parts/12-bights Hansen's bush knot.

The upper two diagrams in Fig. 47 depict respectively the string-run and grid-diagram of a 5-parts/2-bights Casa knot on which the 15-parts/6-bights Hansen's bush knot is based. The lower two diagrams in Fig. 47 depict the string-run and grid-diagram at the end of the first braiding-phase following the completion of the Casa knot.

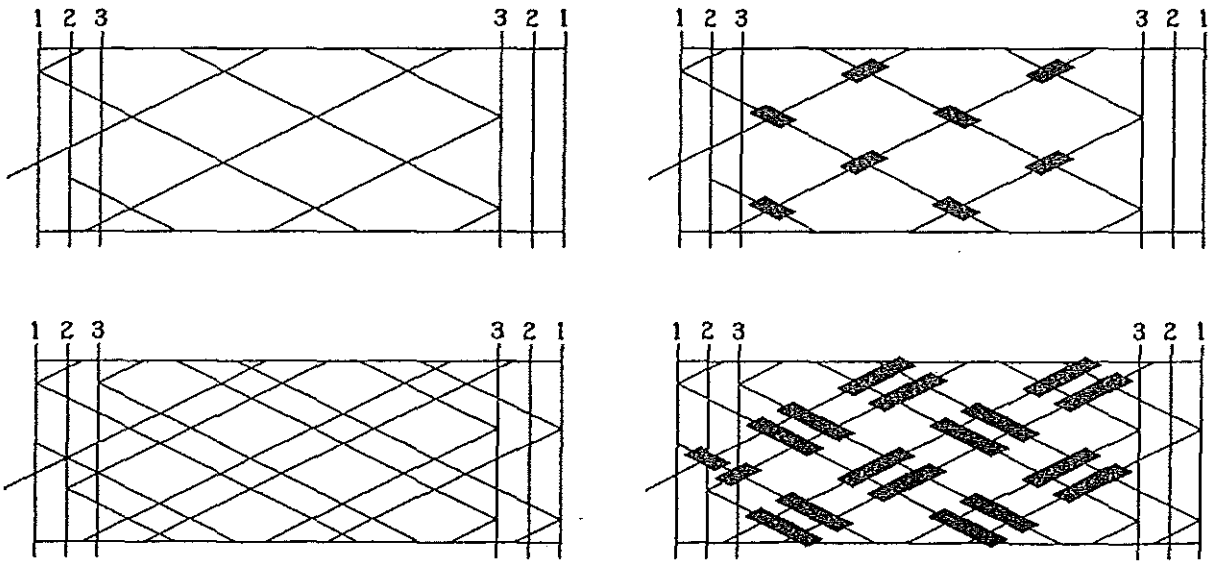


Fig. 47 — The 5-parts/2-bights Casa knot,
end of first braiding-phase following the Casa knot.

The upper two diagrams in Fig. 48 depict respectively the string-run and grid-diagram of a 7-parts/4-bights Casa knot on which the 21-parts/12-bights Hansen's bush knot is based. The lower two diagrams in Fig. 48 depict the string-run and grid-diagram at the end of the first braiding-phase following the completion of the Casa knot.

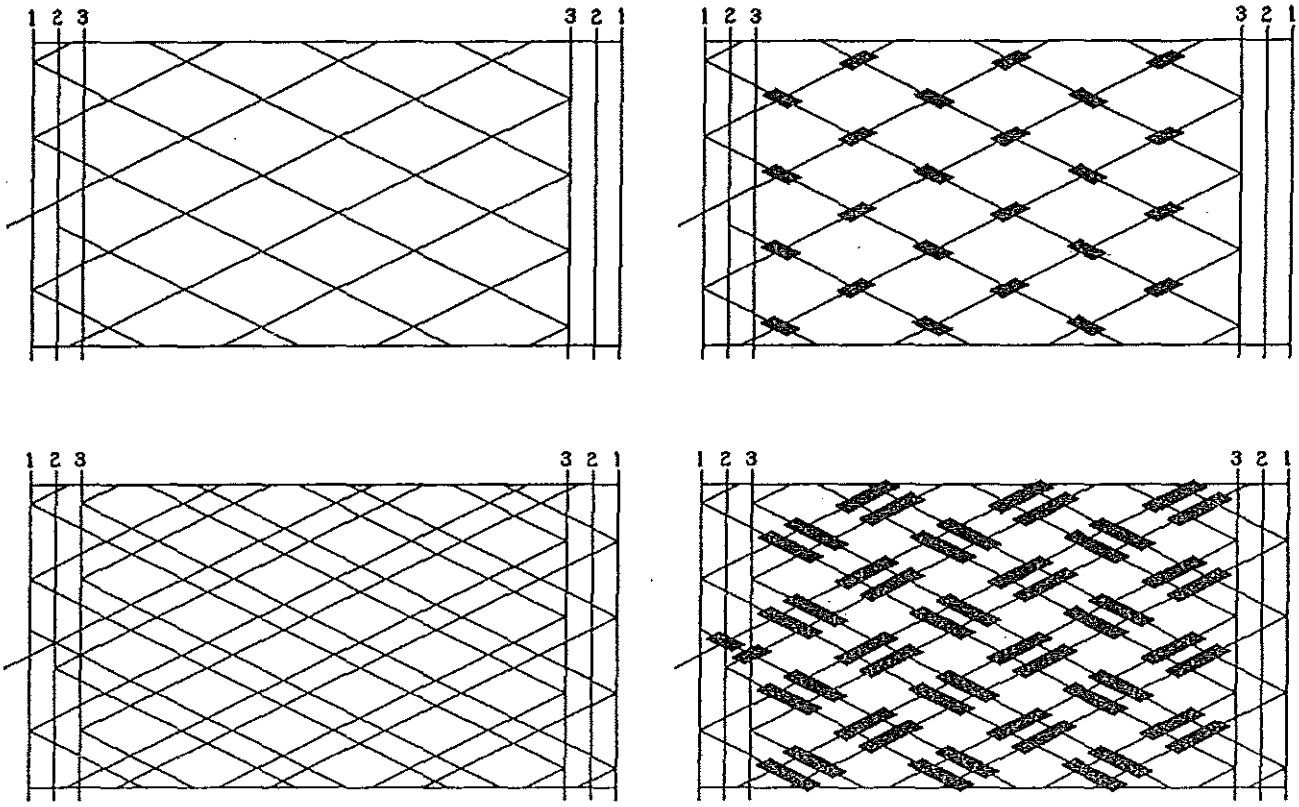


Fig. 48 — The 7-parts/4-bights Casa knot,
end of first braiding-phase following the Casa knot.

At the end of the first braiding-phase, following the completion of the Casa knots, we have produced the common **one-string imitation duplex Casa bush knots** with respectively 10-parts/4-bights and 14-parts/8-bights. Let's first have a look at these bush knots and compare them with the duplex Casa knots and the one-string imitation duplex Casa knots.

The duplex Casa knots with respectively 10-parts/4-bights and 14-parts/8-bights are depicted by their string-runs and grid-diagrams in Fig. 49. It are 2-string Regular Nested Cylindrical Braids (the bights along each of the two bight-edges (the left-hand one and the right-hand one) are regularly nested).

Again, in order to compare the grid-diagrams of Fig. 49 with those of Fig. 50, we have placed in each knot the Standing-end of each of the two strings on the same intersection row, although in practice we should place these Standing-ends at regular intervals along the bight-edge.

If we desire to braid these duplex Casa knots in their single-string imitation form, then we have to modify the codings of the Casa knots on which they are based. If we don't, then we end up with the one-string imitation duplex Casa bush knots. The (proper) one-string imitation duplex Casa knots with respectively 10-parts/4-bights and 14-parts/8-bights are shown in Fig. 50. Note that the base knot in each case is not a Casa knot, since for the one-string 10-parts/4-bights imitation duplex Casa knot the

final crossing on half-cycle 4 must now be an *under*-crossing instead of an over-crossing, and similarly, for the one-string 14-parts/8-bights imitation duplex Casa knot the final crossing on half-cycle 8 must now be an *under*-crossing instead of an over-crossing. It should now be obvious that the first crossing on half-cycle 7, respectively half-cycle 11, becomes an *over*-crossing instead of an under-crossing.

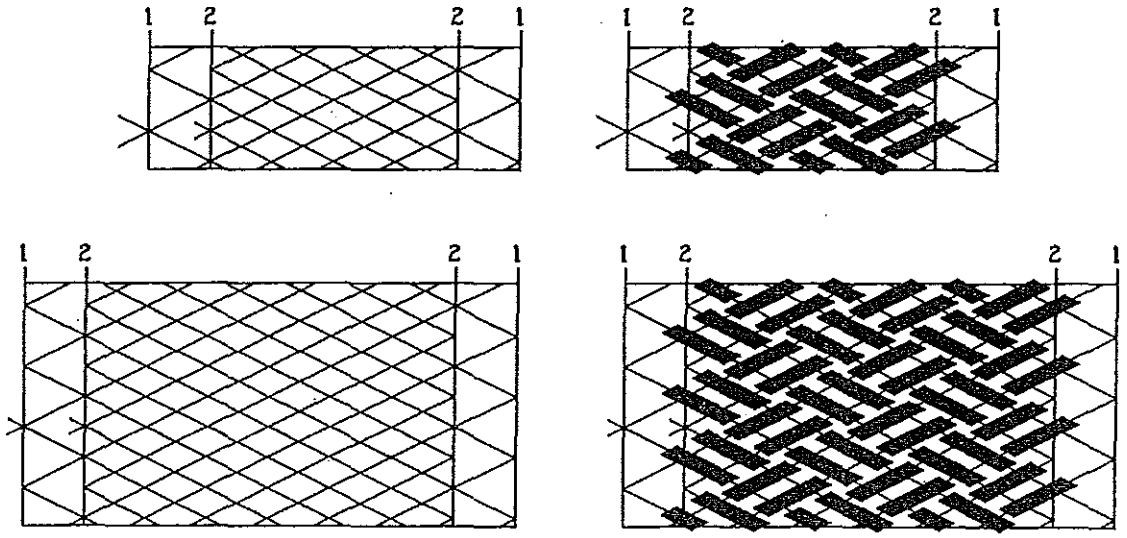


Fig. 49 — The 10-parts/4-bights duplex Casa knot, the 14-parts/8-bights duplex Casa knot.

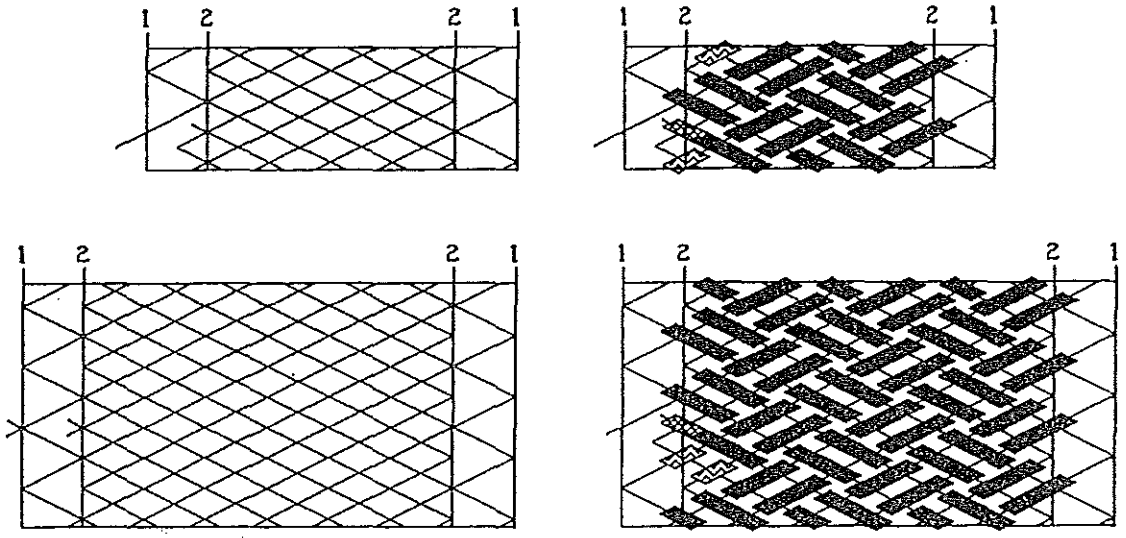


Fig. 50 — The 10-parts/4-bights one-string imitation duplex Casa knot, the 14-parts/8-bights one-string imitation duplex Casa knot.

Finally, the last crossing on the last half-cycle (half-cycle 8, respectively half-cycle 16) must be an *over*-crossing in order to ensure that the ends of the strings may be worked away properly, and hence the associated bights can be formed correctly. In Fig. 51 we have shown this finishing-process in detail.

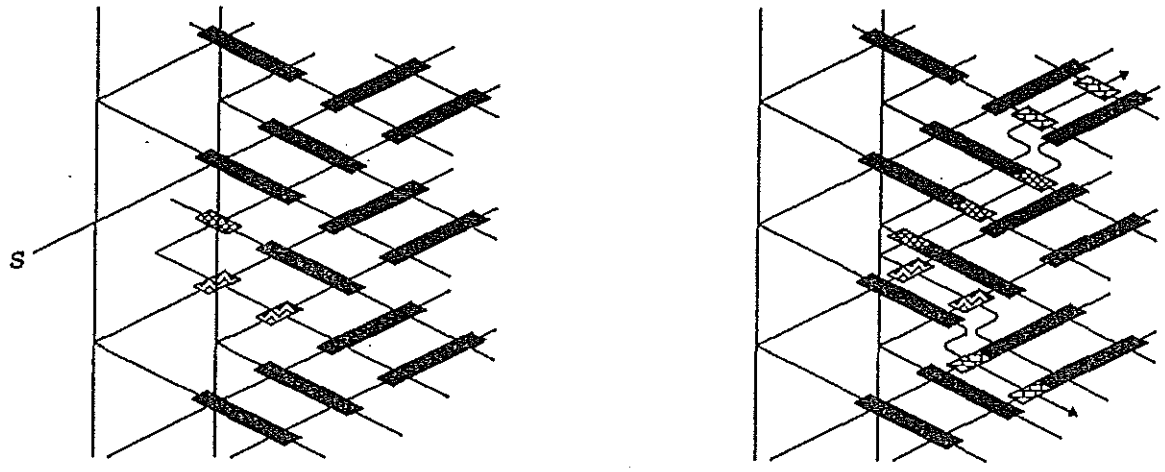


Fig. 51 — Working the string-ends away.

In contrast with Hansen's bush knots, the Hansen type 1 knots, to which we shall restrict our discussion in this article, are knots which require three strings in their construction; it are interbraids of three identical Casa knots with an odd number of parts. The 15-parts/6-bights Hansen type 1 knot is depicted in Fig. 52, and the 21-parts/12-bights Hansen type 1 knot is shown in Fig. 53. The bigger knot gives as usual a much better overview of the braid-form we are dealing with (a good overview is essential for enabling us to generalize a braid-form) .

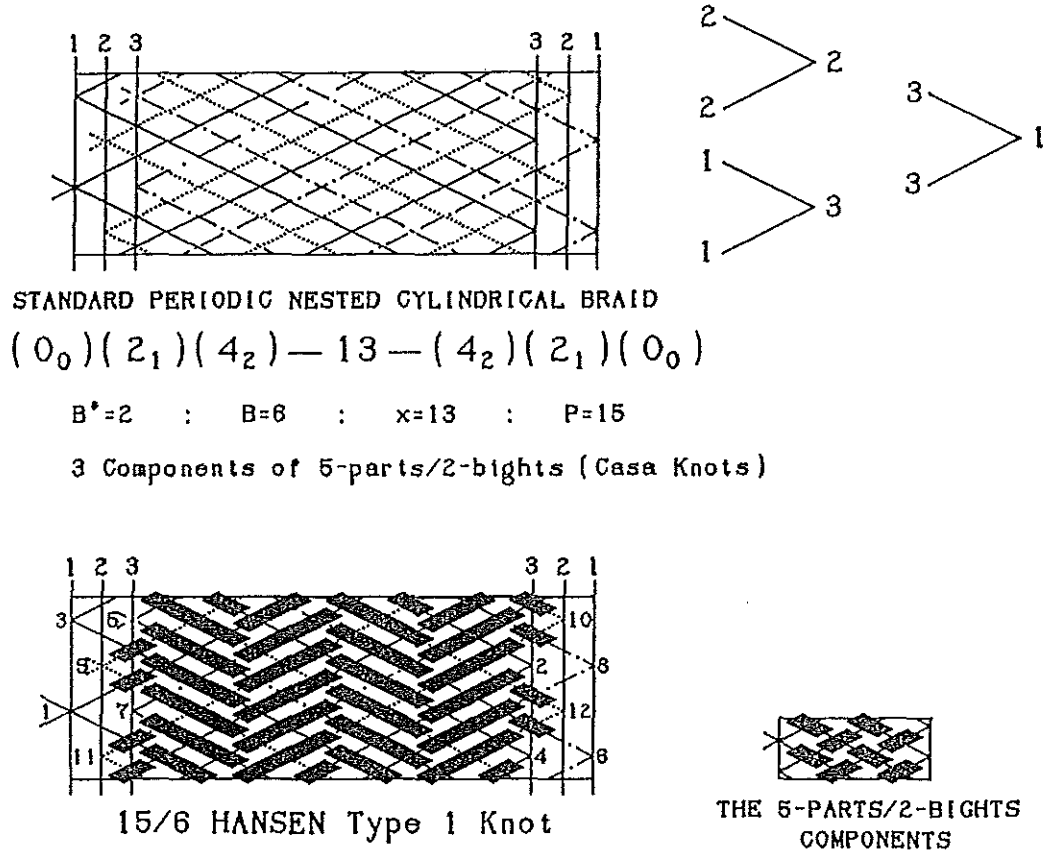


Fig. 52 — String-run and grid-diagram of the 15/6 Hansen type 1 knot.

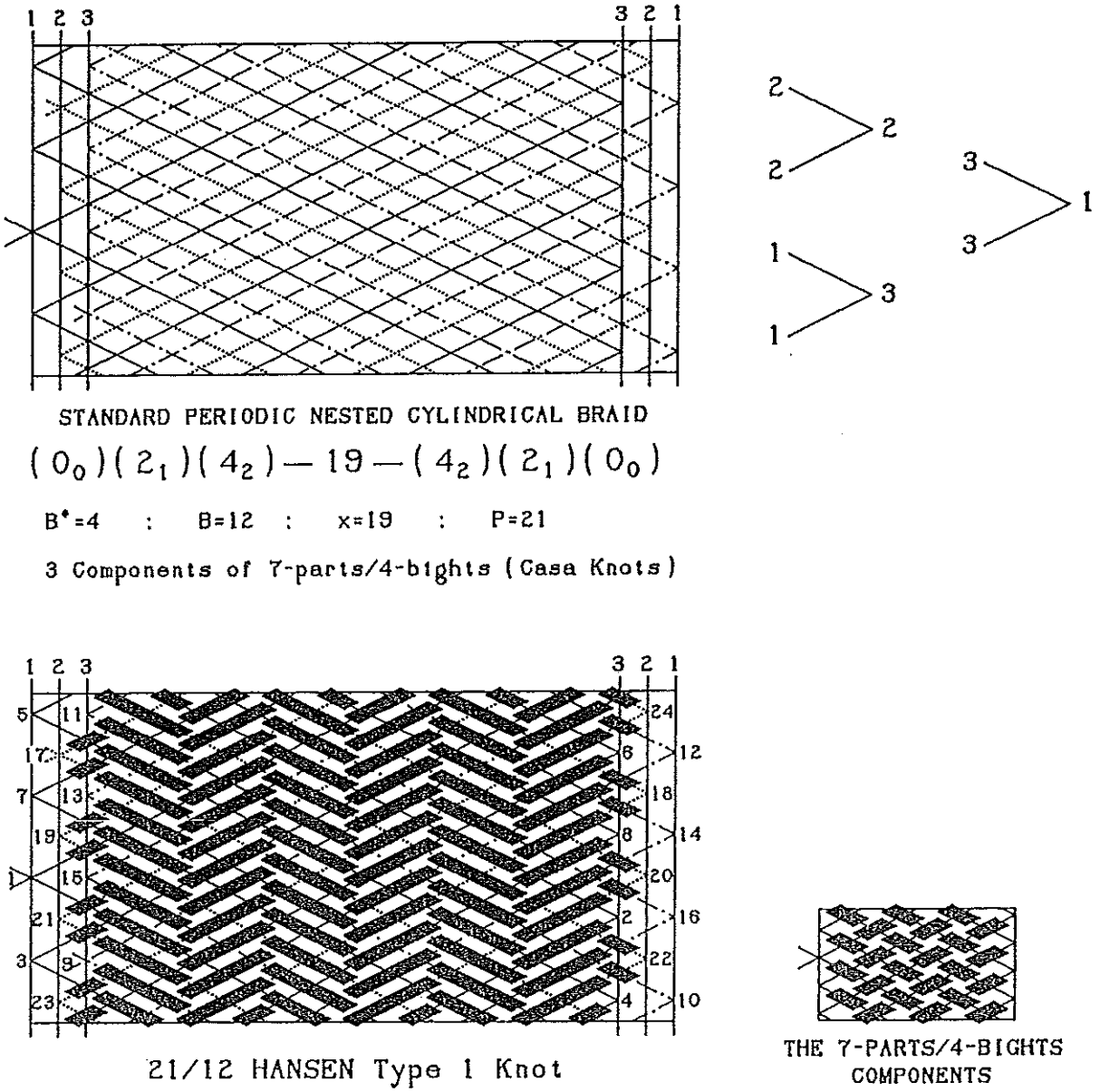


Fig. 53 — String-run and grid-diagram of the 21/12 Hansen type 1 knot.

The Hansen type 1 knots are **Standard Periodic Nested Cylindrical Braids.** Note that the bights along an edge are positioned in a periodic pattern, with three bights per period of which two are nested. Each of the two bight-edges (left-hand one, and right-hand one) has three bight-boundaries, which are numbered 1, 2, and 3. The string-run of one Casa knot runs between the left-hand bight-boundary 1 and the right-hand bight-boundary 3, the string-run of another Casa knot runs between the left-hand bight-boundary 2 and the right-hand bight-boundary 2, and the string-run of a further Casa knot runs between the left-hand bight-boundary 3 and the right-hand bight-boundary 1. This is schematically indicated at the right of the string-run diagram. Below the string-run diagrams we find some notations associated with the string-run; they will be explained in some future issue (they are also discussed in Pamphlet No. 8). For the time being it will suffice to know that B^* indicates the number of periods along a bight-edge, and that x indicates the column-distance between the innermost bight-boundaries

(hence here, between left-hand bight-boundary 3 and right-hand bight-boundary 3).

The half-cycle braiding algorithms for the 15/6 Hansen type 1 knot, depicted by its grid-diagram in Fig. 52, are as follows:

half-cycle:

1. $L_1 \longrightarrow R_3$: Free Run.
2. $R_3 \longrightarrow L_1$: $2o$.
3. $L_1 \longrightarrow R_3$: $2o$.
4. $R_3 \longrightarrow L_1$: $u - o - u - o$.

---

5. $L_3 \longrightarrow R_1$: $u - o - u - o$.
6. $R_1 \longrightarrow L_3$: $u - 2o - u - 2o$.
7. $L_3 \longrightarrow R_1$: $u - 2o - u - 2o$.
8. $R_1 \longrightarrow L_3$: $2u - 2o - 2u - 2o$.

---

9. $L_2 \longrightarrow R_2$: $o - 2u - 2o - 2u - 2o - u$.
10. $R_2 \longrightarrow L_2$: $o - 2u - 3o - 2u - 3o - u$.
11. $L_2 \longrightarrow R_2$: $o - 2u - 3o - 2u - 3o - u$.
12. $R_2 \longrightarrow L_2$: $o - 3u - 3o - 3u - 3o - u$.

The half-cycle braiding algorithms for the 21/12 Hansen type 1 knot, depicted by its grid-diagram in Fig. 53, are as follows:

half-cycle:

1. $L_1 \longrightarrow R_3$: Free Run.
2. $R_3 \longrightarrow L_1$: o.
3. $L_1 \longrightarrow R_3$: o.
4. $R_3 \longrightarrow L_1$: $u - o - u$.
5. $L_1 \longrightarrow R_3$: $u - o - u$.
6. $R_3 \longrightarrow L_1$: $u - 2o - u - o$.
7. $L_1 \longrightarrow R_3$: $u - 2o - u - o$.
8. $R_3 \longrightarrow L_1$: $u - o - u - o - u - o$.

---

9. $L_3 \longrightarrow R_1$: $u - o - u - o - u - o$.
10. $R_1 \longrightarrow L_3$: $u - o - u - 2o - u - o$.
11. $L_3 \longrightarrow R_1$: $u - o - u - 2o - u - o$.
12. $R_1 \longrightarrow L_3$: $2u - o - u - 2o - 2u - o$.
13. $L_3 \longrightarrow R_1$: $2u - o - u - 2o - 2u - o$.
14. $R_1 \longrightarrow L_3$: $2u - 2o - u - 2o - 2u - 2o$.
15. $L_3 \longrightarrow R_1$: $2u - 2o - u - 2o - 2u - 2o$.
16. $R_1 \longrightarrow L_3$: $2u - 2o - 2u - 2o - 2u - 2o$.

---

17. $L_2 \longrightarrow R_2$: $o - 2u - 2o - 2u - 2o - 2u - 2o - u$.
18. $R_2 \longrightarrow L_2$: $o - 2u - 2o - 2u - 3o - 2u - 2o - u$.
19. $L_2 \longrightarrow R_2$: $o - 2u - 2o - 2u - 3o - 2u - 2o - u$.
20. $R_2 \longrightarrow L_2$: $o - 3u - 2o - 2u - 3o - 3u - 2o - u$.
21. $L_2 \longrightarrow R_2$: $o - 3u - 2o - 2u - 3o - 3u - 2o - u$.
22. $R_2 \longrightarrow L_2$: $o - 3u - 3o - 2u - 3o - 3u - 3o - u$.
23. $L_2 \longrightarrow R_2$: $o - 3u - 3o - 2u - 3o - 3u - 3o - u$.
24. $R_2 \longrightarrow L_2$: $o - 3u - 3o - 3u - 3o - 3u - 3o - u$.

With both these knots we first braided the component between the bight-boundaries $1 \longleftrightarrow 3$, then we braided the component between the bight-boundaries $3 \longleftrightarrow 1$, and

finally we braided the component between the bight-boundaries 2 ⟷ 2. We can of course follow any sequence in which we braid the components, the end-result will always be the same. Since the three components in a particular knot are all identical, the half-cycle algorithms for the first component will always be the same, irrespective in which order the components are braided. However, the half-cycle algorithms for the second and third component depend on the order in which the components are braided.

The grid-diagrams of the 1-string imitation forms of the above two knots are presented in Fig. 54.

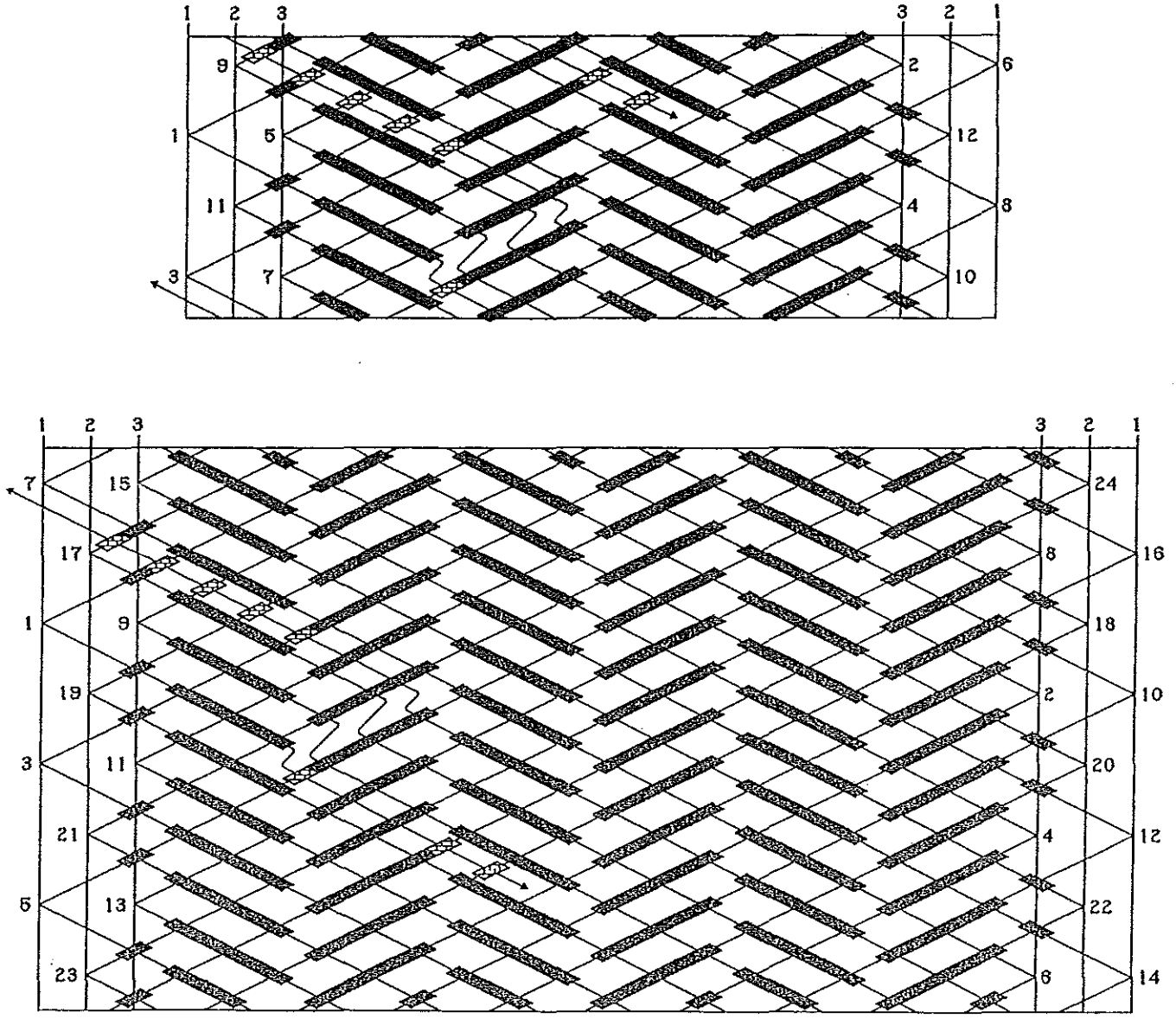


Fig. 54 — The 15-parts/6-bights imitation Hansen type 1 knot, the 21-parts/12-bights imitation Hansen type 1 knot.

In the following half-cycle algorithms for these knots depicted by their grid-diagrams in Fig. 54, it should again be noted that the back-braiding of the string-ends are the last operations in the braiding-process.

The half-cycle algorithms for the 15-parts/6-bights 1-string imitation Hansen type 1 knot in accordance with its grid-diagram in Fig. 54 are as follows:

half-cycle:

1. $L_1 \longrightarrow R_3$: Free Run.
2. $R_3 \longrightarrow L_1$: $2o$.
3. $L_1 \longrightarrow R_3$: $2o$.
4. $R_3 \longrightarrow L_3$: $u - o - u - o$.
5. $L_3 \longrightarrow R_1$: $u - o - u - o$.
6. $R_1 \longrightarrow L_3$: $u - 2o - u - 2o$.
7. $L_3 \longrightarrow R_1$: $u - 2o - u - 2o$.
8. $R_1 \longrightarrow L_2$: $2u - 2o - 3u - 2o - u$.
9. $L_2 \longrightarrow R_2$: $o - u - 3o - 2u - 2o - u$.
10. $R_2 \longrightarrow L_2$: $o - 2u - 3o - 2u - 3o - u$.
11. $L_2 \longrightarrow R_2$: $3u - 3o - 2u - 3o - u$.
12. $R_2 \longrightarrow L$: $o - 3u - 3o - 8u$.

back-braid the Standing-end:

$L_1 \longrightarrow R$: $u - 3o - 11u$.

The half-cycle algorithms for the 21-parts/12-bights 1-string imitation Hansen type 1 knot in accordance with its grid-diagram in Fig. 54 are as follows:

half-cycle:

1. $L_1 \longrightarrow R_3$: Free Run.
2. $R_3 \longrightarrow L_1$: o.
3. $L_1 \longrightarrow R_3$: o.
4. $R_3 \longrightarrow L_1$: $u - o - u$.
5. $L_1 \longrightarrow R_3$: $u - o - u$.
6. $R_3 \longrightarrow L_1$: $u - 2o - u - o$.
7. $L_1 \longrightarrow R_3$: $u - 2o - u - o$.
8. $R_3 \longrightarrow L_3$: $u - o - u - o - u - o$.
9. $L_3 \longrightarrow R_1$: $u - o - u - o - u - o$.
10. $R_1 \longrightarrow L_3$: $u - o - u - 2o - u - o$.
11. $L_3 \longrightarrow R_1$: $u - o - u - 2o - u - o$.
12. $R_1 \longrightarrow L_3$: $2u - o - u - 2o - 2u - o$.
13. $L_3 \longrightarrow R_1$: $2u - o - u - 2o - 2u - o$.
14. $R_1 \longrightarrow L_3$: $2u - 2o - u - 2o - 2u - 2o$.
15. $L_3 \longrightarrow R_1$: $2u - 2o - u - 2o - 2u - 2o$.
16. $R_1 \longrightarrow L_2$: $2u - 2o - 2u - 2o - 3u - 2o - u$.
17. $L_2 \longrightarrow R_2$: $o - 2u - 2o - 2u - 2o - 2u - 2o - u$.
18. $R_2 \longrightarrow L_2$: $o - 2u - 2o - 2u - 3o - 2u - 2o - u$.
19. $L_2 \longrightarrow R_2$: $3u - 2o - 2u - 3o - 2u - 2o - u$.
20. $R_2 \longrightarrow L_2$: $o - 3u - 2o - 2u - 3o - 3u - 2o - u$.
21. $L_2 \longrightarrow R_2$: $o - 2u - 3o - 2u - 3o - 3u - 2o - u$.
22. $R_2 \longrightarrow L_2$: $o - 3u - 3o - 2u - 3o - 3u - 3o - u$.
23. $L_2 \longrightarrow R_2$: $o - 3u - 3o - 2u - 3o - 3u - 3o - u$.
24. $R_2 \longrightarrow L$: $o - 3u - 3o - 3u - 3o - 8u$.

back-braid the Standing-end:

$L_1 \longrightarrow R$: $u - 3o - 17u$.

Another way of braiding the 1-string imitation 15-parts/6-bights and 21-parts/12-bights Hansen knots is shown by the grid-diagrams in Fig. 55. This method is the 1-string braiding adaptation of the braiding procedure for the Hansen knots whereby the first to be braided component runs between the bight-boundaries 1 $\longleftrightarrow$ 3, the second to be braided component runs between the bight-boundaries 2 $\longleftrightarrow$ 2, and the third to be braided component runs between the bight-boundaries 3 $\longleftrightarrow$ 1.

It should again be stessed that it is irrelevant whether we follow the procedures as set out in Fig. 54 with their associated half-cycle algorithms, or follow the procedures as set out in Fig. 55 with their associated half-cycle algorithms. The finished braids will have an identical appearance.

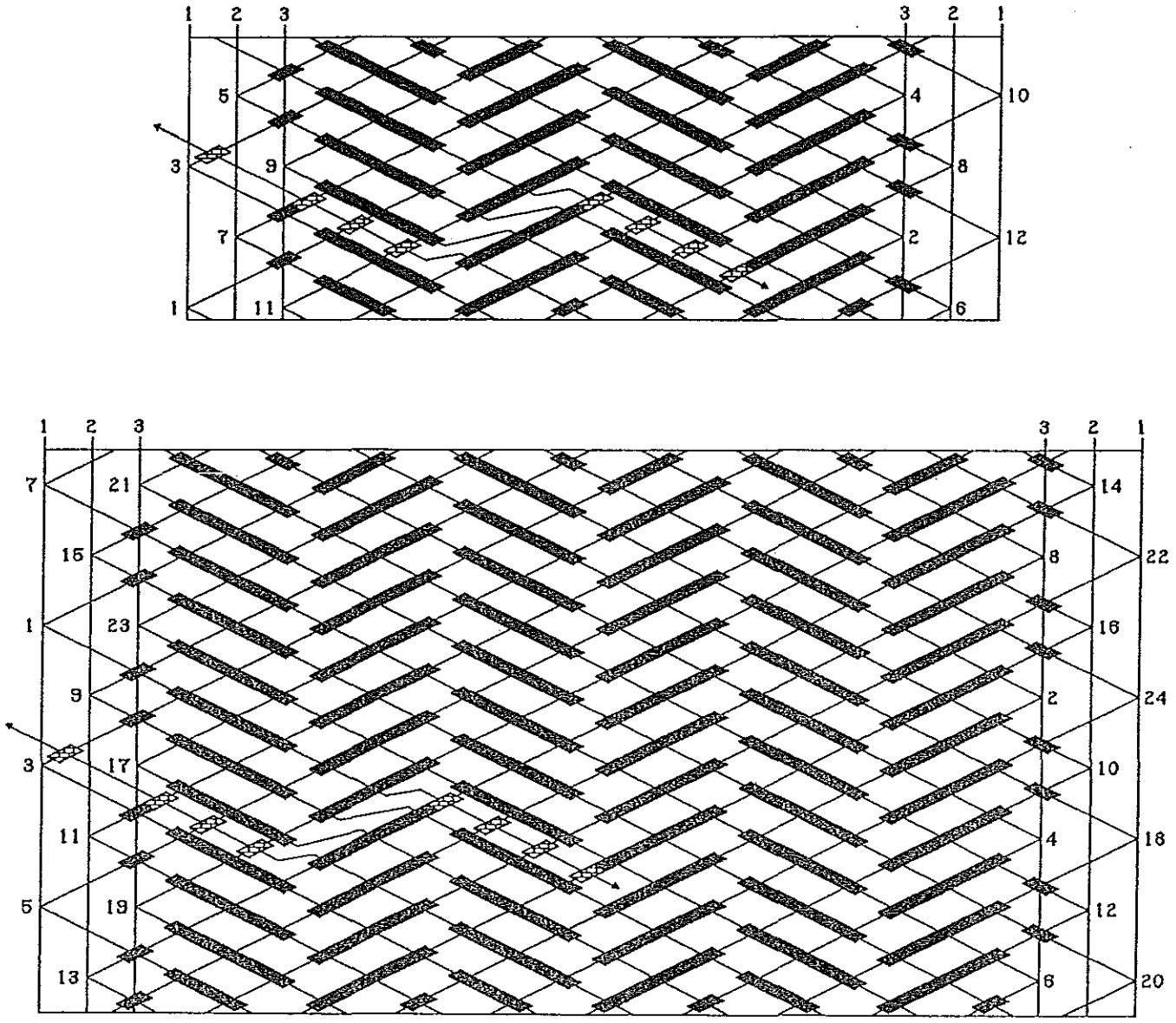


Fig. 55 — The 15-parts/6-bights imitation Hansen type 1 knot, the 21-parts/12-bights imitation Hansen type 1 knot.

The half-cycle braiding algorithms for the 1-string imitation 15/6 and 21/12 Hansen knots, associated with the grid-diagrams in Fig. 55 are presented on pg. 63.

The half-cycle algorithms for the 15-parts/6-bights 1-string imitation Hansen type 1 knot in accordance with its grid-diagram in Fig. 55 are as follows:

half-cycle:

1. $L_1 \longrightarrow R_3$: Free Run.
2. $R_3 \longrightarrow L_1$: $2o.$
3. $L_1 \longrightarrow R_3$: $2o.$
4. $R_3 \longrightarrow L_2$: $u - 2o - u.$
5. $L_2 \longrightarrow R_2$: $u - o - u - o.$
6. $R_2 \longrightarrow L_2$: $u - 2o - u - 2o - u.$
7. $L_2 \longrightarrow R_2$: $o - u - 2o - u - 2o.$
8. $R_2 \longrightarrow L_3$: $2u - 2o - 2u - 2o.$
9. $L_3 \longrightarrow R_1$: $u - 2o - 2u - 2o - u.$
10. $R_1 \longrightarrow L_3$: $o - 2u - 3o - 2u - 3o.$
11. $L_3 \longrightarrow R_1$: $3u - 2o - 2u - 3o - u.$
12. $R_1 \longrightarrow L$: $o - 3u - 3o - 7u.$

back-braid the Standing-end:

$L_3 \longrightarrow R$: $u - 3o - 10u.$

The half-cycle algorithms for the 21-parts/12-bights 1-string imitation Hansen type 1 knot in accordance with its grid-diagram in Fig. 55 are as follows:

half-cycle:

1. $L_1 \longrightarrow R_3$: Free Run.
2. $R_3 \longrightarrow L_1$: $o.$
3. $L_1 \longrightarrow R_3$: $o.$
4. $R_3 \longrightarrow L_1$: $u - o - u.$
5. $L_1 \longrightarrow R_3$: $u - o - u.$
6. $R_3 \longrightarrow L_1$: $u - 2o - u - o.$
7. $L_1 \longrightarrow R_3$: $u - 2o - u - o.$
8. $R_3 \longrightarrow L_2$: $u - o - u - o - u.$
9. $L_2 \longrightarrow R_2$: $u - o - u - o - u - o.$
10. $R_2 \longrightarrow L_2$: $u - o - u - 2o - u - o - u.$
11. $L_2 \longrightarrow R_2$: $o - u - o - u - 2o - u - o.$
12. $R_2 \longrightarrow L_2$: $2u - o - u - 2o - 2u - o - u.$
13. $L_2 \longrightarrow R_2$: $o - 2u - o - u - 2o - 2u - o.$
14. $R_2 \longrightarrow L_2$: $2u - 2o - u - 2o - 2u - 2o - u.$
15. $L_2 \longrightarrow R_2$: $o - 2u - 2o - u - 2o - 2u - 2o.$
16. $R_2 \longrightarrow L_3$: $2u - 2o - 2u - 2o - 2u - 2o.$
17. $L_3 \longrightarrow R_1$: $u - 2o - 2u - 2o - 2u - 2o - u.$
18. $R_1 \longrightarrow L_3$: $o - 2u - 2o - 2u - 3o - 2u - 2o.$
19. $L_3 \longrightarrow R_1$: $3u - o - 2u - 3o - 2u - 2o - u.$
20. $R_1 \longrightarrow L_3$: $o - 3u - 2o - 2u - 3o - 3u - 2o.$
21. $L_3 \longrightarrow R_1$: $3u - 2o - 2u - 3o - 3u - 2o - u.$
22. $R_1 \longrightarrow L_3$: $o - 3u - 3o - 2u - 3o - 3u - 3o.$
23. $L_3 \longrightarrow R_1$: $3u - 3o - 2u - 3o - 3u - 3o - u.$
24. $R_1 \longrightarrow L$: $o - 3u - 3o - 3u - 3o - 7u.$

back-braid the Standing-end:

$L_1 \longrightarrow R$: $u - 3o - 16u.$

The few examples here discussed should clearly show the importance of grid-diagrams in the design of braid-forms. Without a grid-diagram it is impossible to get a proper overview of a braid-form, and hence it is impossible to correct faulty braiding procedures.

It is in fact extremely rare to encounter a properly constructed single-string imitation of a multi-string braid-form. The single-string imitations invariably contain braiding irregularities, which create, due to their positioning, a more or less ugly finish and consequently degrade and spoil the braid-form. We have shown that such irregularities are totally unnecessary since they can readily be avoided by using correct designs instead. Such correct designs, although containing irregularities in their **string-runs**, hide these in such a way that they are totally undetectable in the finished braidwork.

Any properly designed single-string imitation knot requires some string-run and coding modification at the transition-point(s) where one 'component' goes over into another 'component'. Such a transition-point is normally at a considerable distance from the string-ends, and hence corrective action must be taken well before we reach the end of the braiding process. The proper corrective action can only be taken during the design stage, hence well before the commencement of the braiding-process.

In order to achieve a correct design, the braider should ensure that a transition-point **never occurs** near a bight-boundary, and that it will be covered properly by the local weaving-pattern. Some further Examples on this subject are discussed in Pamphlet No. 1 *"Introducing Grid-diagrams in Braiding"*; the reader is advised to study these thoroughly.

We have discussed some mongrel knots which were supposed to look more or less like Gaucho knots (2-pass type), but miserably failed to do so. Furthermore these knots, even in their correct form, do not possess any desirable properties, and hence are not much good for anything; consequently we call them **mongrel knots.**

Then we met some bush knots, which also were supposed to look like Gaucho knots (3-pass type). Although they didn't quite look like Gaucho knots, they are excellent bush knots, and in their proper form they are knots of a high quality. Their single-string imitations are, in their proper form, also excellent knots. Furthermore, by changing the coding of the crossings on the left-hand and right-hand bight-boundaries 3 in such a way that there the string-run between the bight-boundaries 2 $\longleftrightarrow$ 2 makes *under-under* movements with the string-runs between the bight-boundaries 1 $\longleftrightarrow$ 3 and 3 $\longleftrightarrow$ 1, the overall weaving-pattern looks much like a 3-pass Gaucho one. Since these knots in their improper single-string form are very easy to braid, their braiding procedures are such that these may be remembered with ease, and since they offer several excellent proper alternative braid-forms, we call them **bush knots.**

We haven't shown in this article how we obtained the various sets of half-cycle braiding algorithms. Although it is always possible to read the half-cycle algorithms from the grid-diagram, in most cases is this a tedious and hence rarely convenient method. Several much more convenient procedures do exist, but before we can have a look at some of these, we first will have to learn some **natural arithmetic.** Although everybody has come in contact with this form of arithmetic, and in fact uses some of it unconsciously every day, it is not being taught to any extent in schools. This natural arithmetic is called **modular arithmetic.** In the next issue we shall have a look at this arithmetic and see how it is intimately associated with braiding.

THE BRAIDER

No.4 **NOVEMBER 1995.**

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A quarterly publication for the braiding artisan

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THE BRAIDER'S NOTEBOOK

In the previous issue we stressed that at least in the beginning all designs should be tested by means of the actual construction of the designed braid-form. Although the braider will eventually, after gaining sufficient experience in the interpretation of grid-diagrams, be able to dispense with these testing procedures in many cases, he or she will never be able to do without it completely. It is important that the braider is fully aware of the influence which the braiding material may exert on the braid-form under consideration. This influence is caused by two aspects of the braiding material:

(1). The flexibility of the braiding material.
(2). The cross-sectional shape of the braiding material.

(1). The range of different braiding materials has in more recent times greatly increased. Whereas in distant past mainly rawhide or leather was used for flat-string†, today much use is made of the flat tubular braided synthetics. This material is very flexible, and consequently many braid-forms which are highly unsatisfactory with the use of much less flexible braiding material, may seem to be acceptable when the test-braids are made with such flexible material.

It is therefore very important that the tests with flat-string should be carried out with string made from rawhide, leather, or fabric-reinforced plastic (such as "Dura Flex" lace).

(2). Many braid-forms which are suitable for flat-string, are unsuitable for round-string and vice versa. It is therefore important to know the reasons why, and what can be done to alleviate them.

Most braid-forms require **flat**-string as the braiding material since round-string would give a poor appearance. Hence when round-string is being used as braiding material, it is not surprising that many braid-forms are braided in multi-ply form (two or more parallel strings which in essence act as a 'single string'). This gives the multi-ply 'single string' the necessary *flat* appearance for a good looking braid-form.

A multi-ply braid-form, which is a multi-string braid-form‡, is often imitated by a single-string braid which we shall call an **imitation multi-ply braid-form.**

It should be clearly understood that multi-ply braid-forms and imitation multi-ply braid-forms are only intended for round-string braiding material.

Unfortunately some braiders, and especially whipmakers violate this very important rule by using imitation multi-ply knots, braided from *flat*-string, for covering knobs. The reason, no doubt, being that such people only know how to braid a few knots, which are too small in single-ply form to fulfil the requirement of covering the knob. Hence they resort to following the string-run and weaving-pattern of the single-ply knot, one or more times around, until sufficient coverage has been achieved. Irrespective of the

† We use the term *string* for all braiding materials, hence *round-string* for cord, rope or solid round braid; *flat-string* for any flat braiding material whether solid or tubular braid.

‡ Although a multi-ply braid-form is a multi-string braid-form, a multi-string braid-form is not necessarily a multi-ply braid-form.

standard of any further braidwork, any braided article which contains such braids can only be awarded a low grade since obviously a more suitable knot for flat braid material should have been used. Such very commonly encountered imitation multi-ply knots are the 3-parts/4-bights 3-ply Casa Knot and the 5-parts/4-bights 4-ply Casa Knot (the 5-parts/4-bights Casa Knot is also often encountered in its 3-ply form). Although any 4-bights Casa Knot will close completely at a bight-boundary, they should only be used in bush-braidwork, but never in quality braidwork for covering a knob.

When flat-string is used (the cross-sectional width is considerably greater than the cross-sectional height), the parallel strings in a multi-ply braid-form tend to slide over each other since in absolute sense a multi-ply braid-form is not form-fast. When such a braid is properly tightened, then only sufficient cross-sectional height of the strings can prevent this from happening. The closer the ratio (cross-sectional width)/(cross-sectional height) is to 1, the less likely this sliding will occur, and hence the greater the apparent form-fastness will be. Hence round-string (for which this ratio is equal to 1) is eminently suited for multi-ply and imitation multi-ply braid-forms.

A correctly braided *imitation multi-ply braid-form* is rarely encountered. It is normally braided in its bush-form, in which form it has of course no place in quality braidwork of a high standard. Hence in this article we shall have a look how an imitation multi-ply Casa knot should be braided. We shall take for an Example a 7-parts/5-bights Casa Knot.

In Fig. 35 on pg. 35 (*The Braider* issue No. 2) is shown the braiding process of a 7-parts/5-bights 1-ply Casa Knot depicted in Fig. 56 below.

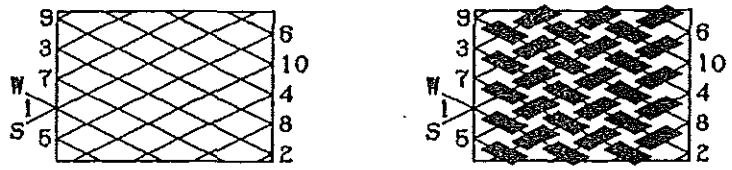


Fig. 56 — String-run and grid-diagram of a 7-parts/5-bights 1-ply Casa Knot.

Fig. 57 shows the string-run and grid-diagram of the imitation 2-ply form of this Casa Knot (7-parts/5-bights one-string imitation duplex Casa Knot). The 'base' knot is here not the 7-parts/5-bights Casa Knot since the last crossing on half-cycle 10 is now an *under*-crossing instead of an *over*-crossing. Recall that we have seen an identical coding-change for the imitation duplex Casa Knots discussed on pgs. 55–57 (*The Braider* issue No. 3). Such braid-forms are the best for imitation 2-ply Casa Knots.

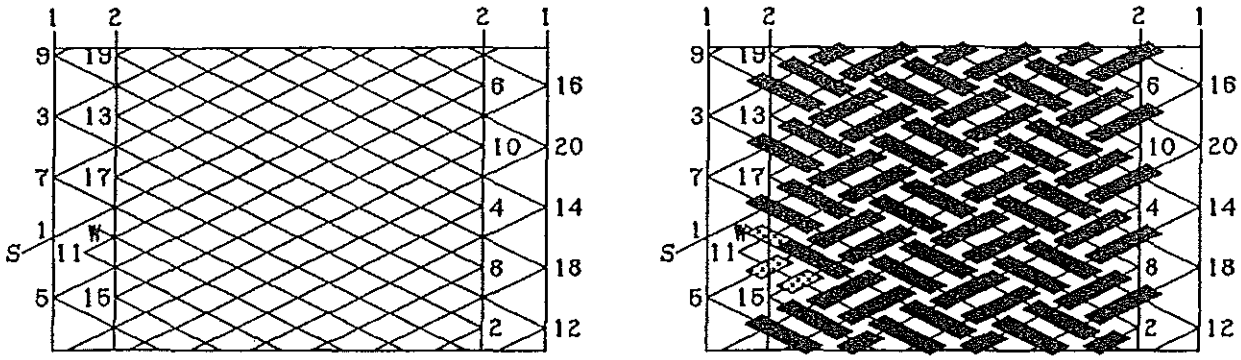


Fig. 57 — 7-parts/5-bights one-string imitation 2-ply (duplex) Casa Knot.

There is an alternative braid-form for the 2-ply imitation Casa Knots, which may be of an equal quality standard, depending on the cross-sectional dimensions of the braiding material which is being used in its construction. This alternative braid-form has the advantage of retaining the 1-ply Casa Knot as its 'base' knot, while a required coding-change takes place towards the end of the very last half-cycle. Hence, especially the novice may find this form easier to braid, although it is in some cases not quite up to the standard of the form depicted in Fig. 57.

This alternative form is for the 7-parts/5-bights one-string imitation 2-ply Casa Knot depicted by the diagrams in Fig. 58.

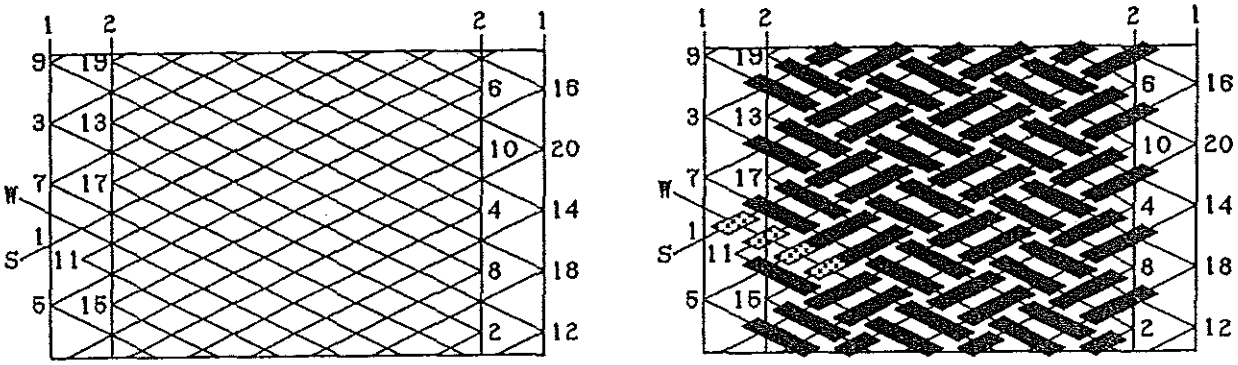


Fig. 58 — The alternative form for the 7-parts/5-bights one-string imitation 2-ply (duplex) Casa Knot.

The string-ends in the braid-forms depicted in Fig. 57 and Fig. 58 are worked away as shown in the respective diagrams 1 and 2 of Fig. 61.

Let's now have a look at a one-string imitation x-ply Casa Knot for which $x > 2$ (x is greater than 2)†. For an Example we shall take a 7-parts/5-bights one-string imitation 3-ply Casa Knot.

When we follow procedures similar to those used in the method associated with Fig. 57, we obtain a form that should however not be used. This form is depicted by the diagrams in Fig. 59. Its associated method for working the string-ends away is shown in diagram 3 of Fig. 61.

⋆⋆‡ *The reader should first try to find, with the aid of the grid-diagram, the reason why the method depicted in Fig. 59 should not be used. Then he or she should braid this form and examine his or her justification for the unsuitability of this form. Care should be taken to ensure that the braid is tightened properly.*

When, on the other hand, we follow procedures similar to those used in the method associated with Fig. 58, we obtain the correct form, and hence this is the form that should only be used. This correct form is depicted by the diagrams in Fig. 60. Its associated method for working the string-ends away is shown in diagram 4 of Fig. 61.

† The symbol $>$ stands for *greater than*, while the symbol $<$ stands for *smaller than*. We shall mainly use these symbols in further articles.

‡ Paragraphs preceded by ⋆⋆ and printed in a slanted roman typeface pose questions with which the reader can test his or her skill in the interpretation of grid-diagrams. The answers to these questions will appear in the issue following.

Due to the special coding-nature of multi-ply Casa Knots, their one-string imitation x-ply form with $x > 2$ should be braided in one way only, while their one-string imitation 2-ply form may be braided in two different ways. One of these two ways (the method depicted in Fig. 57 is generally the preferred method, and this method is essentially different to the method by which the one-string imitation x-ply form with $x > 2$ should be braided.

$\star\star$ *The reader should try to prove, with the aid of the grid-diagrams, why the method depicted in Fig. 57 is essentially different to the method depicted in Fig. 59. Furthermore, he or she should try to justify why the method depicted in Fig. 57 is in generally a better method than the one depicted in Fig. 58, notwithstanding that the method depicted in Fig. 58 is similar to the one depicted in Fig. 60. Finally, the proofs should be confirmed with sample braids in which the appropriate methods are used in accordance with Fig. 61 for working the string-ends away.*

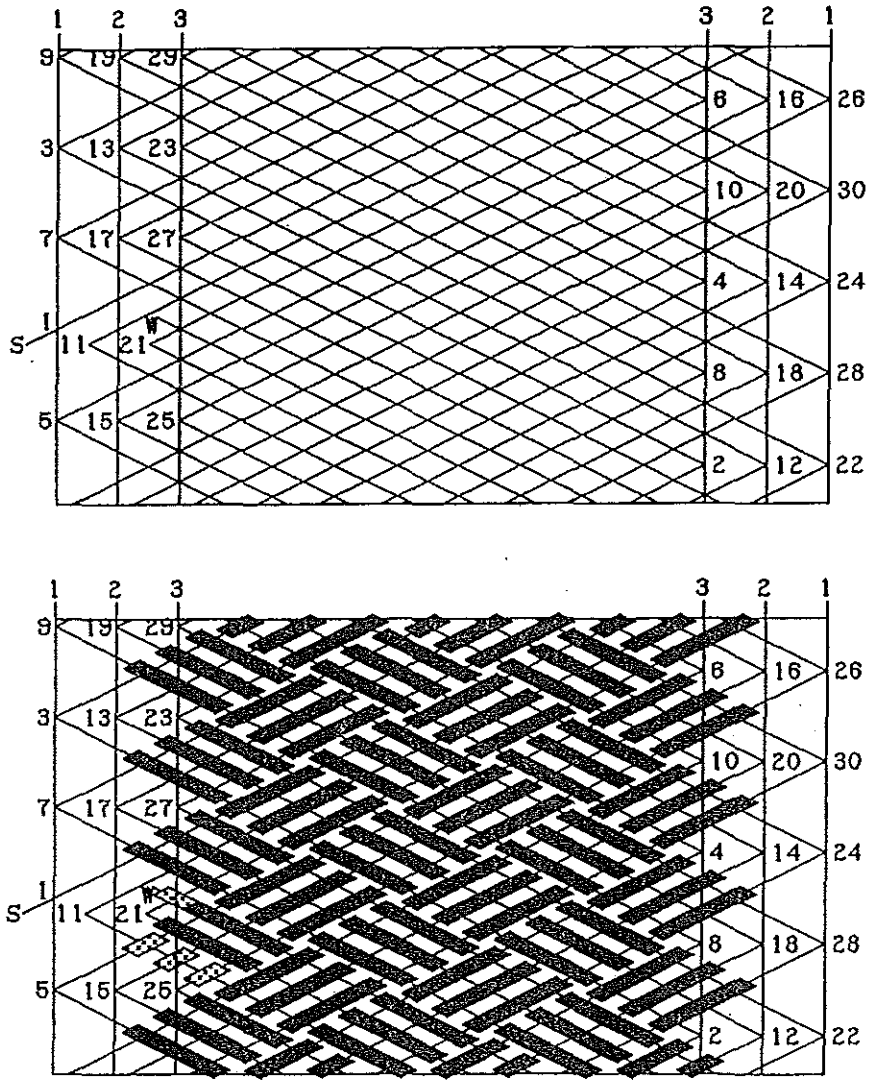


Fig. 59 — 7-parts/5-bights one-string imitation 3-ply (triplex) Casa Knot. This form should not be used!

We can't stress enough that multi-ply knots and one-string imitation multi-ply knots should only be braided from **round** and never from **flat** braiding material. When the

braider desires a checked weaving-pattern in a knot braided from flat-string, then a knot which is suitable for both this flat braiding material and the checked weaving-pattern should be used. Examples of such knots are the Semi-Regular Checked Knots, the one-string imitation form of these knots, and the Checked Pineapple Knots. In due course we shall discuss some of these knots in *The Braider*.

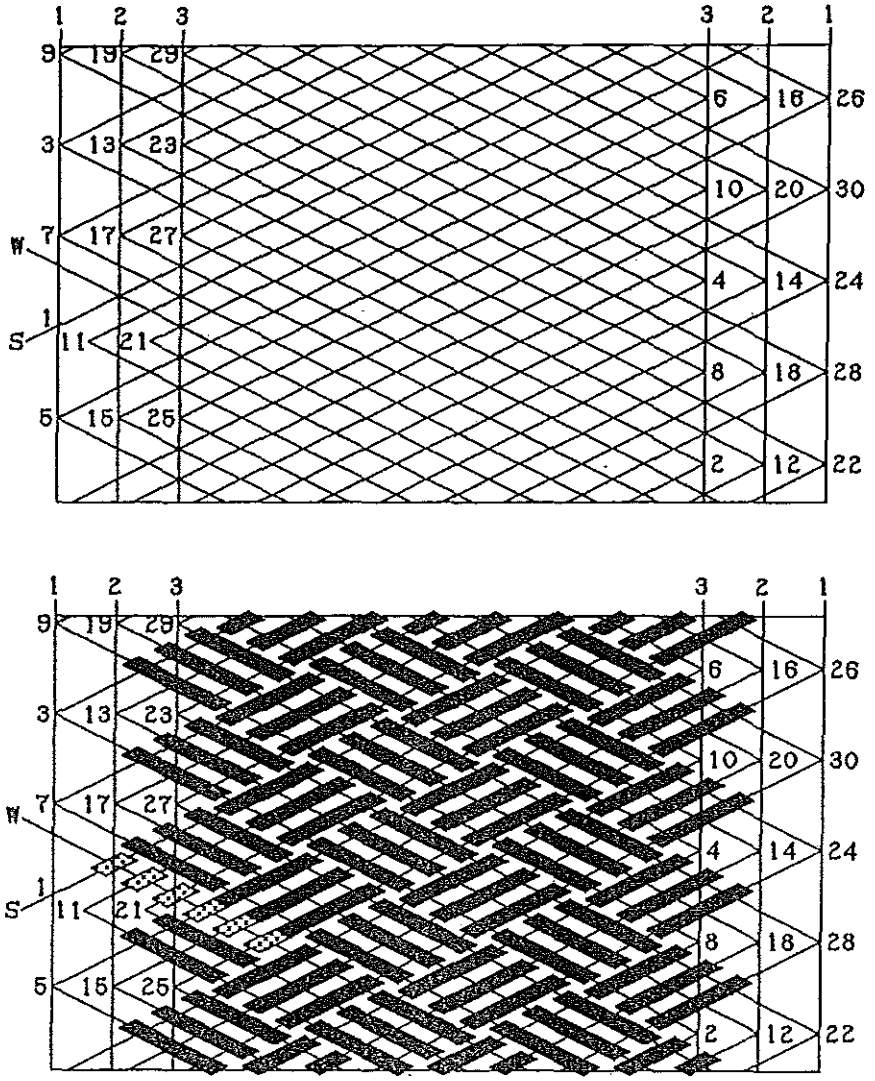


Fig. 60 — 7-parts/5-bights one-string imitation 3-ply (triplex) Casa Knot. This form is the only correct one to be used.

The artisan braider is no doubt well aware of the reason for the existence of multi-ply braid-forms, and will not use them incorrectly. Unfortunately many other braiders seem to be either unaware of the reason for their existence, or have not got the versatility which enables them to use a braid-form appropiate for the intended application. Much otherwise first class braidwork has to be down-graded due to the inappropriate use of multi-ply braid-forms or their one-string imitations. This inappropriate use is normally compounded by the use of incorrect braid-forms for these one-string imitations.

After the braider has gained sufficient experience with the interpretation of grid-diagrams, these diagrams can greatly assist him or her in avoiding incorrect designs.

This in turn will not only enable the braider to use the available time and braiding materials more efficiently, but will also significantly extend his or her braiding knowledge. With this knowledge, the braider will be able to design new braid-forms for the envisaged applications.

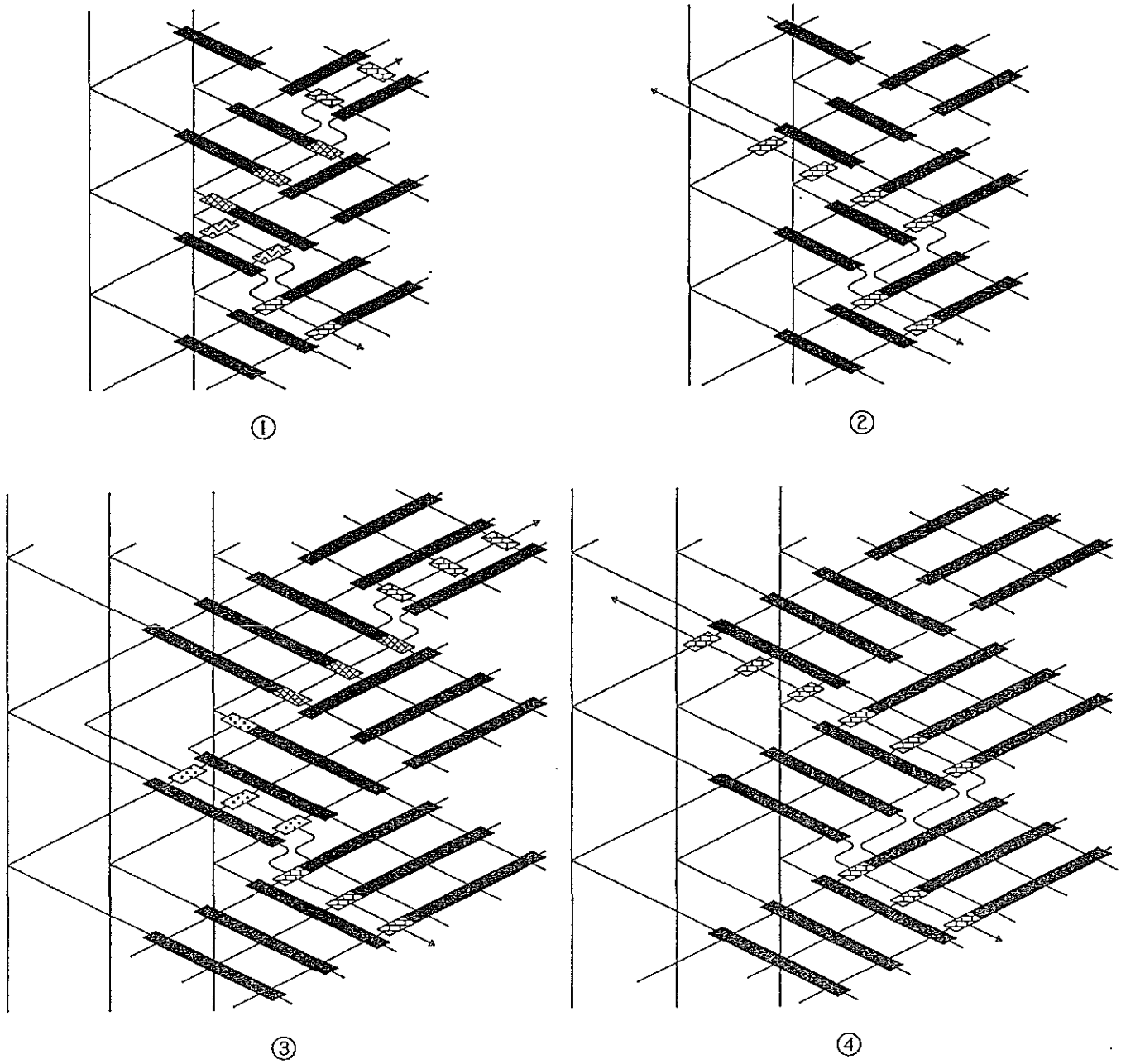


Fig. 61 — The procedures for working away the strings in the braid-forms depicted in the respective Figs. 57, 58, 59, and 60.

Regular Flat Braids, Belts, and Buckles

In the second issue of *The Braider* we have discussed the procedures for braiding a Regular Flat Braid by starting it at the tip of the billet, and in issue three we elaborated

a little more on one of the in issue two discussed tip constructions. Starting the braiding process at the tip of the billet has for a belt the advantage of being able to produce a neat braided billet. If on the other hand we would finish the flat braid at the billet-end, then we will either have to back-braid the thong-ends along the back of the billet, or alternatively at least cover the end of the billet with a piece of thin leather at both sides of the braid, glued and sewn on in order to hide the thong-ends. Neither of these two methods gives however a neat finish to the billet.

When for a belt we start with the braiding process at the tip of the billet, then consequently we will have to finish at the buckle-end, and hence back-braid the ends of the thongs there along the back of the flat braid. Since the attachment of a buckle to a flat braid is an easy and more or less self-evident matter and since the back-braiding involved has been clearly described in many books (see for example: *The Encyclopedia of Rawhide and Leather Braiding* by Bruce Grant, pp. 100-101; or *The Bushman's Handcrafts* by R. M. Williams, pp. 210-211), we shall not waste space in repeating it here. However, the braider should not just push the tongue of the buckle through the braid, but should instead provide a slit in the centre of the braid in order to give the tongue the necessary freedom of movement and hence to eliminate damage to the braid. For this purpose a slit of two or three bights should be sufficient. The position of the slit in colour work should be such that the thong-colour of opposing bights is the same. For the central placing of the slit we obviously require a regular flat braid with an **even** number of strings.

A hook-buckle does not require a slit in the braid, since it has no projecting part which has to penetrate the braid at the place of its attachment.

In contrast with the buckle-end, the billet-end of the braid causes some problems and to these we shall now devote our attention.

The billet has somehow to be fastened to the buckle. This can be done as follows:

(1). Braid holes in the billet for the buckle-tong or buckle-hook.

(2). Use eyelets with on both sides of the braid washers.

(3). Push the buckle-tongue or buckle-hook through the braid.

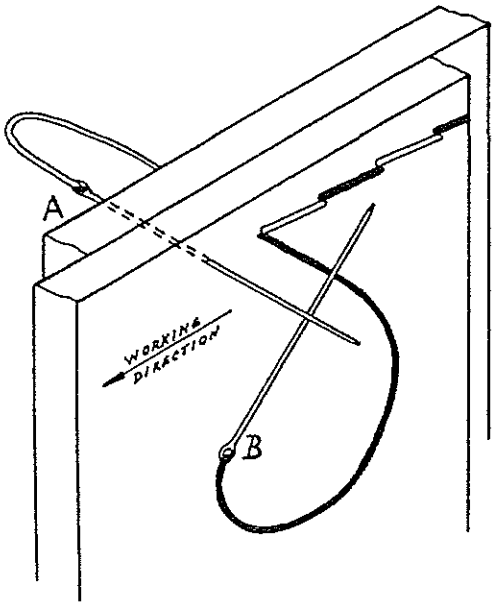
(4). Glue, glue and sew, or sew a strip of leather to the back of the braid over the length which contains the holes. This strip of leather has punched holes which align with the holes in the braid. The holes in the leather strip should be reinforced with eyelets.

(5). A special reinforcing method for the braid over the braid-length containing the holes.

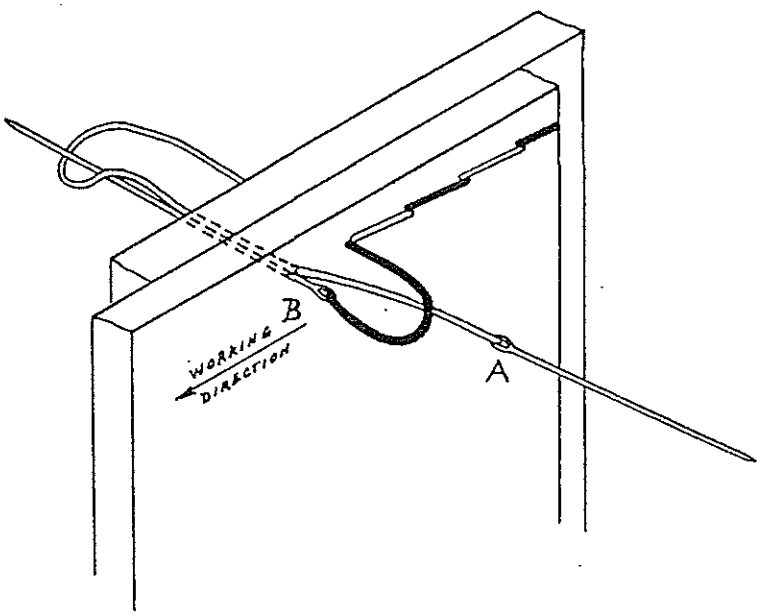
None of the methods (1), (2), (3) are to be recommended Especially the methods (1) and (3) soon destroy the good appearanse of the billet. Although method (4) makes the billet somewhat thicker and less flexible, this can to a large extent be limited by using a not too thick leather strip with eyelets reinforced holes. The attachment of the leather strip by means of glue alone is not only restricted to certain braiding materials, but can generally not be recommended due to its questionable permanence; when possible, the glue and sew method should be employed here.

Any sewing should be done with the **locked saddle stitch**. The procedure involved is set out in Fig. 63 for sewing from right to left, and from left to right. It should be noted here that there is never any justification for using the **common saddle stitch**.

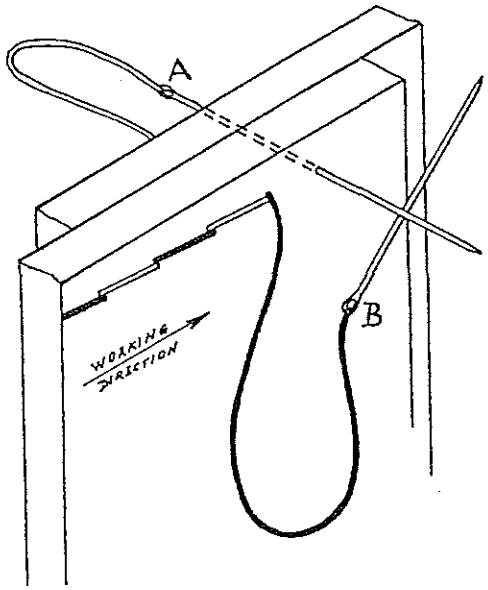
The reason being that it is of an inferior quality and not any easier or quicker to execute; this stitch is depicted in Fig. 65.



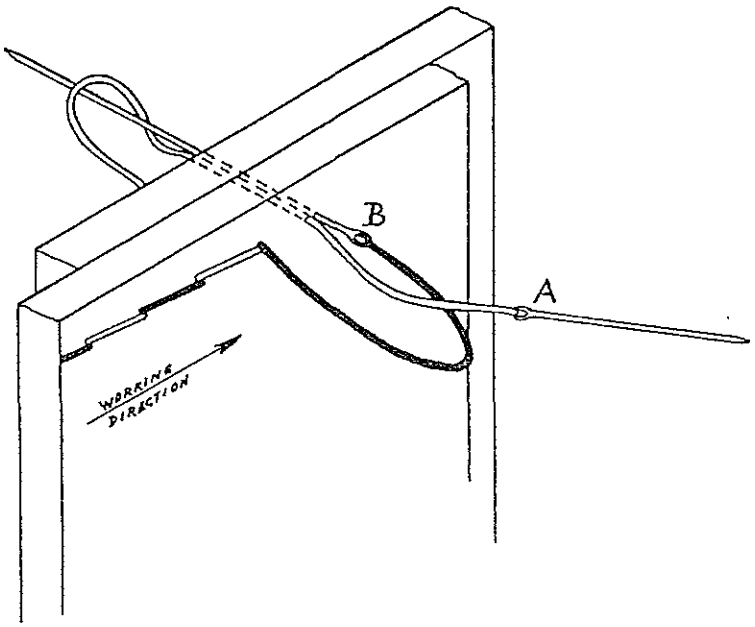
Needle "B" under needle "A".



Insert needle "B" at left of needle "A".



Needle "B" under needle "A".



Insert needle "B" at right of needle "A".

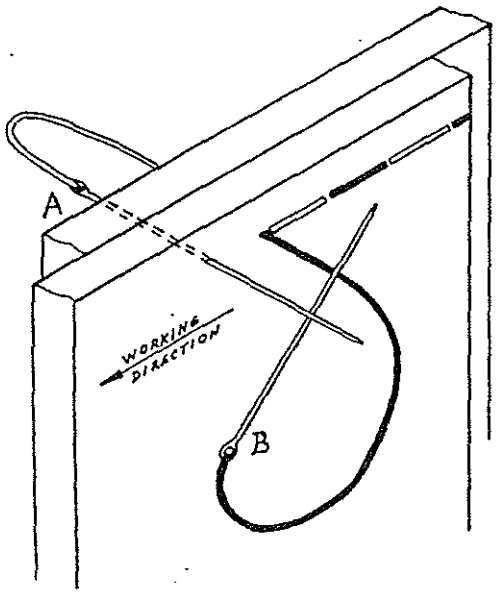
Fig. 63 — The locked saddle stitch.

In cross-section the locked saddle stitch is of the construction as depicted in Fig. 64, whereas the cross-section of the common saddle stitch is of the construction as depicted in Fig. 66.

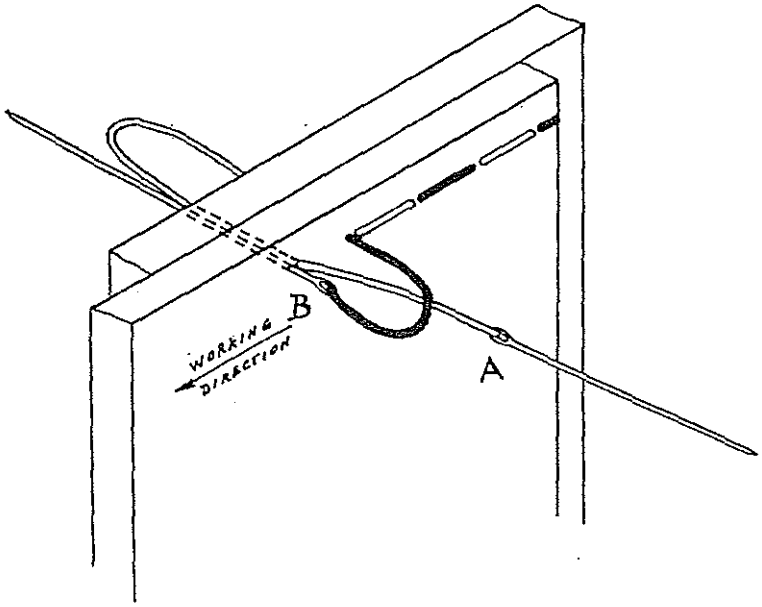


THE LOCKED SADDLE STITCH

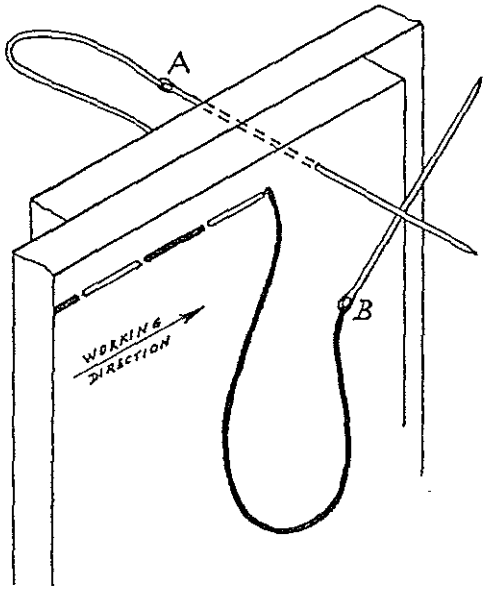
Fig. 64 — A cross-sectional view of the locked saddle stitch.



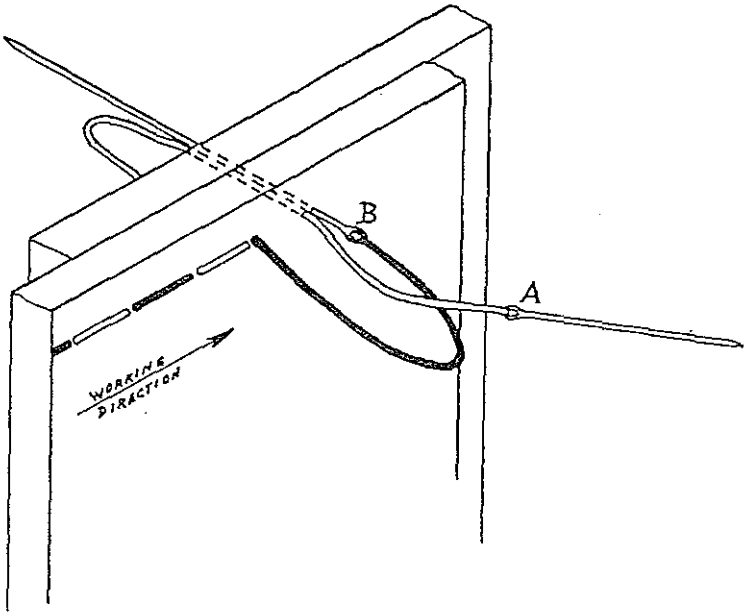
Needle "B" under needle "A".



Insert needle "B" at left of needle "A".



Needle "B" under needle "A".



Insert needle "B" at right of needle "A".

Fig. 65 — The common saddle stitch.



Fig. 66 — A cross-sectional view of the common saddle stitch.

Construction method (4) is depicted in Fig. 67. In this drawing the billet has only three holes, but we can of course have as many holes as we like. The middle drawing shows the sewing line as viewed from the front of the braid, and the bottom drawing shows the back with the the leather strip and eyelets. For wider braids it is advisable to use two sewing lines in order to provide a better reinforcing for the braidwork. The sewing should be done with waxed braided polyester "thread". When for the thonging flat tubular synthetic braid is being used, the length of a stitch can be halved by going through the centre of the crossings; this is especially advisable for wider thonging.

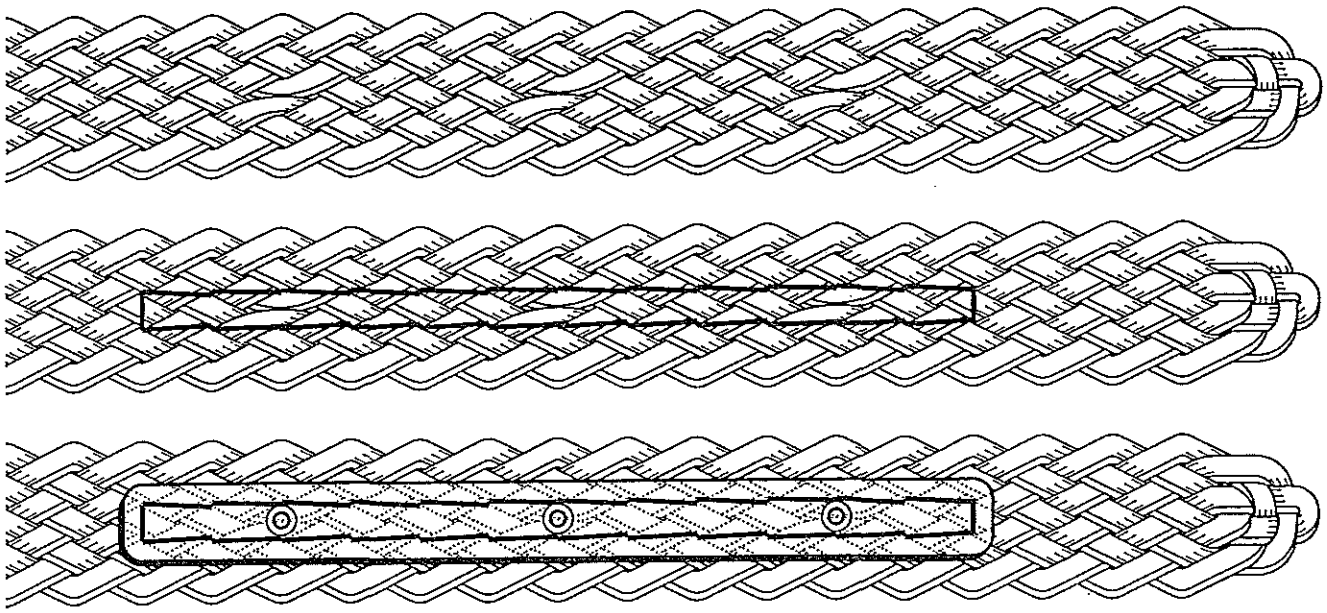


Fig. 67 — Protecting the braid of the billet with a glued and sewn on leather strip.

In method (5) the braid of the billet is reinforced by lines of sewing. Again waxed braided polyester "thread" should be used. The procedure is depicted in Fig. 68. The sewing sequence is indicated in the lower diagram of Fig. 68.

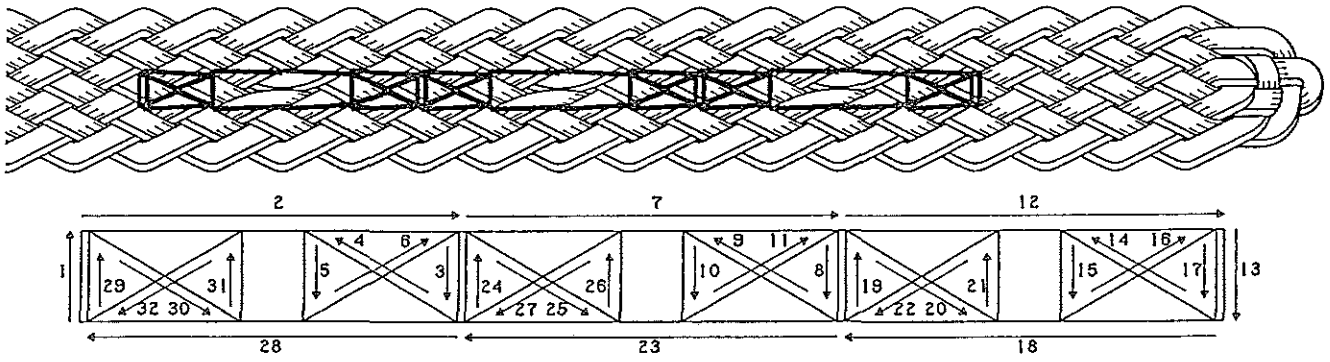


Fig. 68 — Reinforcing the billet (method (5)).

When for the thonging flat tubular synthetic braid is being used, we can again half the stitch-length by going through the centre of the crossings.

When holes are braided in a braid with a colour-pattern, they should be spaced in such a way that at each hole the strings of the opposing bights are of the same colour. This will maintain the colour-pattern of the braid.

Braiding Instructions and their Presentation

Before going over to the **modular arithmetic** and its intimate association with many braid-forms, it will be worth-while to have first a little closer look at the in the previous issue discussed **Hansen Knots** and their imitation forms.

We did define the Hansen Knots discussed as the **Hansen type 1 knots.** Their string-run was that of a **Standard Periodic Nested Cylindrical Braid.** The term '**Standard**' indicates that the knot is an interbraid of Regular Knots; the string-run between one of the left-hand bight-boundaries and one of the right-hand bight-boundaries forms the string-run of one of these interbraided Regular Knots. Since the Hansen Type 1 knots discussed had three left-hand and three right-hand bight-boundaries, they were therefore an interbraid of three Regular Knots, which turned out to be identical.

Since the principal weaving-pattern of these Hansen Knots was that of 3-pass Gaucho Knots, we call them 3-pass Hansen type 1 knots.

We have seen how these knots could be imitated by 1-string knots, however, some modification to the string-run and coding was then necessary. These modifications resulted in a more complicated half-cycle braiding algorithm-table. Hence the question: is there not an easier method or form for a 1-string Hansen Knot? Well, there certainly are easier and much better 1-string forms. These are the Hansen type 1* knots, which come in two forms for the 3-pass variety. For the 21-parts/12-bights 3-pass Hansen type 1* knots the string-runs of these two forms are depicted in the respective diagrams 2 and 3 of Fig. 69.

The diagrams 1 and 1A of Fig. 69 depict respectively the string-run and the grid-diagram of the 21-parts/12-bights 3-pass Hansen type 1 knot.

The only difference in the coding layout is the relative position of the coding on the innermost left-hand and right-hand bight-boundaries.

In some future issue we shall come back to the Hansen Knots and investigate them in greater detail, but for the time being we leave the reader with the following question:

⋆⋆ *We have discussed above the typical string-runs and coding for the 3-pass Hansen Knots of type 1 and type* 1*. *What are the typical string-runs and coding for higher pass Hansen Knots of type 1 and type* 1*? *What can you tell about the passes and about the number of type* 1* *forms? What can you tell about the passes and the number of parts? What can you tell about the relative positions of the adjacent bight-boundaries on the left and right-hand side of the knot? In order to draw an appropriate grid-diagram we need to know the column-distane between the innermost left and right-hand bight-boundaries. Can you find an easy relationship for this distance?*†

† Use isometric graph-paper as an aid in solving these questions.

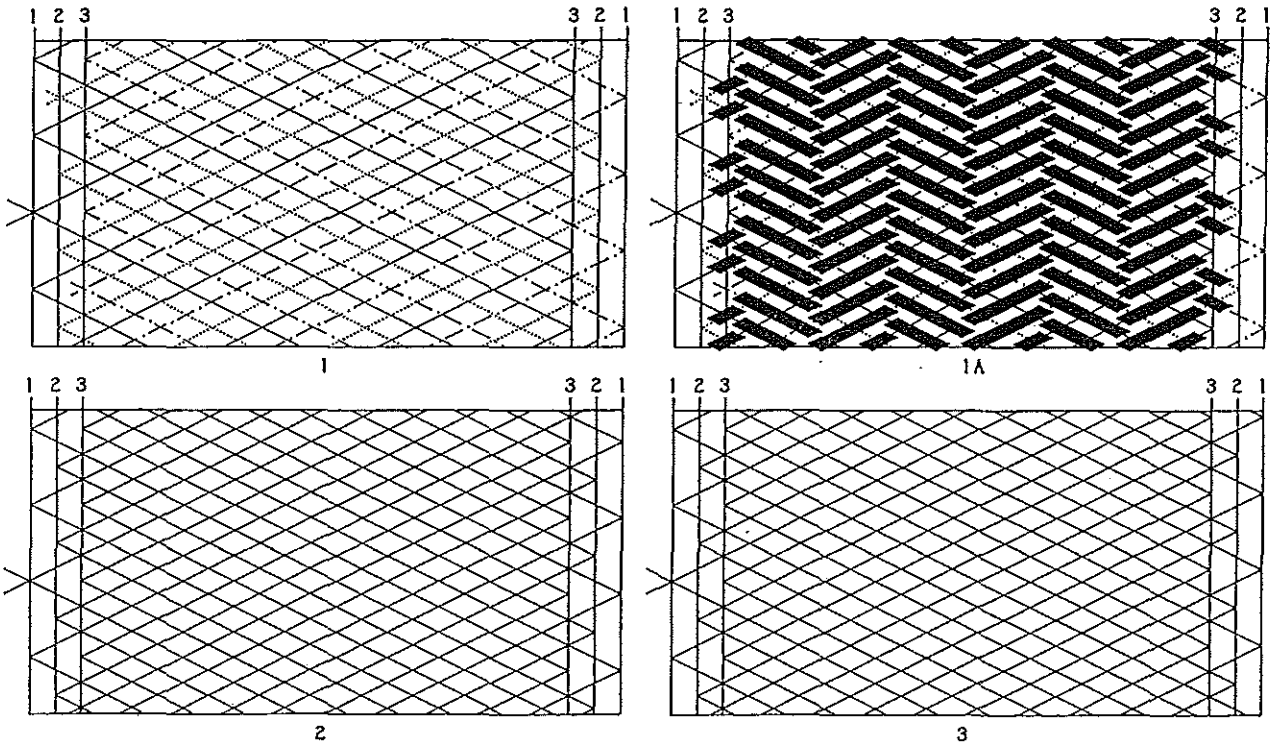


Fig. 69 — The 21-parts/12-bights 3-pass Hansen type 1 and type 1* knots.

We shall now go over to the subject of **modular arithmetic**. Modular arithmetic is a cyclic or clock arithmetic, whereas our "common" arithmetic is a linear arithmetic. Most, if not everything, in the natural world cycles; some cycles are enormously long and some are extremely short. We might not be familiar with many, but we certainly are used to several. One cyclic phenominon we encounter every day is the day itself, and in order to know timewise where we are in that day, many people use a 'clock'. Let's asume that our clock has 12 equal intervals in a cycle: this are the 12 hours in which the dial has been divided. A cycle starts at '0', although on our clock we call it '12'. In fact '0' and '12' are the same, because after having done 12 steps of 1 in a clockwise direction, starting at '0', we reach '12', but are also back at '0'. Of course we are all quite familiar with this, but let's have a look how it all works with additions and subtractions.

Fig. 70, diagram A, depicts our 12 hour clock. Moving in a clockwise direction constitutes addition, and moving in a anticlockwise direction constitutes subtraction.

In diagram B we got 2 and add 7. Of course we can say that we start at 0 and add 2, then we are in 2 (we got 2); then we add 7. Thus as we see: $2 + 7 = 9$.

In diagram C we got 7 and add 9. Again, we can also say that we start at 0 and add 7, then we are in 7 (we got 7); then we add 9. We see that we arrive at 4. In our normal linear arithmetic 7 plus 9 makes 16. However in our cyclic case we start after arriving at 12 with 0 again. If we divide 16 by 12 (the value of a full cycle), then the remainder

---

As mentioned earlier, the answers to the questions in an issue will appear in the issue following. Of course we like to receive your solutions, and any correct solution will be acknowledged in that issue.

is 4, which is the value we obtain by adding 9 to 7 in the modular arithmetic in which the value of a full cycle is 12. It will be obvious that we need some sort of symbolism to write this down in order to get a short, simple and easy to follow mechanism for "cyclic addition". Instead of the symbolism in standard use which is too complicated to be of any practical value in braiding, we shall use the following symbolism:

$$|7+9|_{12} = |16|_{12} = 4.$$

We read this as follows:

7 plus 9 mod 12 is 16 mod 12 is 4.

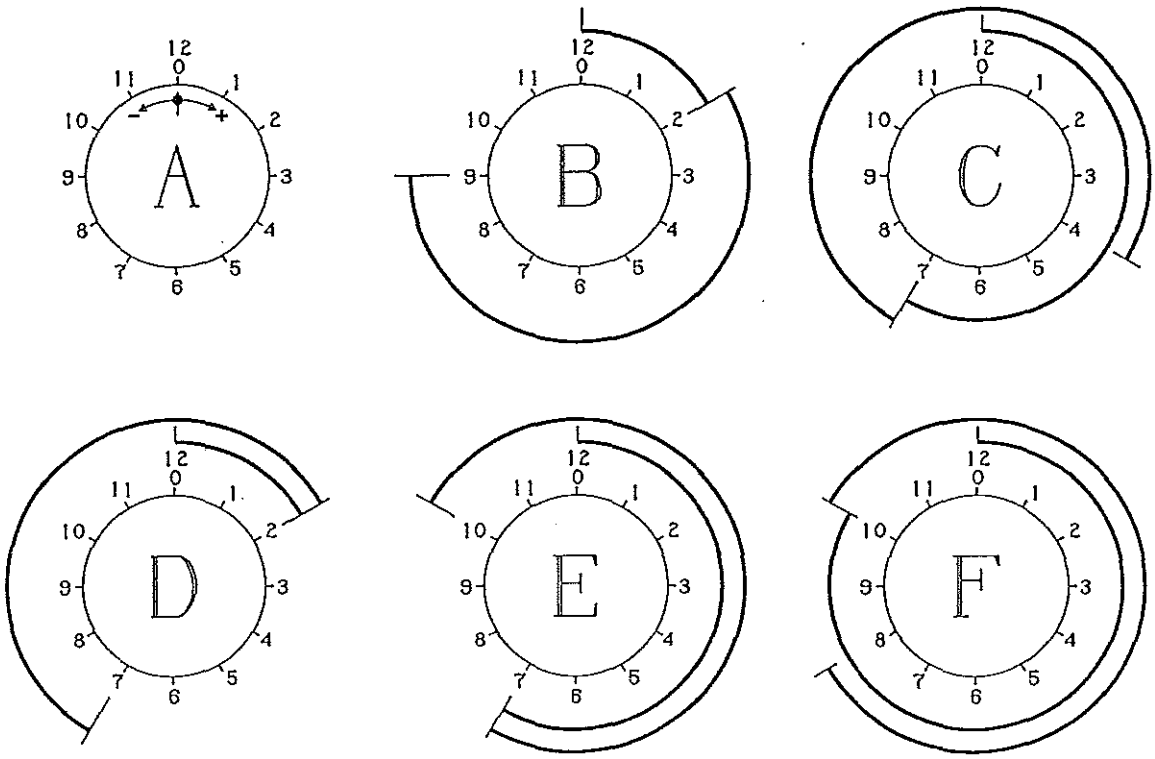


Fig. 70 — The cycle of 12.

In diagram D we got 2 and subtract 7. Again, we can also say that we start at 0 and add 2, then we arrive in 2 (we got 2); then we subtract 7. We see that we arrive at 7. In our normal linear arithmetic 2 minus 7 makes minus 5. Hence from the 0 point position, which is also the 12 point position, we have to subtact 5, thus $12 - 5 = 7$. In our cyclic case we write:

$$|2-7|_{12} = |-5|_{12} = 7.$$

We read this as follows:

2 minus 7 mod 12 is minus 5 mod 12 is 7.

In diagram E we got 7 and subtract 9. Again, we can also say that we start at 0 and add 7, then we arrive in 7 (we got 7); then we subtract 9. We see that we arrive at 10. In our normal linear arithmetic 7 minus 9 makes minus 2. Hence from the 0 point position, which is also the 12 point position, we have to subtact 2, thus $12 - 2 = 10$. In our cyclic case we write:

$$|7-9|_{12} = |-2|_{12} = 10.$$

We read this as follows:

7 minus 9 mod 12 is minus 2 mod 12 is 10.

In diagram F we got 10 and add 10. Again, we can also say that we start at 0 and add 10, then we are in 10 (we got 10); then we add 10. We see that we arrive at 8. In our normal linear arithmetic 10 plus 10 makes 20. However in our cyclic case we start after arriving at 12 with 0 again. If we divide 20 by 12 (the value of a full cycle), then the remainder is 8, which is the value we obtain by adding 10 to 10 in the modular arithmetic in which the value of a full cycle is 12. In our cyclic case we write:

$$|10 + 10|_{12} = |20|_{12} = 8.$$

We read this as follows:

10 plus 10 mod 12 is 20 mod 12 is 8.

Let's now have a look at a few "more complicated" examples:

$$|3 + 23|_{12} = |26|_{12} = 2.$$

$26 \div 12 \longrightarrow$ remainder is 2.

$$|5 + 3 + 43|_{12} = |51|_{12} = 3.$$

$51 \div 12 \longrightarrow$ remainder is 3.

$$|17 - 8 - 43|_{12} = |-34|_{12} = 2.$$

$36 - 34 = 2$. Take the nearest multiple of 12 above 34 and subtract 34.

$$|5 + 3 + 43|_{7} = |51|_{7} = 2.$$

$51 \div 7 \longrightarrow$ remainder is 2.

$$|7 + 9 - 10 + 24|_{13} = |30|_{13} = 4.$$

$30 \div 13 \longrightarrow$ remainder is 4.

$$|7 - 8 - 23|_{5} = |-24|_{5} = 1.$$

$25 - 24 = 1$. Take the nearest multiple of 5 above 24 and subtract 24.

In general notation:

$$|N|_p = r, \quad \text{where}$$

N may be any integer; that is N may be any whole number, positive, negative or zero.

p (which is the modulus) may be any positive integer; that is p may be any positive whole number.

r is an integer which is greater or equal to zero, but smaller than p $(0 \leq r < p)$.

The following Rules are associated with modular arithmetic.

Rules:

(1) If $|A|_p = |B|_p$, and $|C|_p = |D|_p$, then:
$|A + C|_p = |B + D|_p$,
$|A - C|_p = |B - D|_p$,

$$|A \cdot C|_p = |B \cdot D|_p \,.$$

(2) $$|A + B|_p = \Big| |A|_p + |B|_p \Big|_p \,.$$

(3) $$|n \cdot A|_{n \cdot p} = n \cdot |A|_p \,.$$

(4) If $|A|_p = |B|_p$, we may **not** deduce that $A = B$;
we can, however, deduce that $|A - B|_p = 0$.
Conversely, $|A - B|_p = 0$ implies that $|A|_p = |B|_p$.

(5) If $|k \cdot A|_p = |k \cdot B|_p$, then:
$|A|_{\frac{p}{d}} = |B|_{\frac{p}{d}}$, where g.c.d.$(k, p) = d$.
"g.c.d.(k, p)" stands for: "the greatest common divisor of k and p".

To illustrate these rules, say that A is equal to 7, B is equal to 23, C is equal to 15, D is equal to 19, p is equal to 4, n is equal to 5, and k is equal to 6. Then:

(1) $|7|_4 = |23|_4 = 3$, and $|15|_4 = |19|_4 = 3$,
$|7 + 15|_4 = |23 + 19|_4 = 2$,
$|7 - 15|_4 = |23 - 19|_4 = 0$,
$|7 \cdot 15|_4 = |23 \cdot 19|_4 = 1$.

(2) $|7 + 23|_4 = ||7|_4 + |23|_4|_4 = 2$.

(3) $|5 \cdot 7|_{5 \cdot 4} = 5 \cdot |7|_4 = 15$.

(4) $|7|_4 = |23|_4 = 3$, but 7 is not equal to 23;
however, $|7 - 23|_4 = 0$.
Conversely, $|7 - 23|_4 = 0$ implies that $|7|_4 = |23|_4$.

(5) $|6 \cdot 7|_4 = |6 \cdot 23|_4 = 2$,
$|7|_{\frac{4}{2}} = |23|_{\frac{4}{2}} = 1$, where g.c.d.$(6, 4) = d = 2$.
"g.c.d.$(6, 4)$" stands for: "the greatest common divisor of 6 and 4".

Sometimes it is necessary to calculate the **absolute value** of a number. It is the *positive value* of a number. The symbol for the absolute value of a number is a vertical bar on each side of the number.

Examples:

$$|7| = 7 \,.$$
$$|0| = 0 \,.$$
$$|-6| = 6 \,.$$
$$|11 + 4| = |15| = 15 \,.$$
$$|7 - 31| = |-24| = 24 \,.$$
$$|-5 \cdot 7| = |-5 \times 7| = |-35| = 35 \,.$$
$$\left|\frac{-3 \cdot 9 + 3}{6}\right| = \left|\frac{-3 \times 9 + 3}{6}\right| = \left|\frac{-27 + 3}{6}\right| = \left|\frac{-24}{6}\right| = |-4| = 4 \,.$$

Let us now have a look how this modular arithmetic is intimately associated with braiding.

We shall use the string-run of the Casa-coded 7-parts/5-bights Regular Knot. A development of such a knot (cut along a cylinder generating line and rolled out in a flat plane) is illustated by the top-left pictorial drawing of Fig. 71. Below this pictorial drawing we find respectively the string-run diagram and the grid-diagram of this Casa Knot. In the string-run diagram below this grid-diagram we have accentuated the half-cycles which run from lower-left to upper-right, with the columns numbered from left to right. In the bottom string-run diagram we have accentuated the half-cycles which run from lower-right to upper-left, with the columns numbered from right to left.

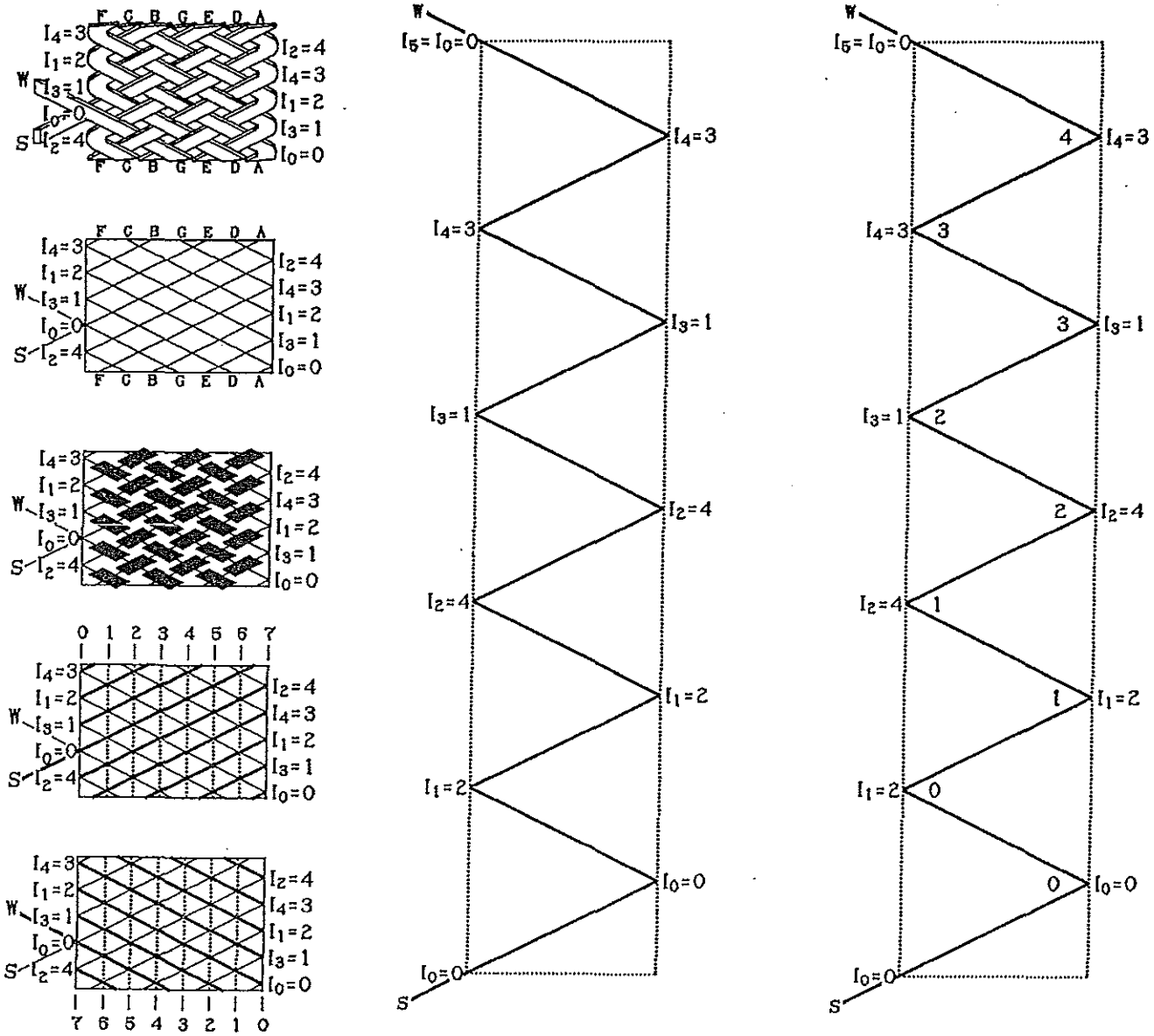


Fig. 71 — Bight-index numbers and bight-numbers.

In these left-hand diagrams we have numbered the bights on the left-hand bight-boundary in an upwards direction in sequential numerical order with bight-index number '0' at the Standing-end where the first half-cycle begins. The bight at the end of the first half-cycle (on the right-hand bight-boundary of the knot) also receives a bight-index number '0'. The remaining bights on the right-hand bight-boundary are also numbered in numerical sequential order upwards. Hence a half-cycle which runs from lower-left to upper-right has at both its ends an identical bight-index number.

In the central diagram and in the right-hand diagram the string-run half-cycles have been rolled off the cylinder in a continuous sequence. Starting with the Standing-end **S** at the left-hand bight-boundary at bight-index number '0' ($= I_0$), the first half-cycle ends at the right-hand bight-boundary at bight-index number '0' ($= I_0$).

The second half-cycle starts at the right-hand bight-boundary at bight-index number '0' ($= I_0$) and ends at the left-hand bight-boundary at bight-index I_1.

From the grid we see that if we move along a half-cycle a distance of **one column**, then we move **one row** along a bight-boundary. The distance between any two adjacent bight-points is equal to **two rows**.

Hence the movement of one column along a half-cycle equals half a bight along a bight-boundary.

The total number of columns we move along any one half-cycle is for Regular Cylindrical Braids always equal to its number of parts p. Hence a (full) cycle (which consists of **two** consecutive half-cycles) results in a $2p$ **row** movement along a bight-boundary which is equal to p **bights**. However, a bight-boundary is a circle and hence moving along the bight-points is a cyclic phenominon, consequently we will have to use modular arithmetic in any calculation which involves such movements.

From bight-index I_0 to bight-index I_1 we thus move p bights, consequently $I_1 = |0 + p|_b = |p|_b$. This gives for our Example: $I_1 = |7|_5 = 2$.

The third half-cycle starts at the left-hand bight-boundary at bight-index number '2' ($= I_1$) and ends at the right-hand bight-boundary at bight-index number '2' ($= I_1$).

The fourth half-cycle starts at the right-hand bight-boundary at bight-index number '2' ($=I_1$) and ends at the left-hand bight-boundary at bight-index I_2.

From bight-index I_0 to bight-index I_2 we thus move $2p$ bights, consequently $I_2 = |0 + 2p|_b = |2p|_b$. This gives for our Example: $I_2 = |2 \cdot 7|_5 = |2 \times 7|_5 = |14|_5 = 4$.

The fifth half-cycle starts at the left-hand bight-boundary at bight-index number '4' ($= I_2$) and ends at the right-hand bight-boundary at bight-index number '4' ($= I_2$).

The sixth half-cycle starts at the right-hand bight-boundary at bight-index number '4' ($=I_2$) and ends at the left-hand bight-boundary at bight-index I_3.

From bight-index I_0 to bight-index I_3 we thus move $3p$ bights, consequently $I_3 = |0 + 3p|_b = |3p|_b$. This gives for our Example: $I_3 = |3 \cdot 7|_5 = |3 \times 7|_5 = |21|_5 = 1$.

The seventh half-cycle starts at the left-hand bight-boundary at bight-index number '1' ($= I_3$) and ends at the right-hand bight-boundary at bight-index number '1' ($= I_3$).

The eighth half-cycle starts at the right-hand bight-boundary at bight-index number '1' ($=I_3$) and ends at the left-hand bight-boundary at bight-index I_4.

From bight-index I_0 to bight-index I_4 we thus move $4p$ bights, consequently $I_4 = |0 + 4p|_b = |4p|_b$. This gives for our Example: $I_4 = |4 \cdot 7|_5 = |4 \times 7|_5 = |28|_5 = 3$.

The ninth half-cycle starts at the left-hand bight-boundary at bight-index number '3' ($= I_4$) and ends at the right-hand bight-boundary at bight-index number '3' ($= I_4$).

The tenth half-cycle starts at the right-hand bight-boundary at bight-index number '3' ($=I_4$) and ends at the left-hand bight-boundary at bight-index I_5.

From bight-index I_0 to bight-index I_5 we thus move $5p$ bights, consequently $I_5 = |0 + 5p|_b = |5p|_b$. This gives for our Example: $I_5 = |5 \cdot 7|_5 = |5 \times 7|_5 = |35|_5 = 0$. Hence we are back at I_0, thus in our Example $I_5 = I_0 = 0$.

In the right-hand diagram of Fig. 71 we find a further set of numbers in the corners

between the successive half-cycles. This are the **bight-numbers**†. Their use will be explained later, but the reader will undoubtedly notice that the half-cycles are related to them. For example the fifth half-cycle is the half-cycle $(2 \cdot 1 + 3)$. In general:

To an **odd**-numbered half-cycle h_o belongs bight-number $i = \dfrac{h_o - 3}{2}$.

The eighth half-cycle, for example, is the half-cycle $(2 \cdot 3 + 2)$. In general:

To an **even**-numbered half-cycle h_e belongs bight-number $i = \dfrac{h_e - 2}{2}$.

In Fig. 72 are depicted the same diagrams as in Fig. 71, but with some additional information. The half-cycles have been numbered in consecutive order, and in the two lower left-hand string-run diagrams the bight-numbers have been added. In the right-hand diagram the bight-index number calculations have been shown.

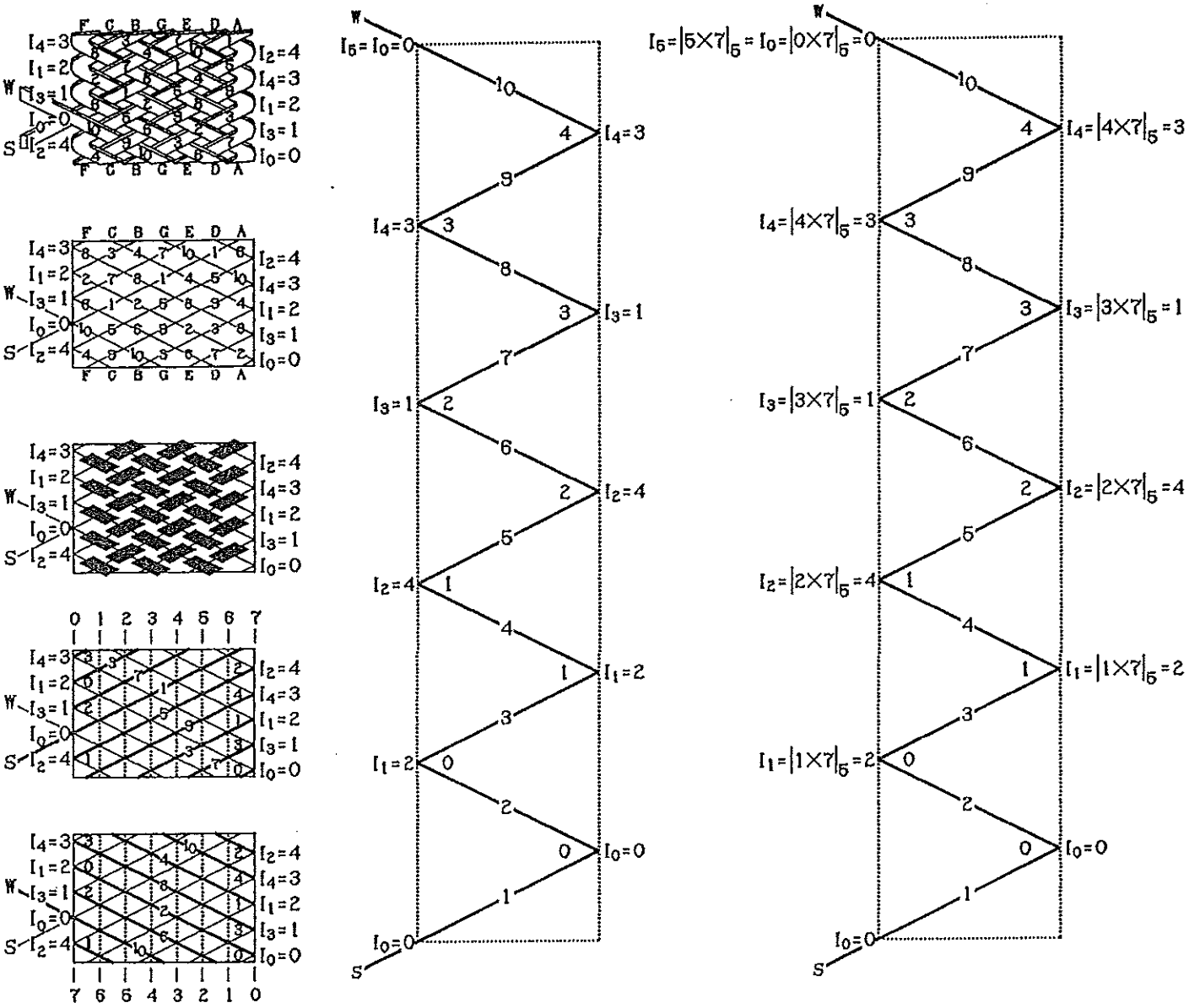


Fig. 72 — Bight-index numbers and bight-numbers.

† Don't confuse *bight-numbers* with *bight-index numbers*. They are quite different entities!!!

In this issue the reader has been given a little glimpse of the connection between modular arithmetic and braiding. It of the utmost importance that those who want to ascent the braiding ladder, must become thoroughly conversant with that what has been discussed here. Some will pick it up in a few minutes and others will have to work their brain and maybe engage it into another gear. However, anybody who has been able to learn the common linear arithmetic taught in shools, should be able to master modular arithmetic. In the next issue we shall continue our discussion and show how we are going to use it in the derivation of half-cycle braiding algorithms, the actual recipes of the half-cycle braiding sequences used in the actual braiding of the braid-form.

Reviews

There is at least one author (Ron Edwards) who apparently very much dislikes our reviews. The review he complains about was the one in issue 2 (pg. 38) in which we reviewed some of his booklets. He considers that review a waste of a full page since some of the reviewed booklets were published more then ten years ago. Furthermore, he is apparently very annoyed about our statement that these pubications fall a very long way short of forming a standard reference or part thereof.

In the reviews the date of a publication is largely irrelevant, since we review the contents and not its birthdate. However, it should be noted that the publications concerned were published in 1984 and 1985 and were afterwards regularly advertised in *"The Australian Whipmaker"*. In fact, they were still being advertised as recently as January 1995 (*The Australian Whipmaker, number 37*). Furthermore, in 1992 three separate issues of *"The Australian Whipmaker"* advertise his book entitled *"Bush Leatherwork"*, which contains the contents of these reviewed booklets, as: *'A standard reference book for all leatherworkers'*. Consequently we included in our review the comment that the contents of these booklets fell a very long way short of forming a standard reference or part thereof.

In our reviews we give credit where credit is deserved, and criticism where criticism is deserved; they are not intended for pleasing the author, but are intended for giving the reader some idea of what he or she is getting for his or her money. The personality of the author whether being of an honest or deceitful nature, the appearance of the author whether having a pretty or ugly face, does not influence our reviews. Only those pubications which are published by commercial publishers will receive comments concerning their physical presentation when necessary. In the case of these booklets, some comment about their physical presentation was therefore required since it was not of a standard that may be expected from a commercial publisher.

With respect to the statement, by the author of these reviewed booklets, of wasting a full page, it might be the opportune moment here to inform subscribers to *"The Braider"* that except for the cover sheet, any supplement and pages which do not display the headline *The Braider* together with a sequential page-number, both in *italic* typeface, do **not** form a part of this publication, and hence are not charged for. This is in stark contrast to some other Journals which contain large amounts of waffle in order to fill their pages. Any *"Editorial"* which is not associated with the subject matters discussed in such a Journal, certainly belongs to the waffle content.

The Scene behind the Stage

In this last issue of its first year's life, we are hopefully allowed to set aside a page in this quarterly Journal *The Braider* in which to give the reader a glimpse at what goes on in its set up and production.

No doubt, the reader will have noticed that *The Braider* differs greatly in many aspects from other Journals not only in the contents *level* of its articles but also in that the author of each article is not specifically mentioned. None of the authors derives a monetary gain from *The Braider*, and in fact each of them subsidizes it by means of their time, postage costs associated with their contributions (which are in many cases quite considerable) or equipment and consumables costs associated with its production. The copyright of each issue is vested **equally** in all its authors which are specified on the back of its cover-page, and they are **equally** the sole proprietors of this publication, provided that any such partaking author fully supports *The Braider* in case of conflict and immediately ceases to submit articles to a conflicting party. Any author who violates this condition will cease to be associated in any way with *The Braider*.

In contrast to the Pamphlet Series and the Book Series (now incorporated under the Pamphlet Series), each of the authors which partakes in *The Braider* contributes technical material such as articles, pictorial drawings and diagrams on a regular basis, although not necessarily in each issue.

A significant difference between *The Braider* and other Journals is its actual presentation in a loose-leaf format. This is done in order to facilitate the assembly of the successive issues into a proper book-form; it gives each reader the opportunity to do this in the by him or her preferred way. It should therefore again be mentioned, that a subscriber can receive *The Braider* in an unfolded condition in a large envelope for an additional charge of NZ$1.00 per issue.

Since we aim to create some general conformity in the layout of the various articles and successive issues, all articles are written in a form which is not person specific, and hence the reader should not try to guess the identity of the actual author of a particular article; in many cases such a guess could turn out to be incorrect.

In order to keep the production costs down, this quarterly will remain in black and white notwithstanding that the software used is able to fulfil any requirement for colour printing. The typesetting system which is being used is TEX (the 'X' in TEX is pronounced as the 'ch' sound in Scottish words like *loch*). It is the most sophisticated computer software typesetting system in existence for the production of technical literature. A manuscript prepared in TEX format can be transformed into pages whose typographic quality is comparable to that of the world's finest printers. Until now, our final output quality, especially with regards the diagrams, left much to be desired. The reason for this was that the masters were printed on a 24-pin dot matrix printer. Since we have now a laser printer, the output quality will be considerably better in future, in fact some of the masters used in this issue were printed on this new laser printer.

We hope that this brief look behind the scene will give you, dear reader, some insight into the workings involved with the production of that little Journal *The Braider* which you are reading now, and which we hope is worth its subscription.

THE BRAIDER

No.5 **FEBRUARY 1996.**

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A quarterly publication for the braiding artisan

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Solutions to the Questions in Issue No. 4

Since each issue of *The Braider* is written more than three months before its publication date, we are not able to acknowledge the solutions received from subscribers in the issue following the one in which the questions concerned did appear, hence in the issue in which the solutions to these questions are presented. Consequently the acknowledgement will take place in the subsequent issue.

The essential parts of the grid-diagrams in Figs. 57, 58, 59 and 60 are depicted below in Fig. 73.

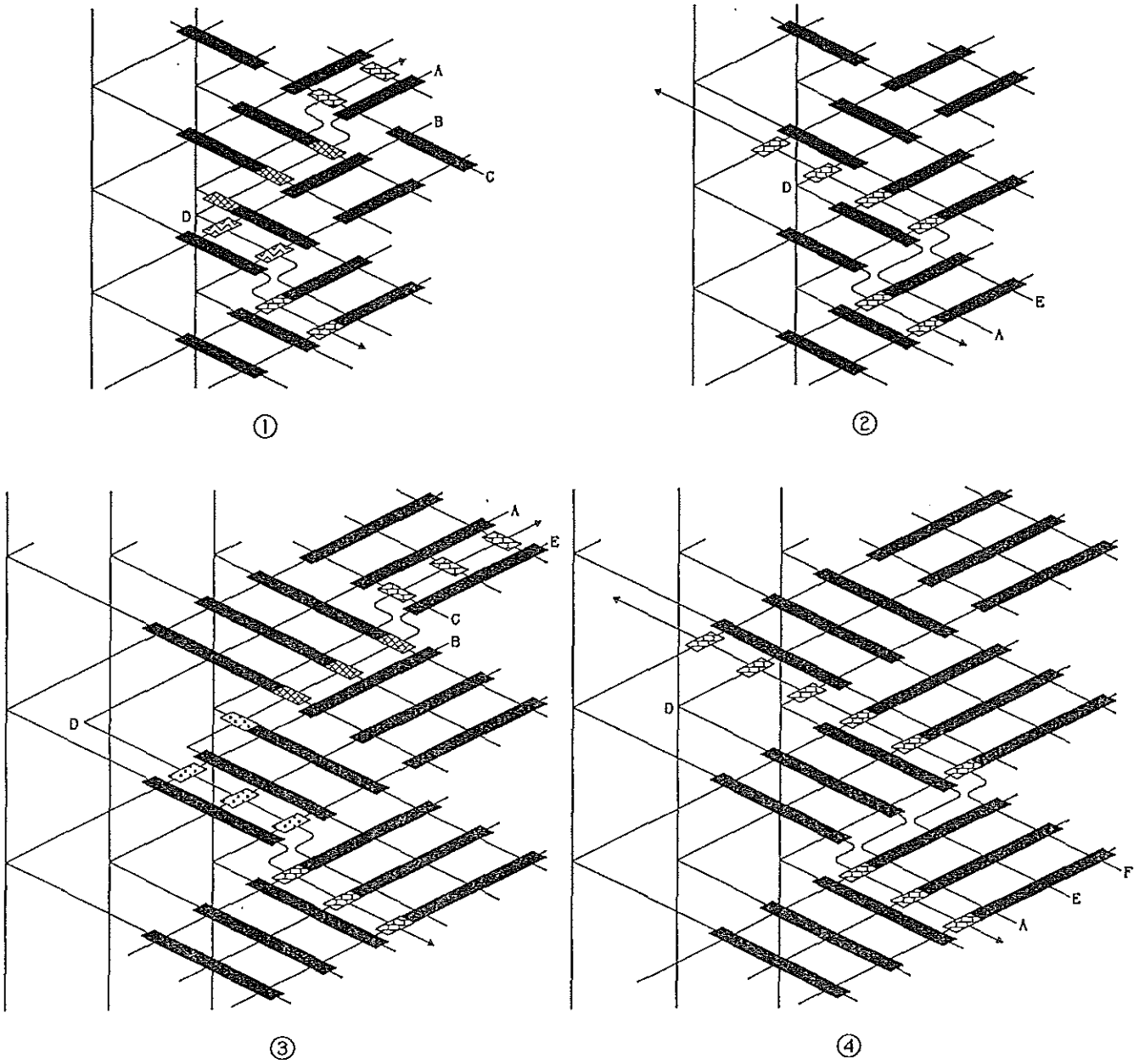


Fig. 73 — The essential parts of the methods depicted by the Figs. 57, 58, 59, 60.

In the method shown by Fig. 59 (diagram ③in Fig. 73), string **A** tends to slide along string **C** and under string **E**. This tendency of string **A** is encouraged by its six undercrossings before bight **D**. In the method shown by Fig. 57 (diagram ①in Fig. 73), string

A is prevented from sliding along string **C** by string **B** since string **A** goes locally from *under* to *over*, while string **B** goes from *over* to *under*. In the method shown by Fig. 58 (diagram ②in Fig. 73), string **A** takes over the place of string **E**; this occurs under the two *unders*. Two *unders* do normally not provide enough cover to hide this changeover completely, while three *unders* give sufficient cover for a complete concealment (the method shown by Fig. 60 (diagram ④in Fig. 73)).

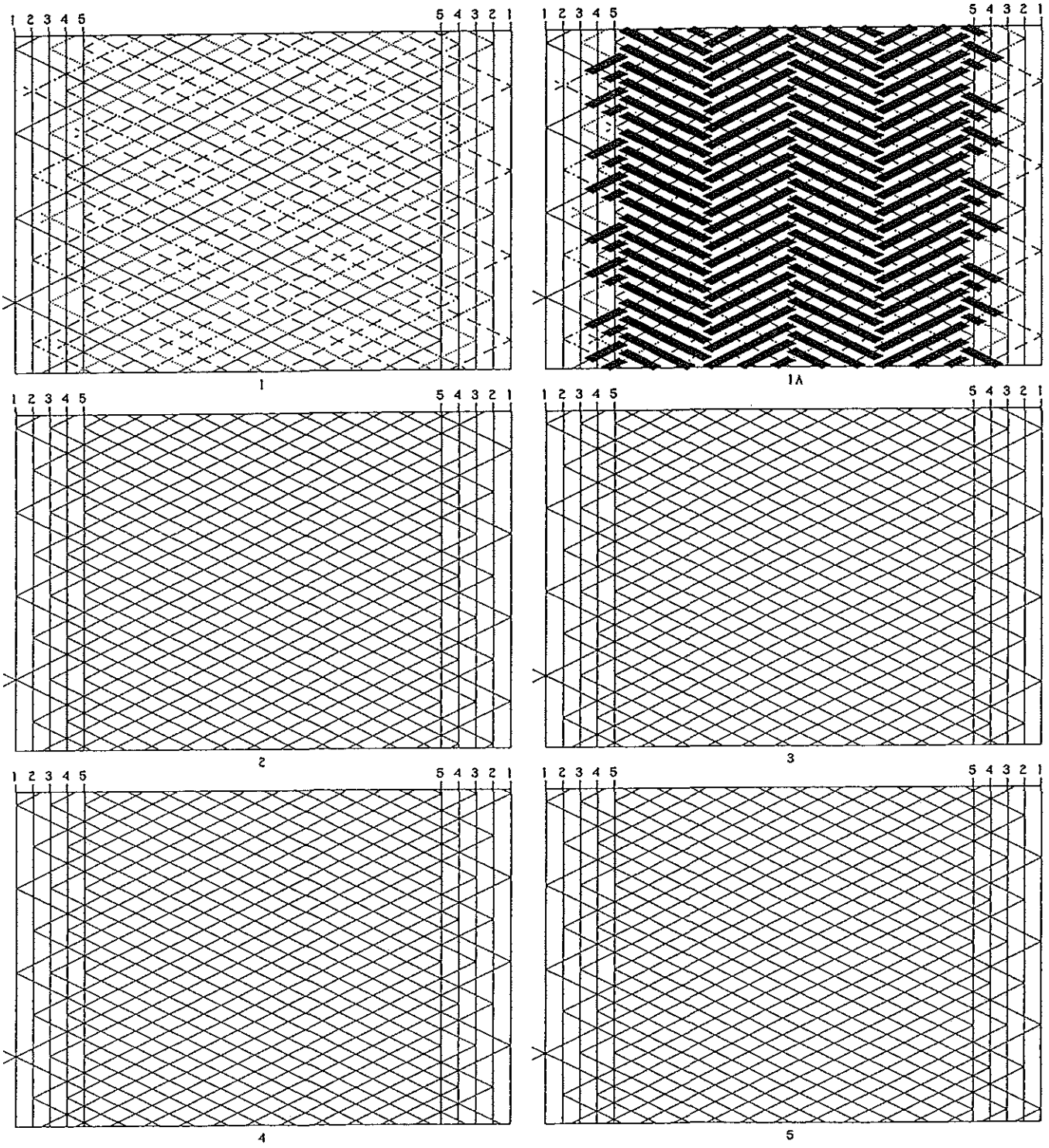


Fig. 74 — The 5-pass Hansen knots of type 1 and 1*.

The reader will no doubt have noticed that the first Hansen Knots of type 1 and

1* were of the 3-pass variety. It should therefore not be surprising that there are no type 1 and 1* Hansen knots with an even number of passes. Thus the next higher pass Hansen knots of type 1 and 1* are the 5-pass ones. The five different forms of these 5-pass Hansen knots are illustrated for a 25-parts/20-bights knot in Fig. 74 (diagrams 1, 2, 3, 4, 5); their coding is depicted in diagram 1*A*.

In general: let n be a positive integer, then there is only one $(2n+1)$-pass Hansen knot of type 1, while there are $2n$ type 1* $(2n+1)$-pass Hansen knots. The $(2n+1)$-pass type 1 Hansen knot is an interweave of $(2n+1)$ identical Casa knots with an **odd** number of parts.

Let m be a positive integer equal to or greater than 1, and let b be a positive integer such that b and $(2m+1)$ are coprime, then the number of parts P of a type 1 and a type 1* Hansen knot with $(2n+1)$ passes is equal to $P = (2n+1)(2m+1)$, while its number of bights B is equal to $B = (2n+1)b$. The relative position of the adjacent bight-boundaries on the left and right-hand side of the knot is such that consecutive bight-boundaries are one grid-column apart; in a $(2n+1)$-pass Hansen knot of type 1 and type 1* there are $(2n+1)$ adjacent bight-boundaries on the left-hand side as well as on the right-hand side of the knot. The column-distance between the innermost left and right-hand bight-boundaries is equal to $x = P - 2n$.

Another imitation three-ply Whippies knot

We mentioned in the previous issue that multi-ply knots or their 1-string imitation forms are only meant to be used for round, but not for a flat, braiding material. We also mentioned that their use for flat braiding material can only be attributed to the lack of braiding knowledge their users possess. This appears typically to be the case with whippies, many of which use multi-ply knots based on 4-bights Casa knots, with 3-parts or 5-parts, for covering their whip-handle knobs.

Since it is more difficult to cover a knob with an 1-string imitation 3-ply knot based on the 5-parts/4-bights Casa knot, the 1-string imitation knot is often encountered in the 4-ply form, which makes the intended result much easier to be achieved.

Just recently in *The Australian Whipmaker* (July 1995, number 39) appeared the description of an 1-string imitation 3-ply knot based on the 5-parts/4-bights Column-coded Regular Knot with the coding *o–u–2o* along the half-cycles from lower-left to upper-right. It is unfortunate that not only errors and misconceptions are apparently not being corrected in its articles, but that in this case an additional error has been introduced by the editor, which undoubtedly will confuse the novice especially even further. It is for this reason that we shall discuss this knot here in some detail.

Due to the two adjacent *overs* in the base knot, it is easy to achieve a good closure at end of the knob with the 1-string imitation 3-ply knot, which is based on the 5-parts/4-bights Column-coded Regular Knot with the above indicated coding. It should be noted that the base knot is a proper 5-parts/4-bights Column-coded Regular Knot and has no resemblance whatsoever to a 4-parts/4-bights Semi-Regular Knot; in fact it cannot even be raised from a smaller Column-coded Regular Knot if we exlude the "trivial" 2-parts/1-bight Regular Knot. Hence the reference to a 4-parts/4-bights (Semi-Regular) Knot should have been deleted. The two drawings of this 1-string imitation 3-ply knot,

apparently inserted in the article by the editor, are not compatible with each other. These two drawings are represented by the drawings A and B in Fig. 75.

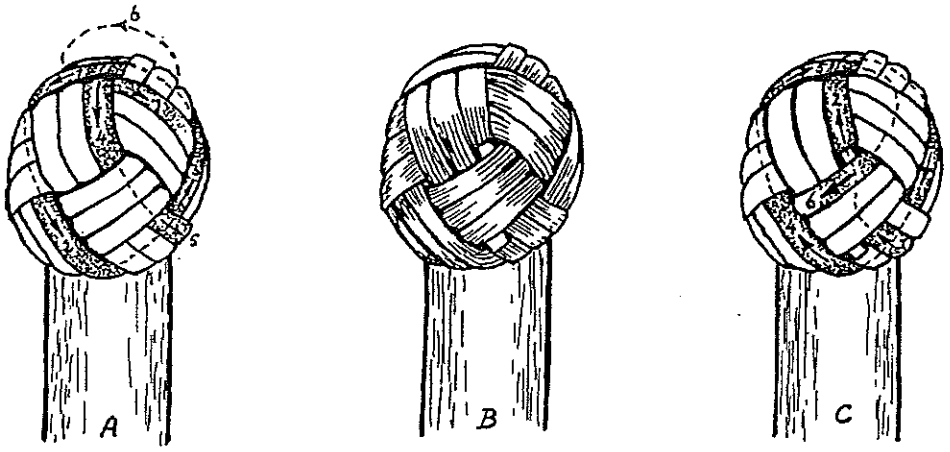


Fig. 75 — Another imitation 3-ply Whippies Knot.

Let's assume that the central drawing (B) in Fig. 75 depicts correctly the finished knot. Then obviously the string could not possibly have been laid down as indicated in the left-hand drawing (A), since it would be impossible to get the string-ends positioned as shown by the central drawing (B). The central drawing (B) clearly indicates, by the position of the string-ends, that the knot has been tied partly one way and partly the other way. It would be normal practice for the braiding-process to start at the mid-length position of the string-length and consequently we could (but don't have to) finish as shown. The braiding process which results in a knot as shown by the central drawing (B) in Fig. 75 is depicted in Fig. 76.

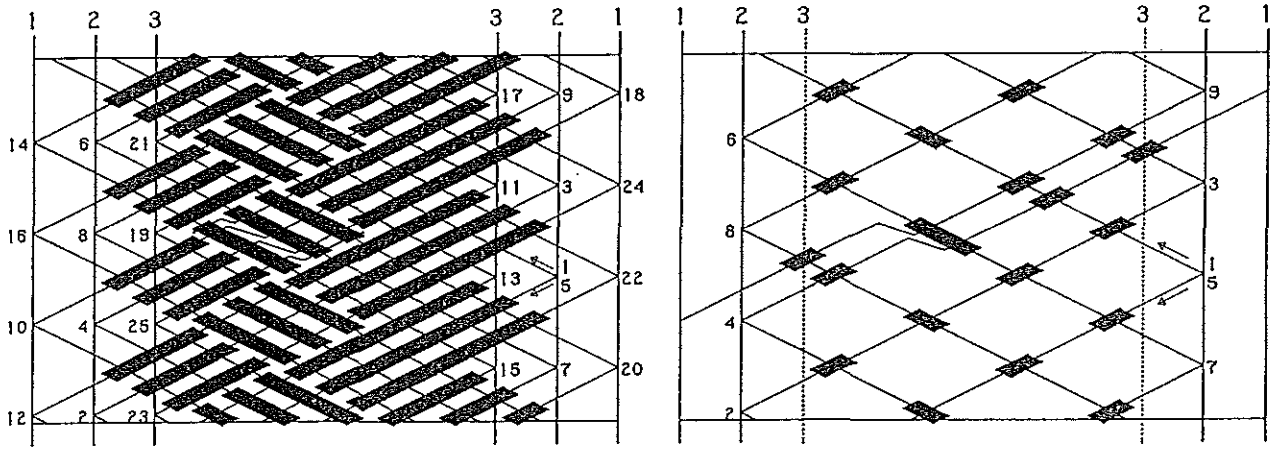


Fig. 76 — The braiding process which produces the knot shown by the central drawing (B) in Fig. 75.

In this process the mid-length position of the string is placed at the bight-point 1, 5 on the right-hand bight-boundary 2. Each half of the string-length is then braided in the direction indicated by its associated arrow. The right-hand drawing (C) in Fig. 75 gives part of the correct string path associated with the cental drawing (B) in this Figure.

The above described braiding method is not the best one for this knot. Instead of

finishing on either side of three *unders*, it is of course much better to finsh at either side of six *unders*. This can be done in two ways as depicted by the diagrams in Fig. 77.

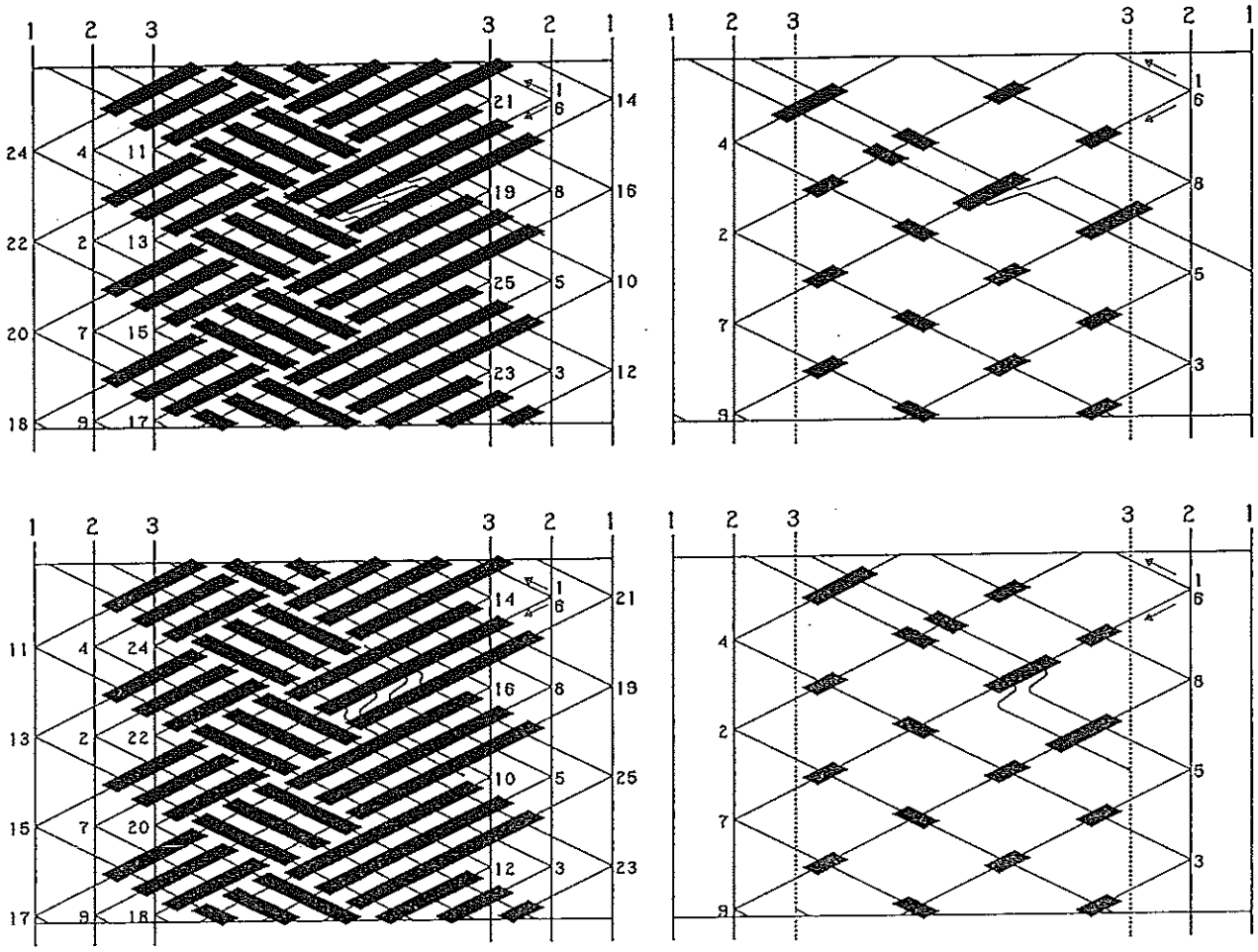


Fig. 77 — The braiding processes for ending at either side of six *unders*.

In both methods, the mid-length position of the string is placed in bight-point 1,6 on the right-hand bight-boundary 2. The method shown in the lower two diagrams of Fig. 77 is to be preferred since it makes working the ends away, so that they cannot be seen, easier.

For those readers who have read the article concerned in *The Australian Whipmaker* it should be mentioned here that the so called "15 bight version of the above knot" is not a version of the knot referred to; it is a totally different Column-coded Regular Knot of 16-parts/15-bights, and is not related in any way to the above discussed Column-coded Regular Knot of 5parts/4-bights. Furthermore it is a knot without any merit due to its lopsided coding (refer to "The Braider's Notebook" in issue 3).

Braiding Instructions and their Presentation

In the previous issue we did encounter the cyclic nature enshrined in the string-run

of a Regular Knot, due to its placement on a cylindrical surface. The cyclic pattern was determined by its number of bights along one of the bight-edges of the knot, and for this reason it is necessary and sufficient to specify the circumferencial size of a Regular Knot by the number of **bights** b along only one of its two bight-edges.

Within the overall cycle of these b bights there is, as we have seen, another cyclic behaviour of the string-run due to its zigzagging between the two parallel bight-edges. We saw that the height of this zigzagging (the distance between the two parallel bight-edges which is specified by the **parts** p of the Regular Knot), within the contraint of the overall cycle of b bights, determined the sequential order in which the string passed through the b bight-points on one of the bight-edges. Finally we saw how all this behaviour could easily be quantified by means of **modular arithmetic**. The **modulus** in this case, determined by the overall cyclic behaviour, was therefore equal to b.

All this gives us automatically **The Law of the Common Divisor** for Regular Knots since, before returning to the bight-point from which the string-run started, the string must have passed through all the other $(b-1)$ bight-points on the bight-edge of its starting-point (automatically it will then also have past through all the bight-points on the other edge); this is again best to be seen from the modular arithmetic involved as follows:

Say the string-run starts at bight-point 0 (bight-index number I_0). After having travelled $2x$ half-cycles (hence x cycles or zigzags) the string arrives in bight-point $|xp|_b$ (bight-index number I_x). In order for the string to return to its starting-point (bight-point 0), the value of $|xp|_b$ must be 0. This should however only happen when $2x = 2b$, hence when $x = b$. Consequently the values for p and b must be coprime (have no divisor other than 1).

$\star\star$ *Say that p and b have a common divisor other than 1. Let their greatest common divisor (g.c.d.) be y. How many strings are required to braid this knot with p-parts and b-bights? How many interwoven knots do we have and what are the parts and bights of each of these knots?*[†]

So far we have seen some general relationships concerning the bight-points on the string-run of a Regular Knot. Now let's have a look at what can be said about the crossing-points on a half-cycle while being laid down. We shall consider the general case[‡]:

Say we lay down, from lower-right to upper-left, the half-cycle which is associated with the right-hand **bight-number** i, hence half-cycle $(2i+2)$. This half-cycle, which will begin in the right-hand bight-point with **bight-index number** $I_i = |ip|_b$, will cross a previous laid down half-cycle $2(j-1)+3 = 2j+1$, associated with the left-hand **bight-number** $(j-1)$, running from lower-left to upper-right. This half-cycle, which did start at the left-hand bight-point with **bight-index number** $I_j = |[(j-1)+1]p|_b = |jp|_b$, will end at the right-hand bight-point with **bight-index number** $I_j = |jp|_b$ (see upper-left diagram in Fig. 78). This right-hand bight-index number I_j lies $|I_j - I_i|_b = |jp - ip|_b = |(j-i)p|_b$ bights (hence $2|(j-i)p|_b$ rows) above the right-hand bight-index

[†] If the reader finds working with the variables p, b, y difficult, then first take some numerical values for these variables.

[‡] Readers who find it difficult to work with the variables p, b, i, j should first take some numerical values for these variables. They should furthermore consult pp. 80–82, and especially Fig. 72 in issue No. 4.

number I_i. Consequently half-cycle $(2i+2)$ crosses half-cycle $(2j+1)$ at $|(j-i)p|_b+nb$ columns left of the right-hand bight-boundary ($n \geq 0$ such that $0 < |(j-i)p|_b+nb < p$).

To braid this half-cycle $(2i+2)$, which runs from lower-right to upper-left, we must know all its crossing-points with all the already laid down half-cycles which run from lower-left to upper-right in order to know whether a particular crossing is an *over* or an *under*. Hence if we know the columns on which these crossing-points lie, then we can assign to each of these crossings its proper crossing movement (*over* or *under*). All the already laid down half-cycles which run from lower-left to upper-right are associated with j-values which range from 0 to i $(0 \leq j \leq i)$, and hence the the value of $(j-i)$ ranges from $-i$ to 0 $(-i \leq j-i \leq 0)$.

Note that the position of the crossings which a half-cycle, running from lower-right to upper-left, makes with the already laid down half-cycles from lower-left to upper-right is measured by their column-distances from the right-hand bight-edge. Because of this the columns are numbered from right to left for the half-cycles which run from lower-right to upper-left.

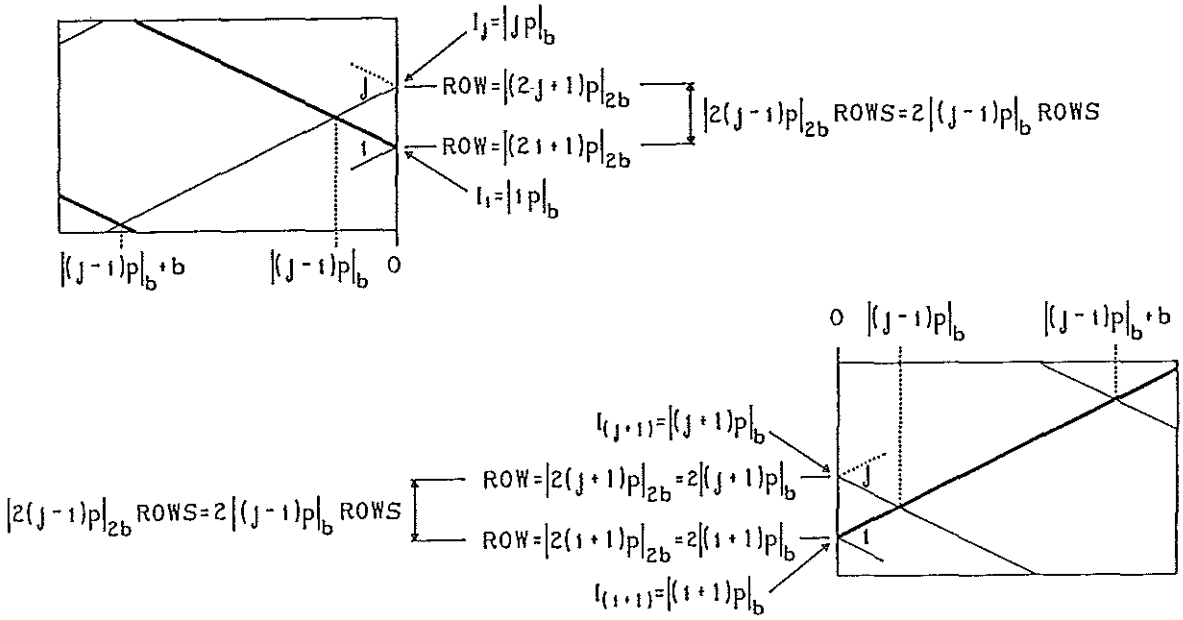


Fig. 78 — Determination of the positions of the crossing-points between half-cycles.

Say we lay down, from lower-left to upper-right, the half-cycle which is associated with the left-hand **bight-number** i, hence half-cycle $(2i+3)$. This half-cycle, which will begin in the left-hand bight-point with **bight-index number** $I_{i+1} = |(i+1)p|_b$, will cross a previous laid down half-cycle $(2j+2)$, associated with the right-hand **bight-number** j, running from lower-right to upper-left. This half-cycle, which did start at the right-hand bight-point with **bight-index number** $I_j = |jp|_b$, will end at the left-hand bight-point with **bight-index number** $I_{j+1} = |(j+1)p|_b$ (see lower-right diagram in Fig. 78). This left-hand bight-index number I_{j+1} lies $|I_{j+1} - I_{i+1}|_b = |(j+1)p-(i+1)p|_b = |(j-i)p|_b$ bights (hence $2|(j-i)p|_b$ rows) above the left-hand bight-index number I_{i+1}. Consequently half-cycle $(2i+3)$ crosses half-cycle $(2j+2)$ at $|(j-i)p|_b + nb$ columns right of the left-hand bight-boundary ($n \geq 0$ such that $0 < |(j-i)p|_b + nb < p$).

To braid this half-cycle $(2i+3)$, which runs from lower-left to upper-right, we must know all its crossing-points with all the already laid down half-cycles which run from

lower-right to upper-left in order to know whether a particular crossing is an *over* or an *under*. Hence if we know the columns on which these crossing-points lie, then we can assign to each of these crossings its proper crossing movement (*over* or *under*). All the already laid down half-cycles which run from lower-right to upper-left are associated with j-values which range from 0 to i $(0 \leq j \leq i)$, and hence the the value of $(j-i)$ ranges from $-i$ to 0 $(-i \leq j-i \leq 0)$.

Note that the position of the crossings which a half-cycle, running from lower-left to upper-right, makes with the already laid down half-cycles from lower-right to upper-left is measured by their column-distances from the left-hand bight-edge. Because of this the columns are numbered from left to right for the half-cycles which run from lower-left to upper-right.

Note also that the formulae which express the column distances of the crossing points associated with the half-cycles $(2i+2)$ and $(2i+3)$ to the respective bight-edges of their starting-points are identical. Hence the column-distances to the right-hand bight-edge of crossing-points on half-cycle $(2i+2)$ are the same as the column-distances to the left-hand bight-edge of crossing-points on half-cycle $(2i+3)$.

Let's now have a look at two intersecting[†] half-cycles for which $j-i=-1$. Their associated bight-points on the bight-edge from which the half-cycle associated with bight-number i leaves, are consecutive bights of the string-run[‡] on this edge. Their first intersection-point is at a distance of $|-p|_b$ columns from the bight-edge on which the starting-point of the half-cycle associated with bight-number i lies. Their second intersection-point is a further distance of b columns from this bight-edge, and so on. Through the term $|-p|_b$ we encounter another cyclic relationship within the overall cycle b, this time involving intersection-columns.

With the above information, we are now able to draw up a scheme from which we can directly read the consecutive intersection-column order, and hence the actual braiding sequence of a half-cycle.

Set off on a horizontal line b bight-points, and mark the leftmost one 0. Calculate the value of $|-p|_b$. Count from bight-point 0 a further $|-p|_b$ bight-points to the right in cyclic fashion[††], and mark the final bight-point 1. Next count from bight-point 1 a further $|-p|_b$ bight-points to the right in cyclic fashion, and mark the final bight-point 2. Keep repeating this process till all bight-points have received a number.

Note that a point which receives the number k is associated with $j-i=-k$. This point is at $|-kp|_b$ points distance from point 0. Recall that the column-distance of the intersection-column, belonging to the first intersection-point between the half-cycles $(2i+2)$ and $(2j+1)$ for which $j-i=-k$, to the bight-boundary from which half-cycle $(2i+2)$ starts, is equal to $|-kp|_b$. Furthermore recall that in order to obtain all the intersection-column distances associated with the first intersection-points on half-cycle $(2i+2)$, we must take for $-k$ the values $-i \leq -k \leq 0$, hence $0 \leq k \leq i$.

---

† Although the two sets of opposing half-cycles do not intersect but rather cross each other, we shall for convenience in future refer to intersecting half-cycles, intersection-points, and intersection columns in lieu of crossing half-cycles, crossing-points, and crossing-columns.

‡ Be careful not to confuse these bights with adjacent bights on the bight-edge. The bights we are here referring to are $|p|_b$ bights apart, and hence are generally not adjacent.

†† In 'cyclic fashion' means that after reaching the rightmost bight-point on the line of bight-points, we **continue** the counting process with the leftmost bight-point.

Example:

Let $b = 5$, and $p = 7$. Then $|-p|_b = |-7|_5 = 3$.

Set off the $b = 5$ bight-points on a horizontal line; mark the leftmost bight-point 0.



Count from bight-point 0 a further $|-p|_b = 3$ bight-points to the right in cyclic fashion, and mark the final bight-point 1.



Count from bight-point 1 a further $|-p|_b = 3$ bight-points to the right in cyclic fashion, and mark the final bight-point 2.



Count from bight-point 2 a further $|-p|_b = 3$ bight-points to the right in cyclic fashion, and mark the final bight-point 3.



Count from bight-point 3 a further $|-p|_b = 3$ bight-points to the right in cyclic fashion, and mark the final bight-point 4.



All the $b = 5$ bight-points have now received a number. We call this scheme **the complementary cyclic bight-number scheme** of the 7-parts/5-bights Regular Knot. With a little contemplation most readers will realize that the numbers are of course the **bight-numbers**, but it are **not** the bight-numbers which belong to the bights (see Fig. 72, issue No. 4, leftmost two bottom diagrams). Nevertheless they are related to the bight-numbers which belong to the bights on the right-hand bight-edge of the knot, which have a scheme as follows:

This scheme, called **the cyclic bight-number scheme**, is obtained in a similar way to that of the complementary cyclic bight-number scheme, but instead of using $|-p|_b = 3$, we use $|p|_b = |7|_5 = 2$. It should now be obvious why we called our earlier discussed scheme the *complementary* cyclic bight-number scheme: each of its numerical values is the *complement* (to $b = 5$) of the corresponding values in the cyclic bight-number scheme.

Since the bight-points in these schemes are the adjacent bights of the knot, what do these schemes tell us?

The cyclic bight-number scheme gives us the order in which the right-hand bights of the knot are traversed by the string-run. Hence first the bight associated with the number 0, then the bight associated with the number 1, then the bight associated with the number 2, then the bight associated with the number 3, and finally the bight associated with the number 4. It is thus clear that these numbers are the bight-numbers belonging to the right-hand bights which are represented by the points (see Fig. 72, issue No. 4).

The complementary cyclic bight-number scheme gives us the sequential columns which are intersection-columns for the half-cycle leaving the bight associated with a certain bight-number. In order to make reading these sequential columns easier, we set

this complementary cyclic bight-number scheme off along the columns. Hence we start by setting off on a horizontal line $(p+1)$ points which represent the two bight-boundaries and the $(p-1)$ columns between them. Thus in our case we set off $p+1=7+1=8$ points.

. . . . . .

The leftmost point represents the left-hand bight-boundary, while the rightmost point represents the right-hand bight-boundary. Above these points we set off from left to right our complementary cyclic bight-number scheme and repeat this scheme as often as required till all the points, except the rightmost one, have received a number. Below the points we set off from right to left our complementary cyclic bight-number scheme and repeat this scheme as often as required till all the points, except the leftmost one, have received a number.

```
0 2 4 1 3 0 2
· · · · · · · ·
  2 0 3 1 4 2 0
```

Next we replace the points by the coding of their columns; the leftmost and the rightmost point are of course not replaced since they represent respectively the left-hand and right-hand bight-boundary. Let the coding of the Regular Knot in our example be that of the Casa knot depicted in Fig. 72, issue No. 4.

```
0 2 4 1 3 0 2
· \ / \ / \ / ·
  2 0 3 1 4 2 0
```

The top-line is associated with the half-cycles which run from lower-left to upper-right, and hence should be read from left to right. The bottom-line is associated with the half-cycles which run from lower-right to upper-left, and hence should be read from right to left.

```
  ⟶
0 2 4 1 3 0 2
· \ / \ / \ / ·
  2 0 3 1 4 2 0
            ⟵
```

Say we want to lay down half-cycle 4. We know that this half-cycle runs from lower right to upper left since it is an **even** numbered half-cycle. Furthermore we know that its associated bight-number is equal to $\frac{4-2}{2}=1$. Thus we read the lower numbers from right to left which are equal or smaller than 1. Hence we read: $u-u$, which is $2u$.

Say we want to lay down half-cycle 7. We know that this half-cycle runs from lower left to upper right since it is an **odd** numbered half-cycle. Furthermore we know that its associated bight-number is equal to $\frac{7-3}{2}=2$. Thus we read the upper numbers from left to right which are equal or smaller than 2. Hence we read: $u-u-u-o$, which is $3u-o$.

We call the scheme

```
  ⟶
0 2 4 1 3 0 2
· \ / \ / \ / ·
  2 0 3 1 4 2 0
            ⟵
```

the algorithm diagram of the 7-parts/5-bights Casa knot in question.

Algorithm diagrams enable us to read the half-cycle braiding sequences readily. They are easy to construct and they eliminate the often lengthy half-cycle algorithm tables.

Here we showed their use with a Column-coded Regular Knot, but they can be adapted for Row-coded Regular Knots, and in fact for several other knots. In the next issue we shall show a few more things these algorithm diagrams provide us with, but for the time being, the reader should have enough material to digest. Some readers will find it all very simple and others might find it difficult, nevertheless, in order to understand braiding processes and consequently be able to obtain some practical advantages, it is most important that the reader studies the foregoing thoroughly. This might take some time of course, but one should try to visualize the actual braiding processes involved, then gradually on all will fall in place and become obvious. If there are any remaining problems, let us know, and we will try to help you in overcoming them in a subsequent issue.

Although much can be done with grid-diagrams, there are times when the use of some mathematics is necessary in order to formulate relationships which will aid us in practice. Again some readers might initially have some difficulties, but with a little perseverance those difficulties should evaporate.

Scenes from the Knot Universe

Under this heading we shall have a look at some of the many scenes from the Universe of Knots a braider will from time to time encounter. In this first article we will look at some **open-enders**. Open-enders are knots in which the ends of the strings are used in the construction of further parts of the article they form a component of.

The open-enders we shall discuss are based on three Casa-coded Regular Cylindrical Braids. They have respectively 3-parts/2-bights, 3-parts/3-bights, and 3-parts/4-bights (see Fig. 79).

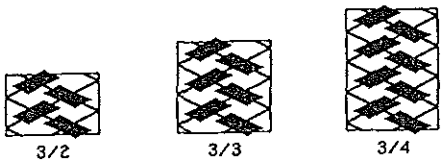


Fig. 79 — The three Casa-coded Regular Cylindrical Braids.

The 2-string open-ender, based on the 3-parts/2-bights Casa-coded Regular Cylindrical Braid, is often used as a **bend**, and in that capacity known as **the Ashley bend**. It is also used as a lanyard knot, normally in its two-ply form.

The 4-string open-ender which is based on the 3-parts/4-bights Casa-coded Regular Cylindrical Braid is commonly used as **Fiador knot** in the **Fiador**. The fiador serves as a throat-latch in the **Hackamore**, a "bridle system" used for horses.

The 2-string open-ender based on the 3-parts/4-bights Casa-coded Regular Cylindrical Braid is used as a lanyard knot, often in its two-ply form.

The 3-string open-ender based on the 3-parts/3-bights Casa-coded Regular Cylindrical Braid is commonly known as **the Footrope knot**.

Since any single-ply n-string (with $n \geq 2$) open-ender, which is based on a 3-parts/n-bights Casa-coded Regular Cylindrical Braid, consists of a **Crown knot** followed by a **Wall knot** under it with its working-ends fed through the eye of the Crown knot, we shall first have a look at Crown knots and Wall knots.

The Crown knot and the Wall knot are described in nearly every book on knots, and consequently every knot-tyer knows how they are tied. Unfortunately their relationship is nearly always described in the same ambiguous manner, for example, Ashley states in his book (*The Ashley Book of Knots*) under #671 (pg. 116) that the Wall knot is the exact reverse of the Crown knot; if either of these knots is turned upside down it becomes the other knot. It will be obvious that the second part of his statement is not compatible with the first part, since by turning a knot physically upside down does not change the knot at all. But also the first part of the statement is incorrect. It is again only the grid-diagram which will be able to tell us what their relationship really is.

The two leftmost diagrams in Fig. 80 depict 4-string Crown knots; the uppermost depicts Crowning to the **right**, and the lowermost depicts Crowning to the **left**. The central two diagrams depict 4-string Wall knots; the uppermost depicts Walling to the **left**, and the lowermost depicts Walling to the **right** (the two uppermost right-hand diagrams are equivalent (one is turned upside down relative to the other), and the two lowermost right-hand diagrams are equivalent (one is turned upside down relative to the other); the equivalency is indicated by the equivalent sign $\equiv$).

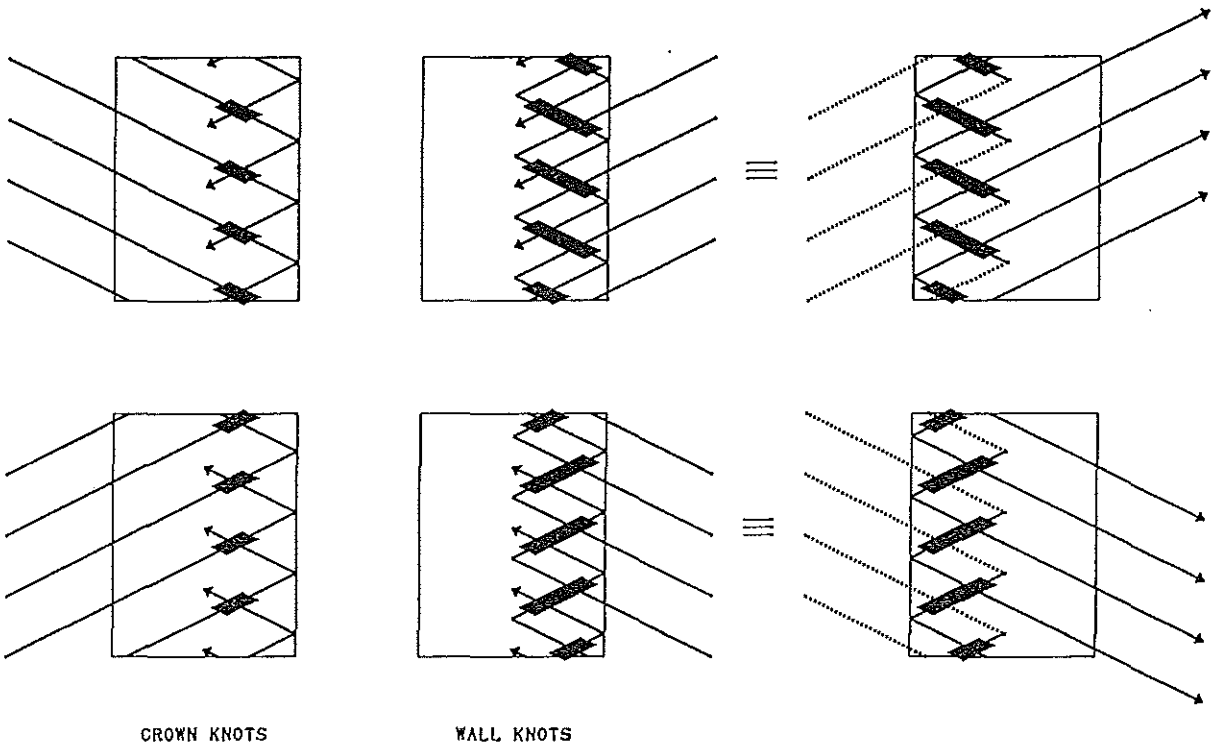


Fig. 80 — The relationship between Crown knots and Wall knots.

If we feed the standing-ends of the depicted Crown knots through the eye of their Crown, we obtain the depicted Wall knots. Hence:

A Wall knot is a Crown knot in which the standing-ends are fed through the eye of its Crown. A Crown knot with Crowning to the right gives then a Wall knot with Walling to the left, and a Crown knot with Crowning to the left gives then a Wall knot with Walling to the right.

We see that the Wall knot is **not** the reverse (Ashley) nor the converse (Curus L. Day) of the Crown knot, neither is it a turned upside down (Ashley) Crown knot. The reader may wonder how they came to their erroneous statements, and why similar statements may be found in other books on knots. The reason is a rather simple one: it is namely quite common that two totally different entities are mixed up; these entities being **the knots** and **the construction methods.** The **construction method** for the Wall knot is the **converse** of the construction method for the Crown knot. It is, however, incorrect to say that the construction method for the Wall knot is the reverse of that for the Crown knot. Nevertheless, we can see what Ashley meant by his statements. One should be aware of the pitfalls associated with mixing up **knot structures** and **knot construction methods.**

If we make a Crown knot with Crowning to the right, and then a Wall knot under it with Walling to the right, and then feed the working-ends through the eye of the Crown, we obtain an **open-ender** based on a 3-parts Casa-coded Regular Cylindrical Braid. This process has been depicted in Fig. 81 for two strings, three strings, and four strings (for convenience the standing-end of the wall knots is not depicted in the Wall knot diagrams).

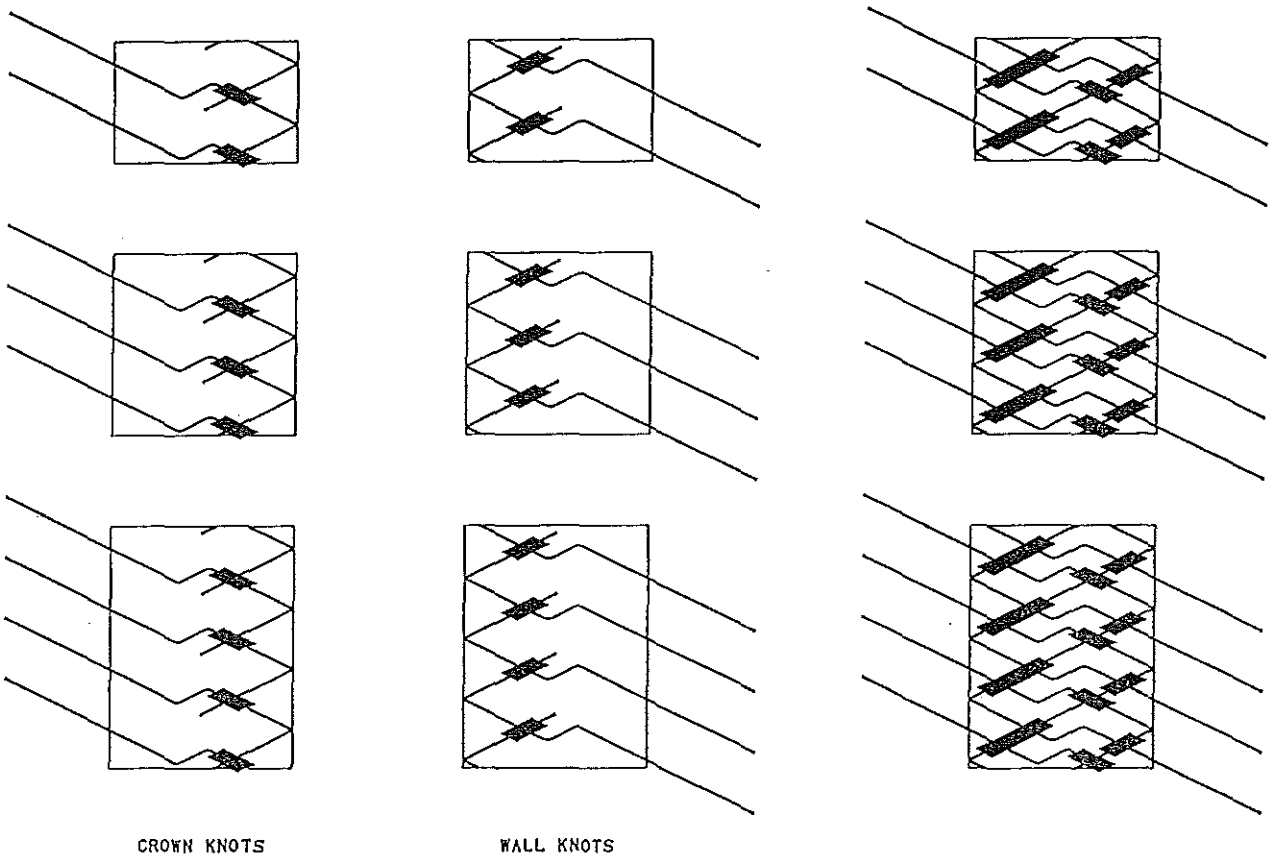


Fig. 81 — 3-parts Casa-coded Regular Cylindrical Braids as open-enders. Standing-ends at left-hand side; working-ends at right-hand side.

We could of course have made the Crown knot with Crowning to the left, and the Wall knot with Walling to the left, but it is more common to perform these processes to the right (more suitable for the common right-handed rope).

Without changing the positions of the string-ends, these knots are readily 'rolled over' into their **reversed** form. In this process we roll the right-hand bights over to the left (only the Crown can be 'rolled over'). The knots obtained are shown by the grid-diagrams in Fig. 82.

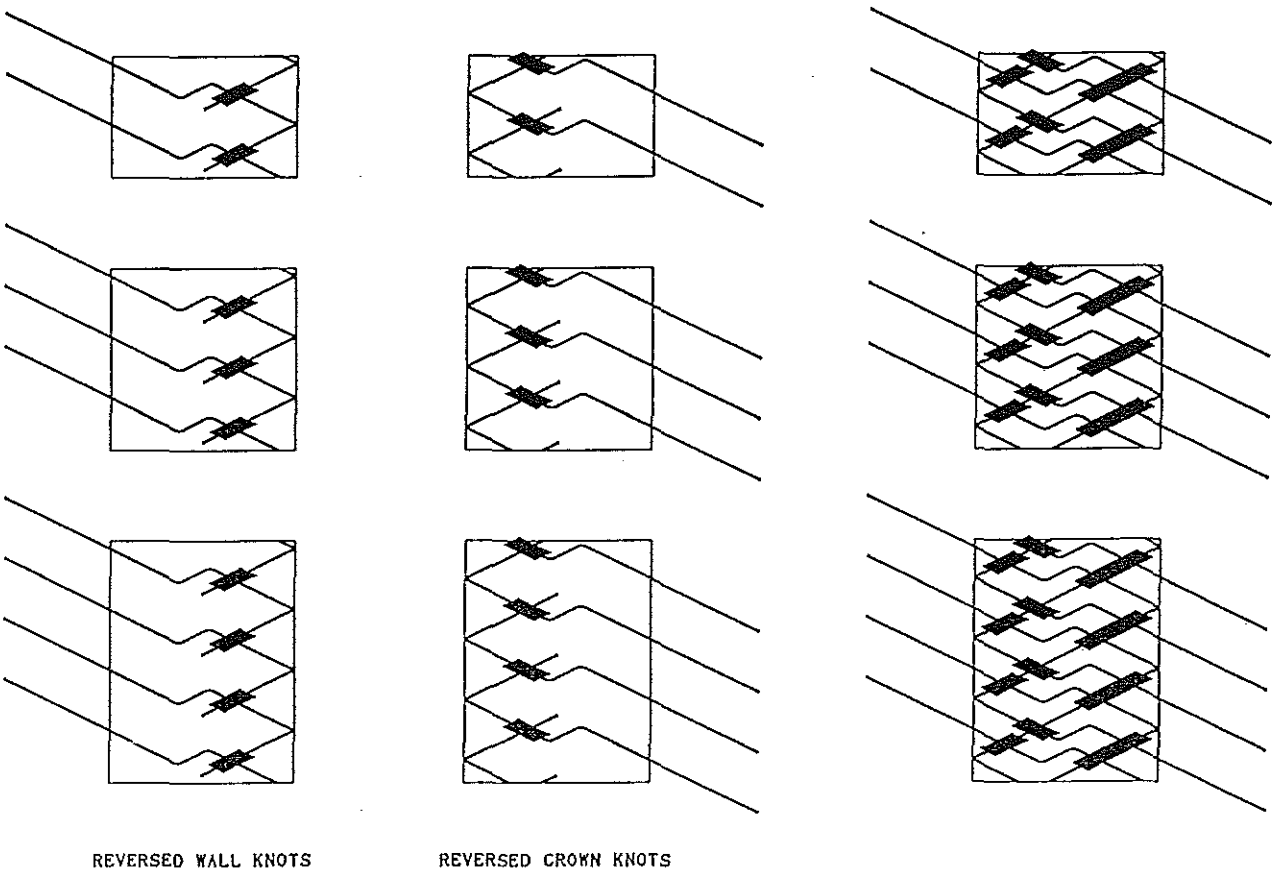


Fig. 82 — The 'rolled over' knots of Fig. 81.
Standing-ends at left-hand side; working-ends at right-hand side.

It is important to note that in these 'rolled over' knots the position of the string-ends has not changed, and hence their standing-ends (left-hand side) and working-ends (right-hand side) have not changed from those in Fig. 81. We cannot stress this important fact enough since similar diagrams are obtained by turning the diagrams in Fig. 81 upside down, however the position of the string-ends has then changed, and consequently their standing-ends and working-ends are then interchanged.

The 2-ply form of the right-hand knots in Fig. 81 are depicted in Fig. 83, and the 2-ply form of the right-hand knots in Fig. 82 are shown in Fig. 84.

The 2-string open-ender based on the 3-parts/4-bights Casa-coded Regular Cylindrical Braid we mentioned at the beginning is depicted by the left-hand grid-diagram in Fig. 85; the right-hand grid-diagram in this Fig. shows its 2-ply form. Note that the important aspect of symmetry is maintained in each of these knots.

This single-ply 2-string knot is often employed as a lanyard knot, unfortunately in the incorrect way. Ashley describes its incorrect use in his book under #787, pg. 141; the necklace part should be formed from the working-ends and not from the standing-ends. By using the loop (the standing-ends) as a necklace, its strings tend to spread the knot.

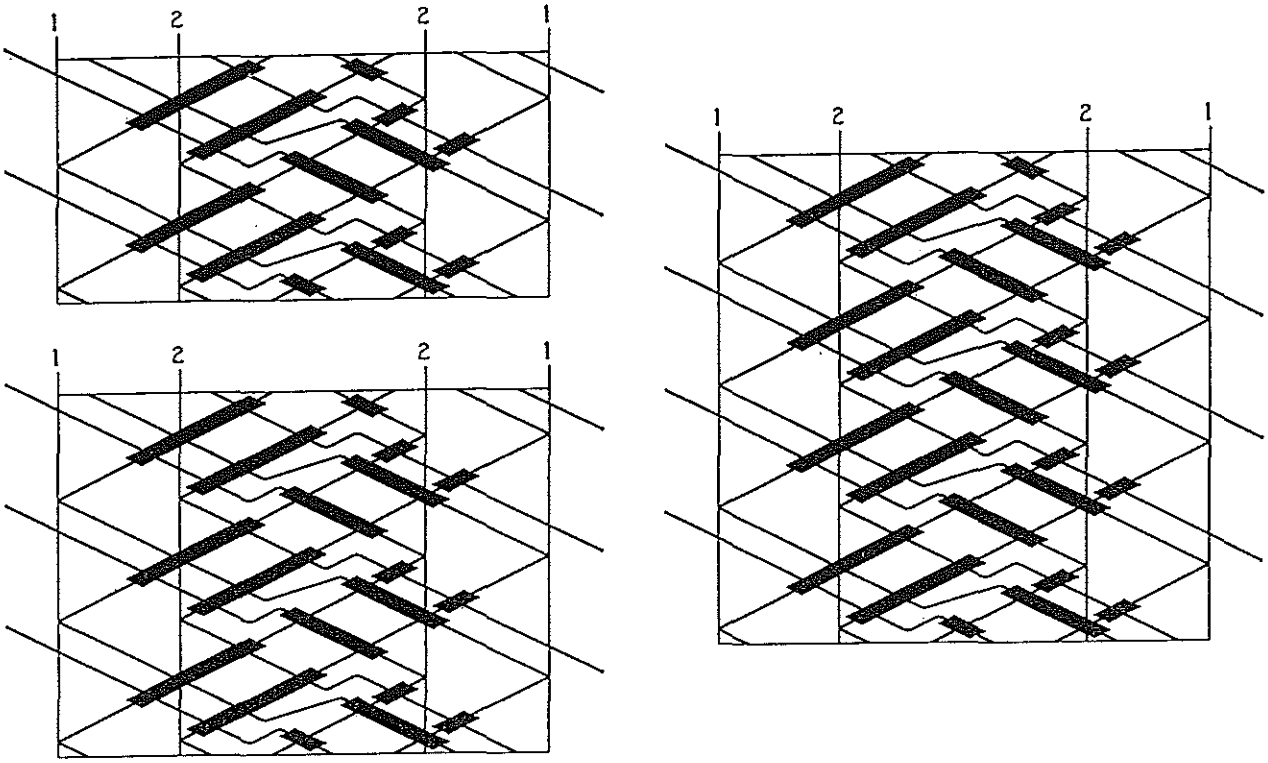


Fig. 83 — The 2-ply form of the knots in Fig. 81.
Standing-ends at left-hand side; working-ends at right-hand side.

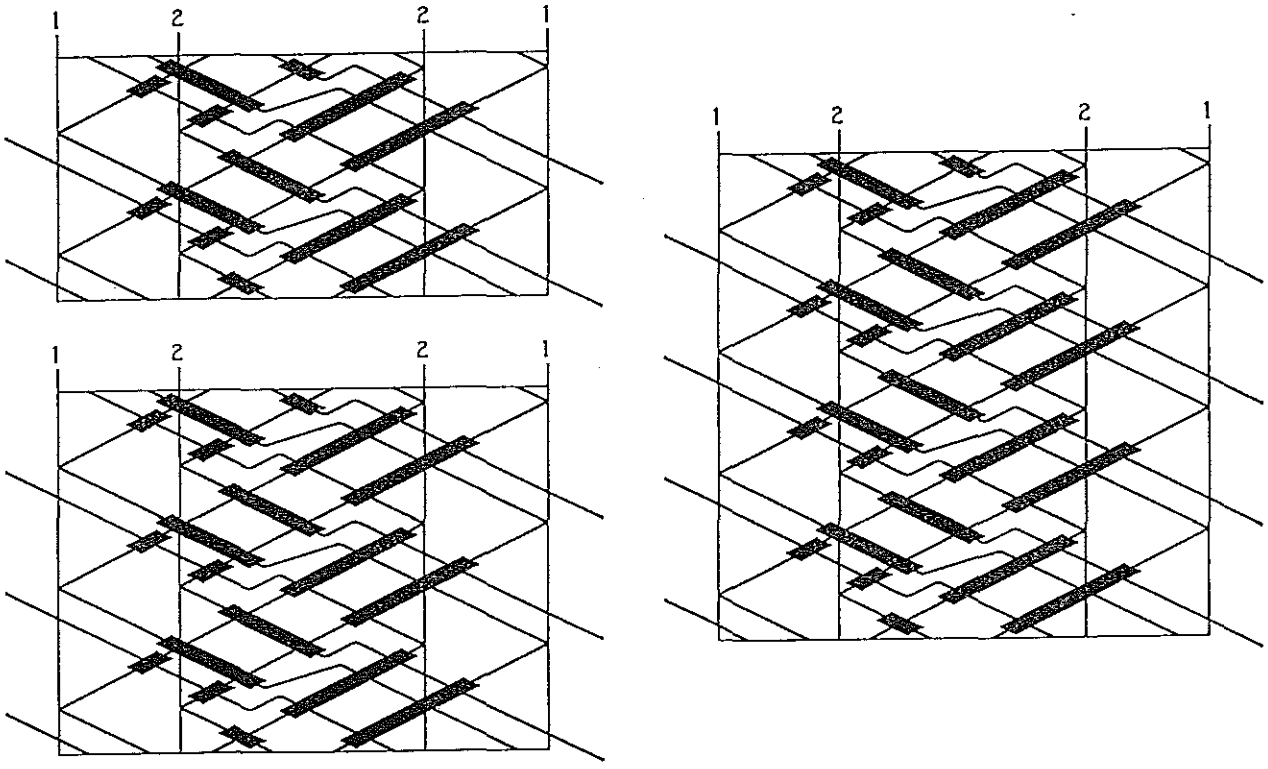


Fig. 84 — The 2-ply form of the knots in Fig. 82.
Standing-ends at left-hand side; working-ends at right-hand side.

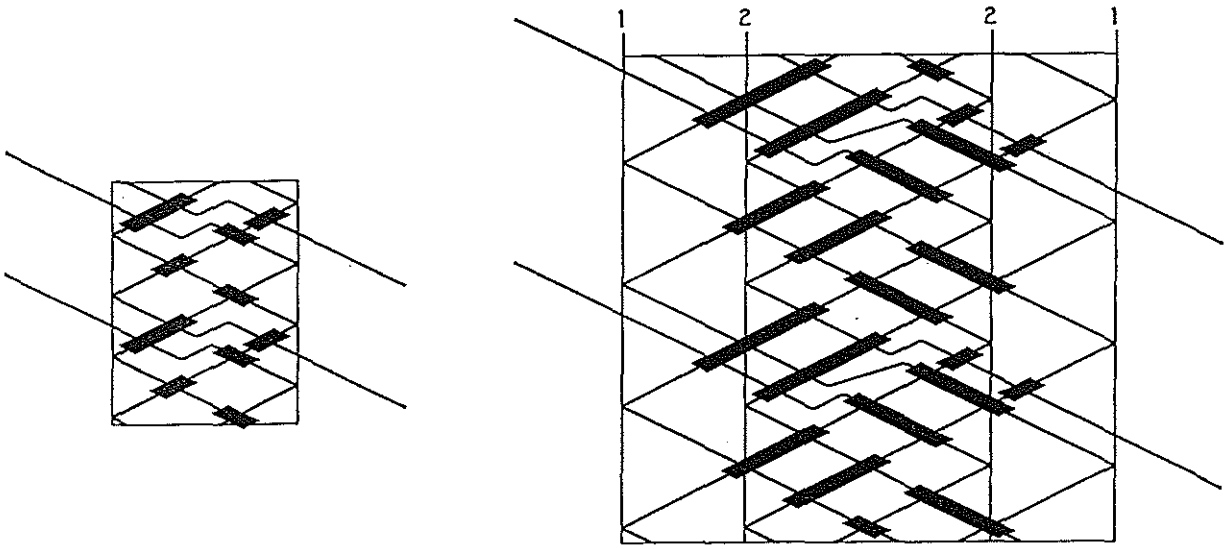


Fig. 85 — The 2-string open-ender based on the 3-parts/4-bights Casa-coded Regular Cylindrical Braid. Single-ply form at left; 2-ply form at right. Standing-ends at the left-hand side of the diagrams.

We mentioned at the beginning that the single-ply 2-string open-ender, based on the 3-parts/2-bights Casa-coded Regular Cylindrical Braid, is often used as a **bend**, and in that capacity known as **the Ashley bend**. It is depicted by the uppermost right-hand diagram in Fig. 81. When the two standing-ends are pulled apart, the **Ashley bend** (named as such by Curus L. Day) is obtained. Ashley describes this bend in his book under #1452, pg. 264, but fails to associate it with the knot under #783, pg. 141. This bend can be tied in several different ways, but it appears that it has not been recognised for what it really is. This is not only testified by the lack of reference to its actual form, but also by the way it has been depicted in the knotting literature. When this bend has been tightened properly, its appearance is as depicted in Fig. 86, and its superior qualities should become apparent.

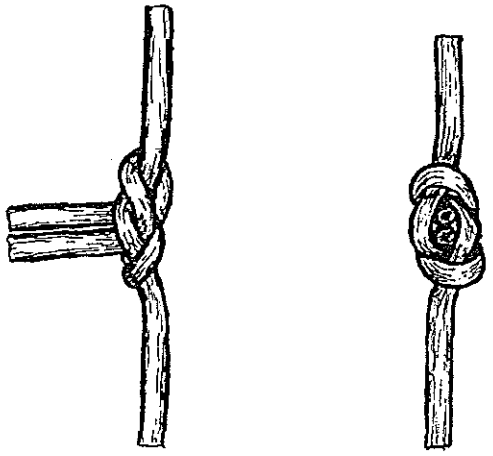


Fig. 86 — The Ashley bend.

There is no doubt that this bend is considerably older than claimed. There is also no doubt that many people will have discovered this bend at different times and places, and that its discovery will continue as long as there are beings which are capable of doing this and do not already know its existence. It is furthermore most unlikely that there were no people who didn't know the real nature of this bend; it appears that their know-how was unfortunately not recorded. This is of course not strange at all, since the recorded knowledge is only a fraction of the existing knowledge at the time. A substantial amount of this existent knowledge will never be recorded or be generally dispersed. Hence very many things will be discovered many times over by different people at different place and time. It is for these reasons that it is totally irrelevant whether a discovery is a genuine first (which of course nobody will ever know to be the case), or the millionth and hence historical dates are of little if any significance.

As mentioned at the beginning, this 2-string open-ender, based on the 3-parts/2-bights Casa-coded Regular Cylindrical Braid, is also used as a lanyard knot. Again one should be careful and use it the correct way. That is, the working-ends should be used for the necklace part in order to prevent the knot from spreading. Its single-ply form is depicted in Fig. 87.

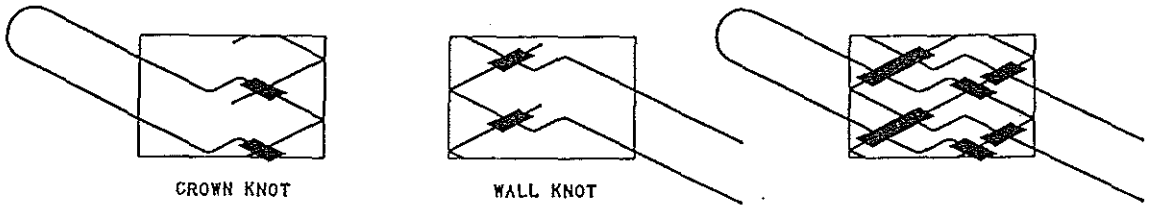


Fig. 87 — The 2-string single-ply Casa-coded 3/2 open-ender as a lanyard knot.

We mentioned at the beginning that the 4-string Casa-coded 3/4 open-ender is much used as a fiador knot in the construction of the fiador. The uppermost left-hand grid-diagram in Fig. 88 shows the layout of the fiador in which the single-ply version of this open-ender is used. The two loops at the left-hand side are used in the formation of the **double Hackamore knot** which secures the fiador to the **bosal**. The loop and two loose rope ends at the right-hand side provide the adjustable fastening of the fiador around the horse's neck with the aid of the **becket hitch**.

The knot may be tied in several different ways, but the method illustrated in Fig. 88 complies with the one depicted in Fig. 81, hence first we form the Crown knot, then the Wall knot under it, and feed the single loop and two rope ends through the eye of the Crown knot. Diagram 1 shows how to fold the rope; the two loops at the left-hand side act as the standing-ends. The diagrams $2, 2A, 3, 4$ depict the successive stages in the Crowning operation. The diagrams $6, 6A, 6B, 7, 8$ depict the successive stages in the Walling and feeding through the eye of the Crown operation.

Since the fiador is made from one length of rope, the strings are coupled as shown, and hence during the Crowning operation we have to reposition the string which was active in diagram 2. This repositioning is done in diagram $2A$. Similarly, in order to get the Walling done correctly and to assure that the working-ends are correctly fed through the eye of the Crown, we have to perform some repositioning after the operation in diagram 6. This repositioning is done in diagrams $6A, 6B$.

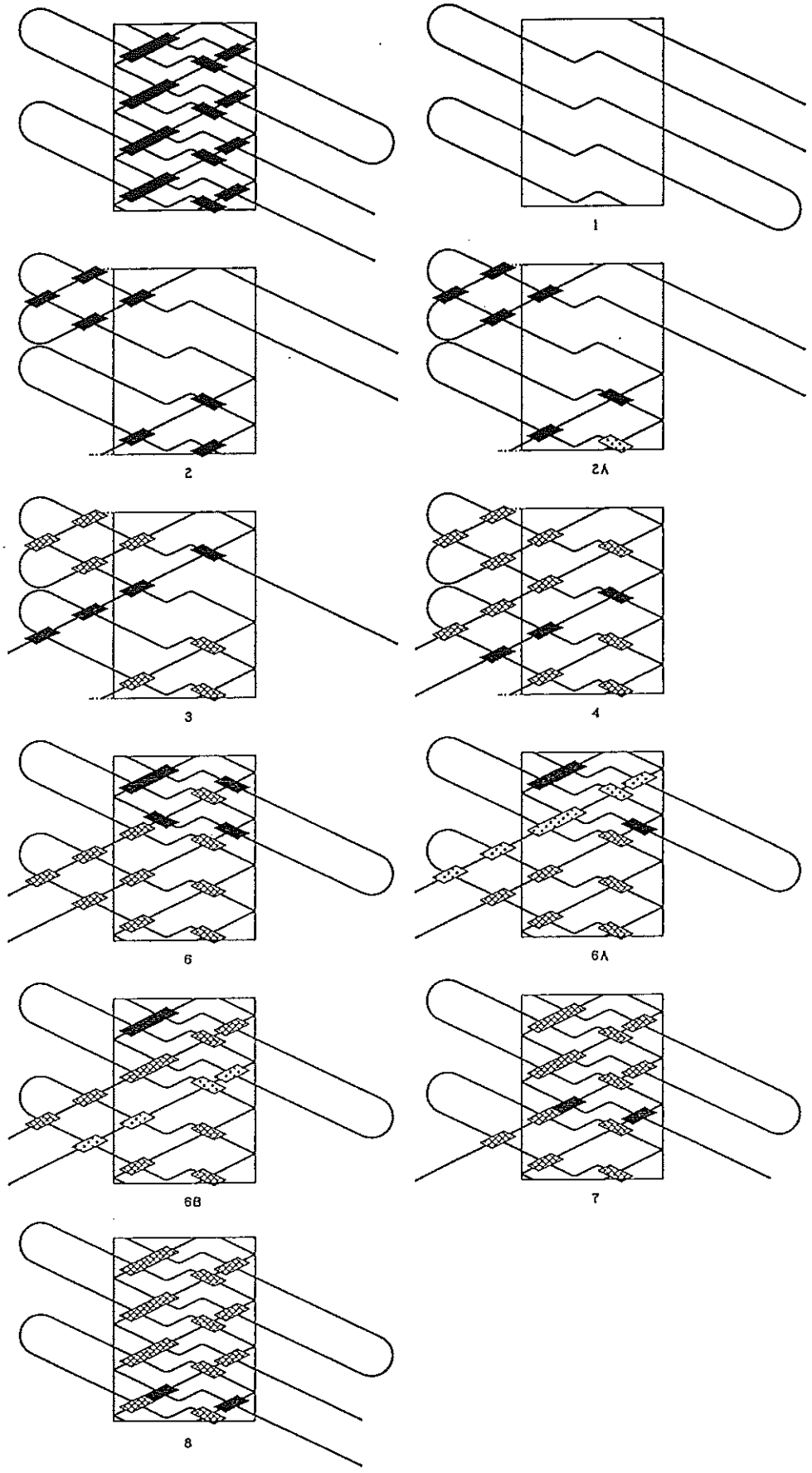


Fig. 88 — The 4-string single-ply Casa-coded 3/4 open-ender as a Fiador knot.

Since these 3-parts open-enders 'roll over' very easily, it is not surprising that we often encounter the 'rolled over' form. However, in order to prevent in applications the knot from spreading, the string-ends which may tend to spread must come through the eye of the Crown; their 'rolled over' form should not be used. Since there are much better open-enders which can be employed as fiador knots, it is advisable to use those instead; we shall discuss a few in the next issue.

The braiding of fiador knots as well as other open-enders with two or four bights can be simplified considerably by using the simple **fiador knot mandrel** depicted in Fig. 89.

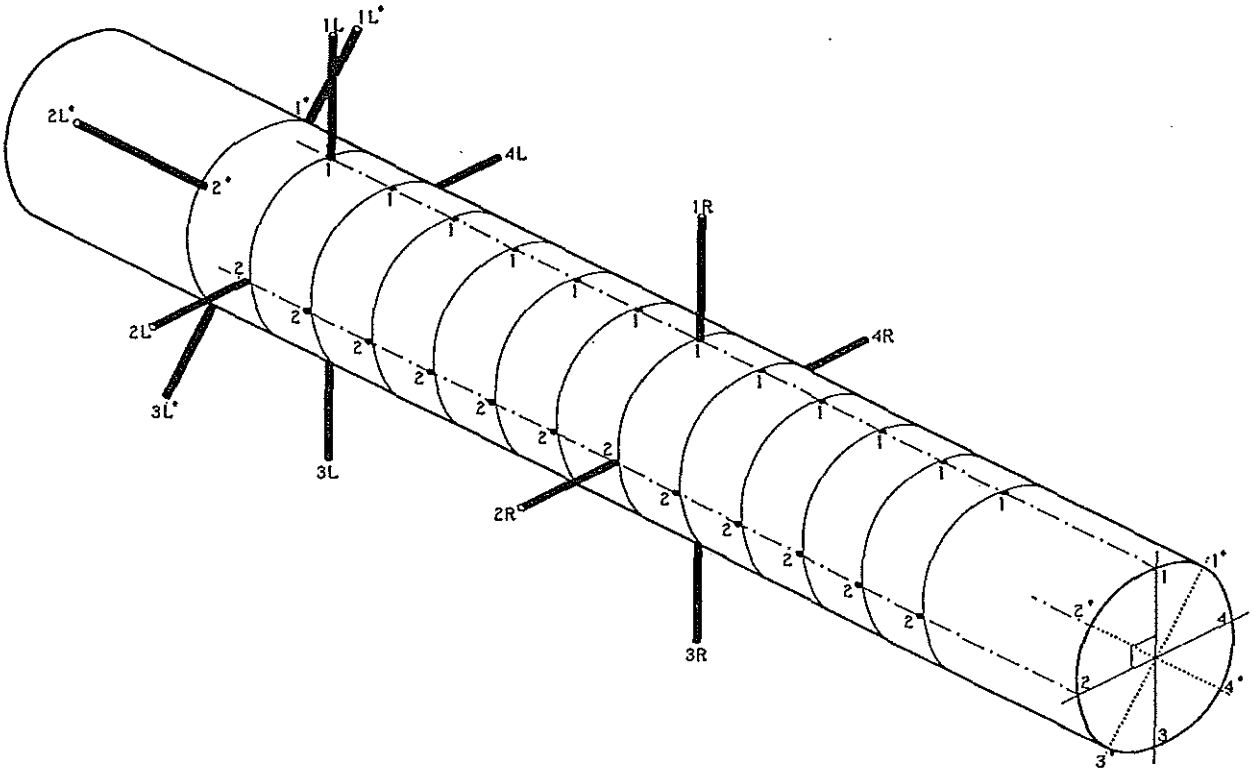


Fig. 89 — A simple fiador knot mandrel.

The mandrel is made from a wooden dowel of about 16mm. diameter and of about 200mm. in length. The equispaced circumscribed lines are 10mm. apart. The holes for the pins are of such a diameter that the pins have a push fit, and are of course offset from the circumscribing lines in order to allow the insertion of the pins. The pins, are cut from 3.5mm. diameter stainless-steel welding-rod, and are about 90mm. long.

The following three exercises should make the reader fully acquainted with its use.

Exercise 1 :

The braiding of the lanyard knot, depicted by the left-hand grid-diagram of Fig. 90, starts by placing the pins $1L^*$-$3L^*$, $2L^*$-$4L^*$, and the pins $1R$-$3R$, $2R$-$4R$ in the mandrel (hence the pins $1L$-$3L$, $2L$-$4L$ are not used). The pins are now in the correct relative positions of the bights as indicated in the grid-diagram.

Next we fold the cord in half and place one half **below** left-pin 4 (hence between the left-pins 4 and 1; the half-cycle **enters** the knot from the **left**, **below** a pin). Run the cord around right-pin 1, then around left-pin 3, then around right-pin 4. Note that all these three half-cycles are free runs. The next half-cycle runs from right-pin 4 to left-pin

2 and makes an *under*-crossing on its way. The next half-cycle runs from left-pin 2 to **above** right-pin 3 (hence between the left-pins 3 and 2; the half-cycle **leaves** the knot towards the **right**, **above** a pin), and on its way it makes an *under*-crossing.

Then we braid the other half of the cord. It enters the knot from the left **below** left-pin 2 (hence between the left-pins 2 and 3; the half-cycle **enters** the knot from the **left**, **below** a pin). The half-cycle runs to right-pin 3, and on its way it makes an *under*-crossing. The next half-cycle runs from right-pin 3 to left-pin 1, and on its way it makes consecutively an *under*-crossing and an *over*-crossing. The next half-cycle runs from left-pin 1 to right-pin 2, and on its way it makes consecutively an *under*-crossing and an *over*-crossing. The next half-cycle runs from right-pin 2 to left-pin 4, and on its way it makes consecutively an *over*-crossing, an *under*-crossing and an *over*-crossing. The next half-cycle runs from left-pin 4 to **above** right-pin 1 (hence between the left-pins 1 and 4; the half-cycle **leaves** the knot towards the **right**, **above** a pin), and on its way it makes two *under*-crossings.

Pull the pins out and slide the knot off the mandrel, then tighten it carefully. The necklace part should be made with the working-ends leaving the knot on the right-hand side.

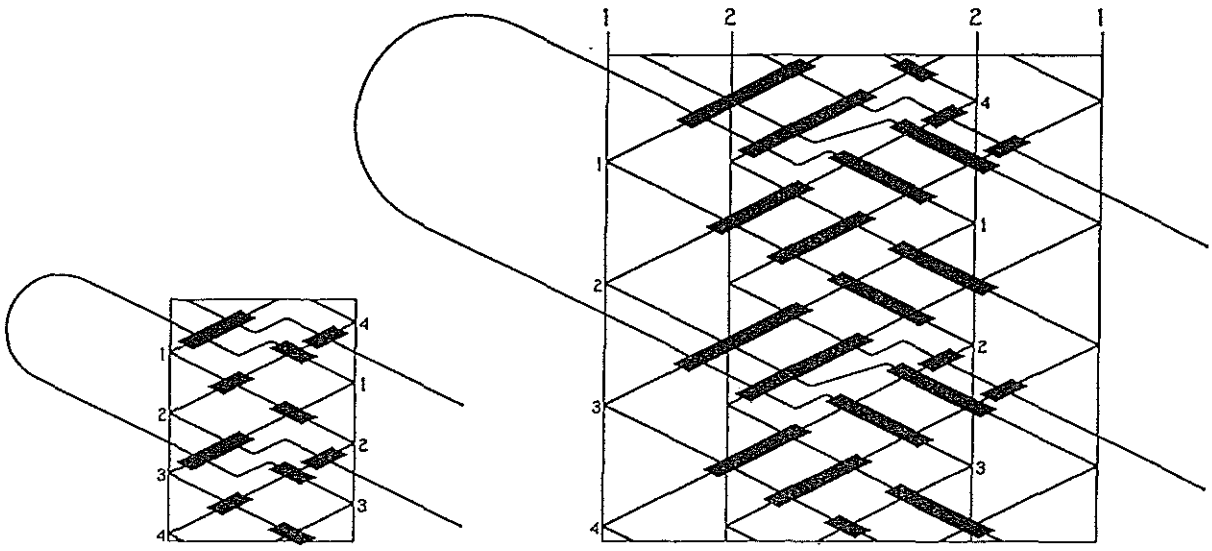


Fig. 90 — The lanyard knots in Exercise 1 and Exercise 2.

Exercise 2:

The 2-ply version of the lanyard knot in Exercise 1 is depicted by the right-hand grid-diagram of Fig. 90. Each pin will finally contain two stacked bights as shown in the diagram. Care should be taken with the positioning of the "ends" as shown.

Exercise 3:

In this Exercise, which is a little harder than the previous two, we braid the earlier discussed fiador knot in the 2-ply form. Its grid-diagram is depicted in Fig. 91.

We start with the single loop at the right-hand side, and braid its two ends till they exit on the left-hand side. Hence we start with the half-cycle which "enters" from the right below right-pin 3 and runs to left-pin 3, where it forms the **inner bight**. Then we lay down the half-cyle which "enters" from the right below right-pin 4 and runs to left-pin 4, where it forms the **inner bight**. Next follows the half-cycle which runs from left-pin 3 (inner bight) to right-pin 1, where it forms the **outer bight**. Next follows

the half-cycle which runs from left-pin 4 (inner bight) to right-pin 2, where it forms the **outer bight**; on its way it makes an *over*-crossing. Next the half-cycle which runs from right-pin 1 (outer bight) to left-pin 4, where it forms the **outer bight**. Next the half-cycle which runs from right-pin 2 (outer bight) to left-pin 1, where it forms the **outer bight**; on its way it makes an *over*-crossing. Then follows the half-cycle which runs from left-pin 4 (outer bight) to right-pin 2, where it forms the **inner bight**; on its way it makes an *over*-crossing. Next follows the half-cycle which runs from left-pin 1 (outer bight) to right-pin 3, where it forms the **inner bight**; on its way it makes three *over*-crossings. Next the half-cycle which runs from right-pin 2 to **below** left-pin 1 where it leaves the knot; on its way it makes an *over*-crossing. Next the half-cycle which runs from right-pin 3 to **below** left-pin 2 where it leaves the knot; on its way it makes in consecutive order two *over*-crossings and one *under*-crossing.

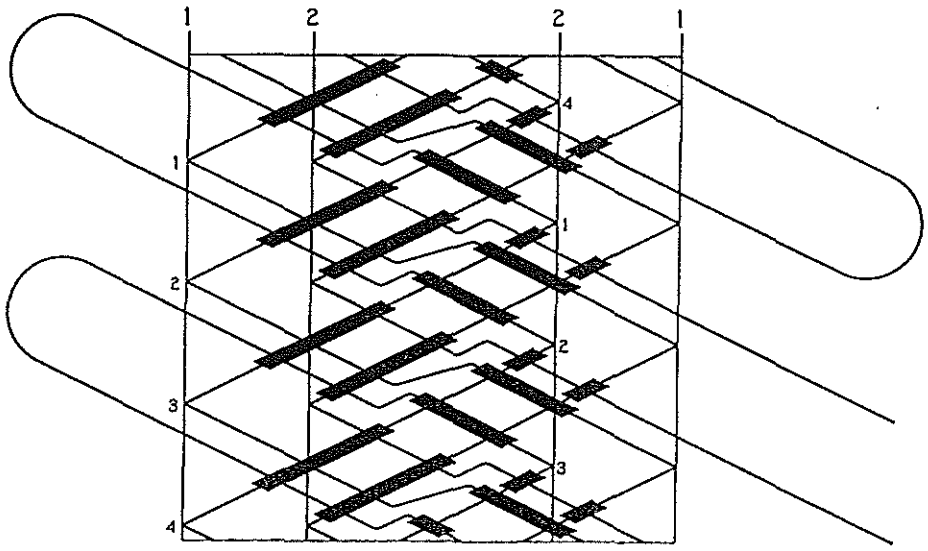


Fig. 91 — The 2-ply fiador knot in Exercise 3.

From now on we braid each cord-part to completion, hence till it leaves the knot on the right-hand side.

First we braid to completion the cord-part which left the knot on the left-hand side below pin 1. Hence we form the upper-left loop and re-enter the cord into the knot from the left **below** the outer bight around left-pin 4. The half-cycle runs then to the right-pin 1, where it forms the **inner bight**; on its way it makes an *under*-crossing. Next follows the half-cycle which runs from right-pin 1 (inner bight) to left-pin 3, where it forms the **outer bight**; on its way it makes in consecutive order two *under*-crossings and one *over*-crossing. Next comes the half-cycle which runs from left-pin 3 (outer bight) to right-pin 4, where it forms the **outer bight**; on its way it makes in consecutive order two *under*-crossings and one *over*-crossing. Next the half-cycle which runs from right-pin 4 (outer bight) to left-pin 2, where it forms the **inner bight**; on its way it makes in consecutive order one *over*-crossing, two *under*-crossings and two *over*-crossings. Next the half-cycle which runs from left-pin 2 (inner bight) to **above** the bight around right-pin 3 where it leaves the knot; on its way it makes four *under*-crossings.

Then we form the lower-left loop and re-enter the cord into the knot from the left **below** the outer bight around left-pin 3. The half-cycle runs then to the right-pin 4, where it forms the **inner bight**; on its way it makes in consecutive order two *under*-crossings and one *over*-crossing. Next follows the half-cycle which runs from right-pin

4 (inner bight) to left-pin 2, where it forms the **outer bight**; on its way it makes in consecutive order one *over*-crossing, two *under*-crossings and two *over*-crossing. Next comes the half-cycle which runs from left-pin 2 (outer bight) to right-pin 3, where it forms the **outer bight**; on its way it makes in consecutive order two *under*-crossings and two *over*-crossings. Next the half-cycle which runs from right-pin 3 (outer bight) to left-pin 1, where it forms the **inner bight**; on its way it makes in consecutive order one *over*-crossing, two *under*-crossings and three *over*-crossings. Next the half-cycle which runs from left-pin 1 (inner bight) to **above** the outer bight around right-pin 2 where it leaves the knot; on its way it makes four *under*-crossings.

Pull the pins out and slide the knot off the mandrel, then tighten it carefully.

Depending on the locallity the open-enders we discussed above have a multitude of names, but it is of some interest to note that the open-enders in Fig. 81 are commonly refered to as **Footrope knots**, while their 'rolled over' forms in Fig. 82 are commonly refered to as **Diamond knots**; however, the fiador knot based on the 3/4 open-ender in Fig. 81 is commonly known as the **Diamond fiador knot**. Where and when this interchange of name occurred is unknown, but there is some likelyhood that the "diamond" fiador knot did exist well before the "footrope" knot. It is, however, fairly certain that the 'rolling over' affair did play a role in this name interchange.

Reviews

The Hackamore Diamond Knot — plus The Knot in a Bottle;
by Eugene Ulrich; 44 pages, 220×140mm., spiral bound.

In this privately published booklet, the author describes **four** methods of tying the single-ply fiador knot we discussed in the previous article. The reader is gently introduced to some relevant open-enders before going over to the tying of the fiador knot. The text is accompanied by good illustrations of the consecutive tying stages.

Of special interest to most readers will be the section which describes how to tie this knot in a small bottle, a genuinely unique article which most readers undoubtedly will want to make. It is the counter-part to a ship in a bottle.

The whole construction process is clearly outlined, and attention has been drawn to the various pitfalls along the way.

This booklet may be obtained from: Mr. Eugene Ulrich,
Box 272,
Faith, SD 57626,
U.S.A.

Isometric Graph papers

Isometric graph papers which are available in the U.S.A.:

No. 341-I Dietzgen Isometric Graph Paper. A very good graph paper.
40-1301 Isometric-Orthographic; L.L.Ridgway Company, Inc. Good but not the best.

THE BRAIDER

No.6 **MAY 1996.**

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A quarterly publication for the braiding artisan

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Solution to the Question in Issue No. 5

Since the g.c.d. of p and b is equal to y, we can write:

$$p = y \cdot p'$$
$$b = y \cdot b'$$

Say the string-run starts at bight-point 0 (bight-index number I_0). After having travelled $2x$ half-cycles (hence x cycles or zigzags) the string arrives in bight-point $|xp|_b = |xyp'|_{yb'}$ (bight-index number I_x). In order for the string to return to its starting-point (bight-point 0), the value of $|xyp'|_{yb'}$ must be 0. This will be the case when $x = b'$ since p' and b' are coprime. However, the string-run has only passed, on each bight-boundary, through $x = b'$ bight-points. Hence in order to pass through all b bight-points on each of the two bight-boundaries we require $\frac{b}{b'} = y$ strings.

Since $|xp|_b = |xyp'|_{yb'} = y|xp'|_{b'}$ [†], the distance on a bight-boundary between any two adjacent bight-points on the same string is equal to y bights. Hence if the first string starts at bight-index number 0 (left bight-boundary), then the second string can start at bight-index number 1 (left bight-boundary), the third string can start at bight-index number 2 (left bight-boundary), the fourth string can start at bight-index number 3 (left bight-boundary), ..., the yth string can start at bight-index number $(y-1)$ (left bight-boundary). Thus we have y interwoven Regular Knots, each of which has $\frac{p}{y} = p'$ parts and $\frac{b}{y} = b'$ bights.

The Regular Knots

In *The Braider* No. 2, page 30, we defined the Regular Knot, and saw that its string-run was fully defined by its number of parts p and its number of bights b. In *The Braider* No. 5, page 90, we showed that the values of p and b had to be coprime (have no divisor other than 1).

We shall now discuss the relationships between the string-runs of different Regular Knots and obtain a graphical overview of the Regular Knots. This overview not only enables us to form a better understanding of the properties of Regular Knots, but is also of great practical importance as we shall see later.

The string-run of a Regular Knot (hence a p/b combination in which p and b are coprime) can be enlarged by means of two **specific** starting methods only[‡]:

(I) in accordance with the Enlargement Method I,
(II) in accordance with the Enlargement Method II.

The start of these specific enlargement methods are depicted by the dotted line in the string-run diagrams of Fig. 92.

[†] See *The Braider* No. 4, page 79, Rule 3.

[‡] There are some enlargement processes which may appear to be different to the uninitiated, however, all these seemingly different processes disguise in some way the only two true methods which exist. These seemingly different enlargement methods will in due course be discussed under *Scenes from the Knot Universe.*

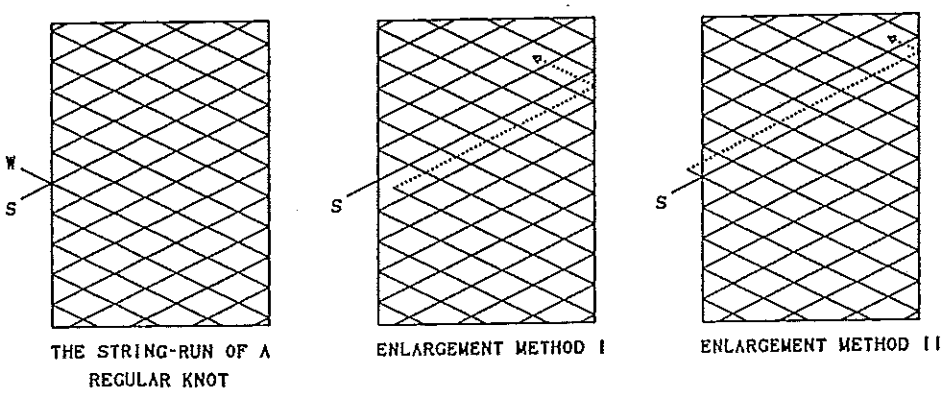


Fig. 92 — The start of Enlargement Method I and of Enlargement Method II.

In Fig. 93 is depicted the string-run diagram of a Regular Knot. To its right are depicted the 'rolled out' cycles of its string-run, and some relationships between its various parameters are indicated.

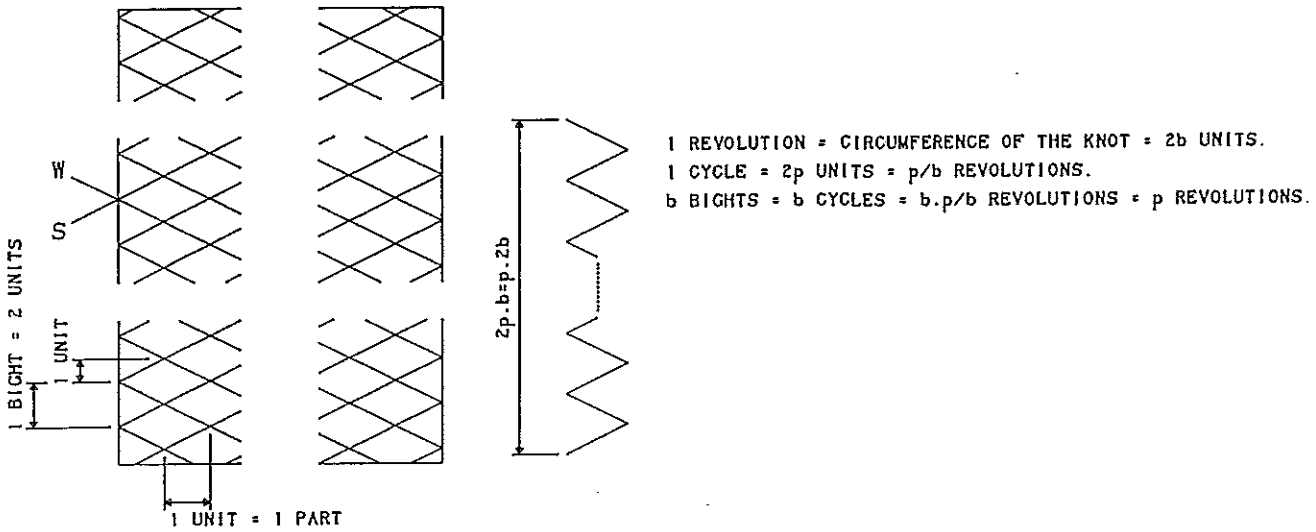

W
S
1 BIGHT = 2 UNITS
1 UNIT
2p.b=p.2b
1 UNIT = 1 PART
1 REVOLUTION = CIRCUMFERENCE OF THE KNOT = 2b UNITS.
1 CYCLE = 2p UNITS = p/b REVOLUTIONS.
b BIGHTS = b CYCLES = b.p/b REVOLUTIONS = p REVOLUTIONS.

Fig. 93 — The string-run of a Regular Knot, and its contained relationships.

The Enlargement Method I process is governed by the Diophantine equation:

$$p \cdot \Delta b = b \cdot \Delta p + 1\,,$$

and the Enlargement Method II process is governed by the Diophantine equation:

$$p \cdot \Delta b = b \cdot \Delta p - 1\,.$$

In these equations Δp is the increase in parts, and Δb is the increase in bights due to the enlargement procedure involved. These Diophantine equations can readily be derived in various ways. For each of the Enlargement Methods I and II, two ways are presented in each of the Figs. 94 and 95. In each of these Figs. the diagrams present a graphical derivation, while on the extreme right is presented an analytical derivation.

For a given Enlargement Method acting on a given p/b combination (with p and b coprime), there are an infinite number of $\Delta p/\Delta b$ combinations which fulfil the associated Diophantine equation. In these $\Delta p/\Delta b$ combinations the Δp and Δb values may be expressed in the following way:

$$\left.\begin{aligned}\Delta p &= |\Delta p|_p + np \\ \Delta b &= |\Delta b|_b + nb\end{aligned}\right\} \quad \text{where } n = 0, 1, 2, \ldots$$

The values of $|\Delta p|_p$ and $|\Delta b|_b$ are the smallest whole numbers† for Δp and Δb respectively (hence $n = 0$) which fulfil the associated Diophantine equation. Since the $|\Delta p|_p$ and $|\Delta b|_b$ values are the smallest whole number solutions for Δp and Δb ($n = 0$), we call their associated Enlargement Method the **fundamental** Enlargement Method, and the $|\Delta p|_p / |\Delta b|_b$ combination the **first order** solution of the associated Diophantine equation.

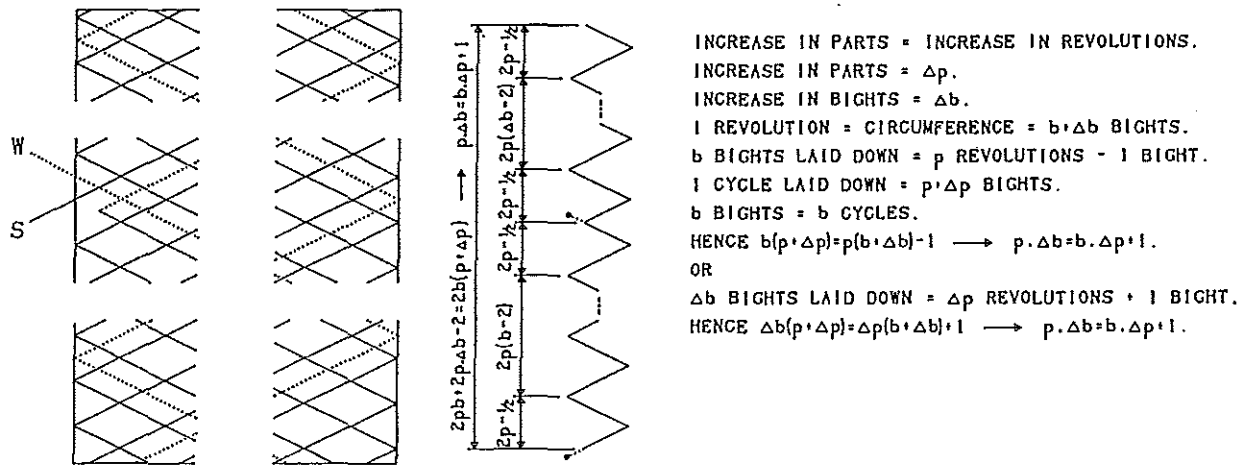


Fig. 94 — The Diophantine equation $p \cdot \Delta b = b \cdot \Delta p + 1$; Enlargement Method I.

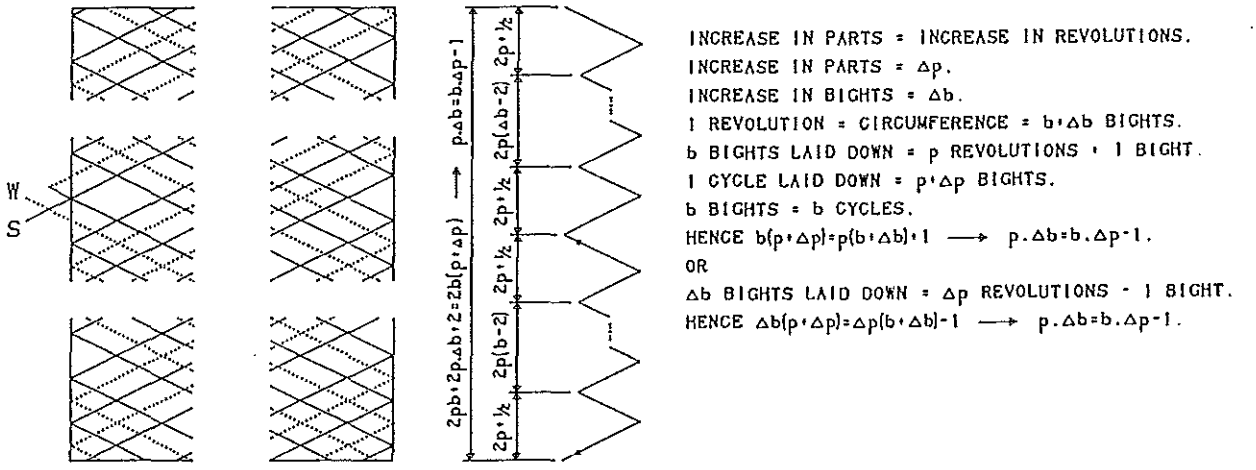


Fig. 95 — The Diophantine equation $p \cdot \Delta b = b \cdot \Delta p - 1$; Enlargement Method II.

A **second order** Enlargement is associated with $n = 1$, hence with $\Delta p = |\Delta p|_p + p$, and $\Delta b = |\Delta b|_b + b$.

A **third order** Enlargement is associated with $n = 2$, hence with $\Delta p = |\Delta p|_p + 2p$, and $\Delta b = |\Delta b|_b + 2b$.

And so on.

† Whole numbers are the numbers $0, 1, 2, \ldots$.

The Enlargement processes can be represented in a **tree**, called the **Regular Knot Tree (RKT)**, wherein each node bears a p/b-value corresponding to a particular string-run (see Fig. 96).

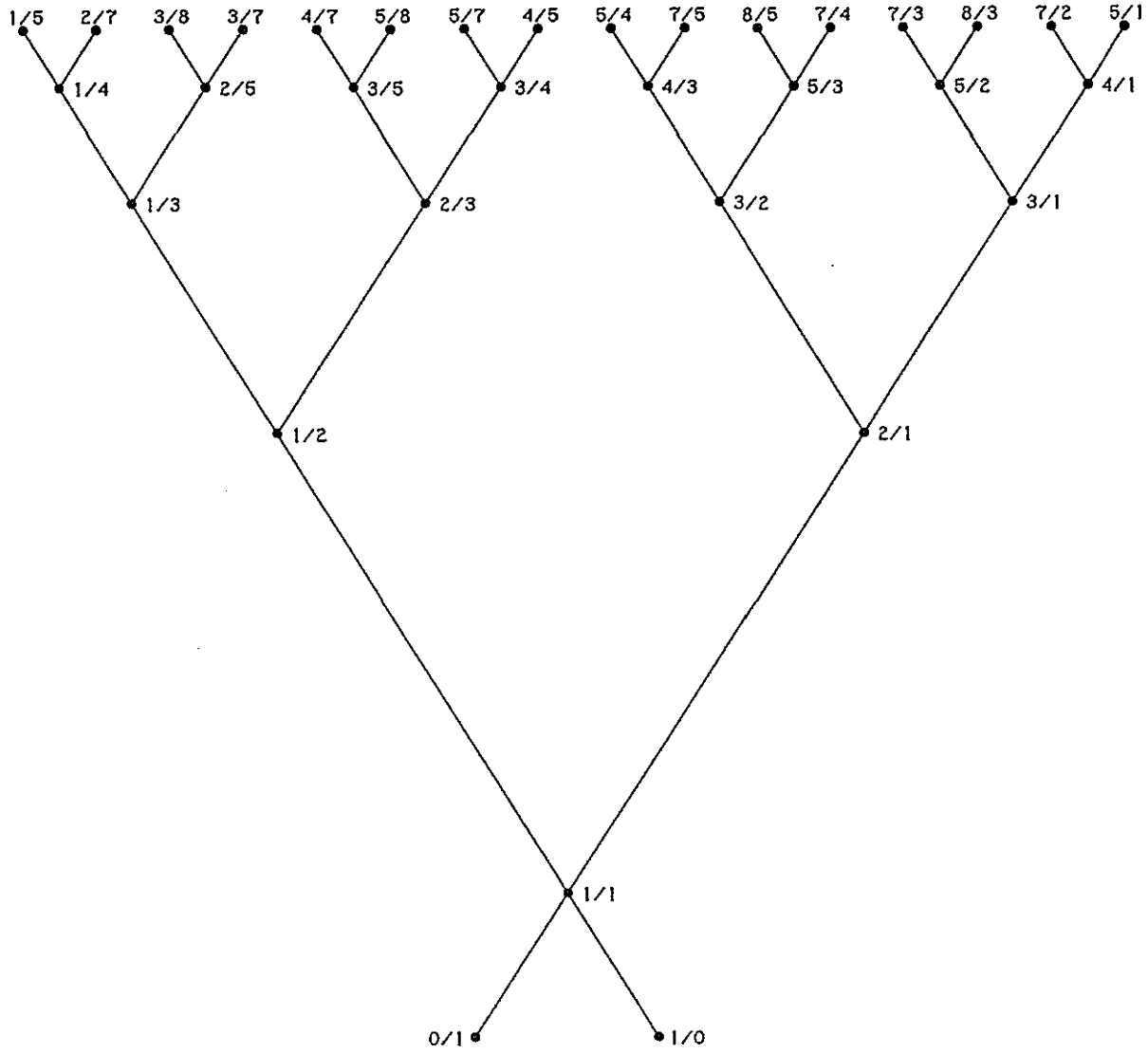


Fig. 96 — The first 4 levels of the RKT above the trunk-node 1/1.

The RKT has two **root-nodes**, bearing the p/b-values 0/1 and 1/0 respectively. These may be thought to correspond to two degenerate knots:

0/1 has no parts and one bight; so has the notial diagram $\langle\ \rangle$,
1/0 has one part and no bights; so has the notial diagram /.

Both of these nodes have a branch upward to the **trunk-node**, which bears the p/b-value 1/1. This value corresponds to the "loop" knot ◯.

Then two upward branches leave 1/1, and similarly two leave every subsequent node above 1/1.

> *A branch to the left is associated with a fundamental Enlargement Method I process, whilst a branch to the right is associated with a fundamental Enlargement Method II process.*

The p/b-values of the nodes above 1/1 are assigned according to the rules as given in Fig. 97.

By means of these rules we can construct the whole RKT from the two root-nodes and the trunk-node.

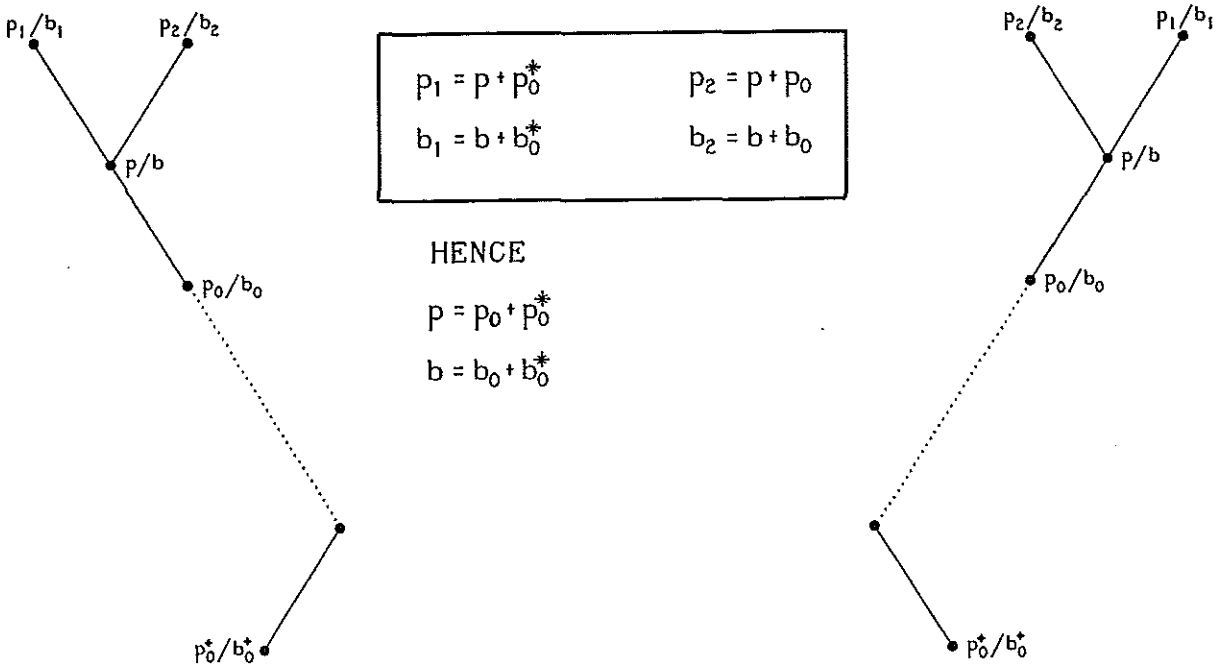


Fig. 97 — The construction rules for the RKT.

Hence by applying the fundamental Enlargement Method I once, an 1/1 string-run enlarges to an 1/2 string-run since 1/2=(1+0)/(1+1); a 3/5 string-run enlarges to a 4/7 string-run since 4/7=(3+1)/(5+2); a 4/5 string-run enlarges to a 7/9 string-run since 7/9=(4+3)/(5+4); a 2/1 string-run enlarges to a 3/2 string-run since 3/2=(2+1)/(1+1); a 7/3 string-run enlarges to a 9/4 string-run since 9/4=(7+2)/(3+1); and so on.

By applying the fundamental Enlargement Method II once, an 1/1 string-run enlarges to a 2/1 string-run since 2/1=(1+1)/(1+0); a 3/5 string-run enlarges to a 5/8 string-run since 5/8=(3+2)/(5+3); a 4/5 string-run enlarges to a 5/6 string-run since 5/6=(4+1)/(5+1); a 2/1 string-run enlarges to a 3/1 string-run since 3/1=(2+1)/(1+0); a 7/3 string-run enlarges to a 12/5 string-run since 12/5=(7+5)/(3+2); and so on.

It will be observed that the tree can be divided into two parts:

1. The left sub-tree in which for all the nodes, except the trunk-node 1/1, the number of parts is smaller than the number of bights.
2. The right sub-tree in which for all the nodes, except the trunk-node 1/1, the number of parts is greater than the number of bights.

It will also be observed that a node in the tree can be reached from the trunk-node 1/1 via a path which is unique to that node, hence we can identify a node by its path from 1/1.

The path from the trunk-node 1/1 to a node is written in the following form:

For node 7/5 the path is $[1;2,1,1]$; that is 1 step to the right, followed by 2 steps to the left, followed by 1 step to the right. *Note that the last 1 in the path-formula is neglected; in fact all path-formulae end with a 1, which is always neglected.* The reason for this seemingly odd way of writing the path-formulae will be explained later on. Thus the string-run of a 7 part/5 bight Regular Knot is obtained by enlarging the string-run of the "loop" knot by one fundamental Enlargement Method II, followed

by two consecutive fundamental Enlargement Methods I, followed by one fundamental Enlargement Method II.

For node 4/7 the path is $[0;1,1,2,1]$; that is 0 steps to the right, followed by 1 step to the left, followed by 1 step to the right, followed by 2 steps to the left.

Thus the string-run of a 4 part/7 bight Regular Knot is obtained by enlarging the string-run of the "loop" knot by one fundamental Enlargement Method I, followed by one fundamental Enlargement Methods II, followed by two consecutive fundamental Enlargement Methods I. Note that the initial 0 in the path-formula indicates that there is at the beginning no fundamental Enlargement Method II.

As mentioned before, the reason for writing the path-formulae in such a rather strange way will be explained later on, but in the meantime it will suffice to note that all path-formulae in the left sub-tree start with a zero before the semi-colon, and that all path-formulae in the right sub-tree start with a positive integer before the semi-colon. Furthermore we observe that the entries in a path-formula are associated with the fundamental Enlargement Methods $[\mathrm{II};\mathrm{I},\mathrm{II},\mathrm{I},\mathrm{II},\mathrm{I},\mathrm{II},\mathrm{I},\mathrm{II},\cdots,1]$.

Say the path-formula of a Regular Knot is $[2;3,4,1]$, then the number of parts of the consecutive nodes on this path are:
$1 \to 1+1=2 \to 2+1=3 \to 3+2=5 \to 5+2=7 \to 7+2=9 \to 9+7=16 \to 16+7=23 \to 23+7=30 \to 30+7=37.$
The number of bights of the consecutive nodes on this path are:
$1 \to 1+0=1 \to 1+0=1 \to 1+1=2 \to 2+1=3 \to 3+1=4 \to 4+3=7 \to 7+3=10 \to 10+3=13 \to 13+3=16.$

Say we want to enlarge the string-run of this 37 parts/16 bights Regular Knot by applying the fundamental Enlargement Method I once. Then the path-formula of this enlarged knot becomes $[2;3,4,1,1]$, and hence its number of parts will be $37+30=67$, and its number of bights will be $16+13=29$.

Instead of applying the fundamental Enlargement Method I once, say we decide to enlarge the string-run of the 37 parts/16 bights Regular Knot by applying the fundamental Enlargement Method II once. Then the path-formula of this enlarged knot becomes $[2;3,5,1]$, and hence its number of parts will be $37+7=44$, and its number of bights will be $16+3=19$.

Say the path-formula of a Regular Knot is $[0;2,3,4,1]$, then the number of parts of the consecutive nodes on this path are:
$1 \to 1+0=1 \to 1+0=1 \to 1+1=2 \to 2+1=3 \to 3+1=4 \to 4+3=7 \to 7+3=10 \to 10+3=13 \to 13+3=16.$
The number of bights of the consecutive nodes on this path are:
$1 \to 1+1=2 \to 2+1=3 \to 3+2=5 \to 5+2=7 \to 7+2=9 \to 9+7=16 \to 16+7=23 \to 23+7=30 \to 30+7=37.$

Say we want to enlarge the string-run of this 16 parts/37 bights Regular Knot by applying the fundamental Enlargement Method II once. Then the path-formula of this enlarged knot becomes $[0;2,3,4,1,1]$, and hence its number of parts will be $16+13=29$, and its number of bights will be $37+30=67$.

Instead of applying the fundamental Enlargement Method II once, say we decide to enlarge the string-run of the 16 parts/37 bights Regular Knot by applying the fundamental Enlargement Method I once. Then the path-formula of this enlarged knot becomes $[0;2,3,5,1]$, and hence its number of parts will be $16+3=19$, and its number of bights will be $37+7=44$.

The reader is advised to work out some other examples in order to become familiar with the system. A thorough familiarization is essential since this will enable the braider to utilize it in important practical applications which will otherwise fall outside the scope of a braider's ability.

Recall from page 109 the formulae:

$$\left.\begin{aligned}\Delta p &= |\Delta p|_p + np \\ \Delta b &= |\Delta b|_b + nb\end{aligned}\right\} \quad \text{where } n = 0, 1, 2, \ldots$$

Furthermore recall from page 109 that for $n = 1$, hence $\Delta p = |\Delta p|_p + p$; $\Delta b = |\Delta b|_b + b$, we obtained a *second order* enlargement. Hence when this enlargement is a second order Enlargement Method I, then it consists of a fundamental Enlargement Method I followed by a fundamental Enlargement Method II, and when this enlargement is a second order Enlargement Method II, then it consists of a fundamental Enlargement Method II followed by a fundamental Enlargement Method I.

For $n = 2$, hence $\Delta p = |\Delta p|_p + 2p$; $\Delta b = |\Delta b|_b + 2b$, we obtained a *third order* enlargement. Hence when this enlargement is a third order Enlargement Method I, then it consists of a fundamental Enlargement Method I followed by two consecutive fundamental Enlargements Method II, and when this enlargement is a third order Enlargement Method II, then it consists of a fundamental Enlargement Method II followed by two consecutive fundamental Enlargements Method I.

The enlargement paths in the RKT, associated with these Enlargements, are shown in Fig. 98 below.

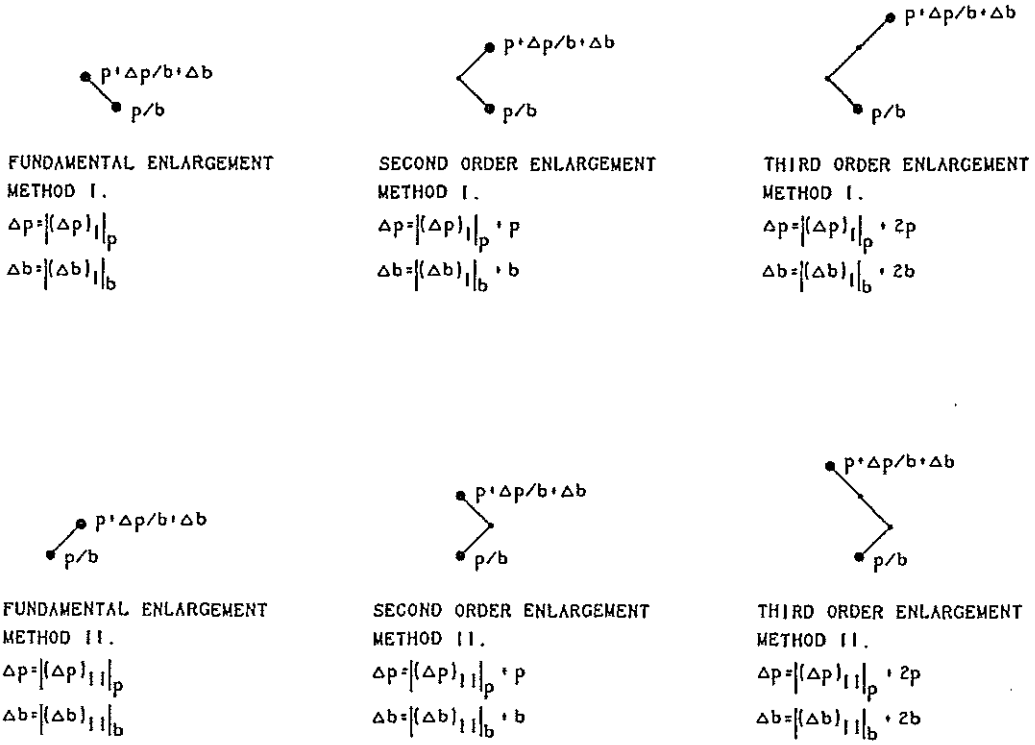


Fig. 98 — Enlargement paths in the RKT.

The essential parts of the string-run diagrams, which are associated with these enlargements, are depicted in Fig. 99.

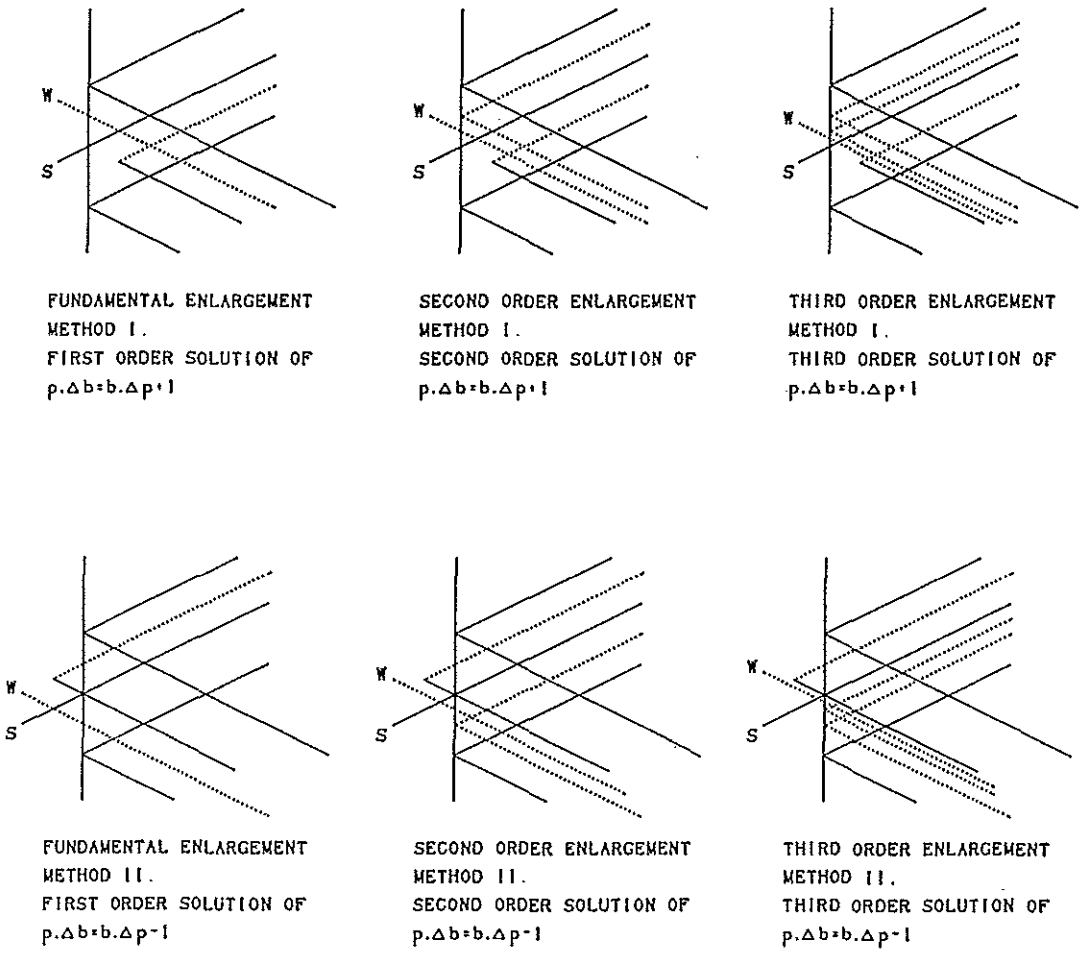


Fig. 99 — The essential parts of the string-run diagram.

Braiding Instructions and their Presentation

In the previous issue, *The Braider* No. 5, we discussed how **the cyclic bight-number scheme** and **the complementary cyclic bight-number scheme** could be constructed. Although the construction procedures were simple in principle, they were nevertheless quite laborious due to the necessary counting involved. The construction of these schemes would therefore be much quicker accomplished if we could number the consecutive points to the right, which follow point 0, in consecutive order. This can readily be done with the aid of the Δ-value for **the cyclic bight-number scheme** and with the Δ^*-value for **the complementary cyclic bight-number scheme.** Δ and Δ^* are the difference, *in cyclic value*, between any two adjacent points of their respective schemes. They are calculated as follows†:

† Note that the formulae for Δ and Δ^* follow directly fom our earlier discussed construction procedures ($\left|\Delta \cdot |p|_b\right|_b = 1$ and $\left|\Delta^* \cdot |-p|_b\right|_b = 1$).

$\Delta = \dfrac{n \cdot b + 1}{|p|_b}$ where n is the smallest positive integer such that Δ is an integer.

$\Delta^* = \dfrac{n \cdot b + 1}{|-p|_b}$ where n is the smallest positive integer such that Δ^* is an integer.

Hence in **the cyclic bight-number scheme** we obtain for the consecutive points from left to right the values:

$$0\,,\ \Delta\,,\ |2\Delta|_b\,,\ |3\Delta|_b\,,\ |4\Delta|_b\,,\ \ldots\,,$$

and in **the complementary cyclic bight-number scheme** we obtain for the consecutive points from left to right the values:

$$0\,,\ \Delta^*\,,\ |2\Delta^*|_b\,,\ |3\Delta^*|_b\,,\ |4\Delta^*|_b\,,\ \ldots\,.$$

⋆⋆ *Prove that we can also express* Δ^* *in the form* $\Delta^* = \dfrac{n^* \cdot b - 1}{|p|_b}$, *where* n^* *is the smallest positive integer such that* Δ^* *is an integer.*

In *The Braider* No. 5, pg. 92, we showed that in the **complementary cyclic bight-number scheme** the point which receives the number k is associated with $j - i = -k$, and that this point is at $|-kp|_b$ point-distance from point 0. Furthermore recall from pp. 90 - 91 that the half-cycle which runs from lower-right to upper-left, starting at the right-hand bight-number i (at bight-index number $|ip|_b$), crosses the half-cycle which runs from lower-left to upper-right, ending at the right-hand bight-number j (at bight-index number $|jp|_b$), at points which are at column-distances of $|(j-i)p|_b + nb$ (where $n = 0, 1, 2, \ldots$ and $0 < |(j-i)p|_b + nb < p$) from the right-hand bight-boundary. Hence for the **first** half-cycle, the half-cycle associated with the **Standing End**, the value of j is equal to 0. Consequently, half-cycle $(2i+2)$, which starts at the right-hand bight-number i (at bight-index number $|ip|_b$), crosses the first half-cycle at points which are at column-distances of $|-ip|_b + nb$ (where $n = 0, 1, 2, \ldots$ and $0 < |-ip|_b + nb < p$) from the right-hand bight-boundary. These column-distances correspond exactly to the point-distances in the **algorithm diagram** from point 0 to the points which carry the value i. Hence the crossings which half-cycle $(2i + 2)$ makes with the Standing End half-cycle (the first half-cycle laid down) are indicated by the points in the algorithm diagram which carry the value i. This supplies us with a very easy and convenient check on the actual braiding progress of every even half-cycle.

Example:

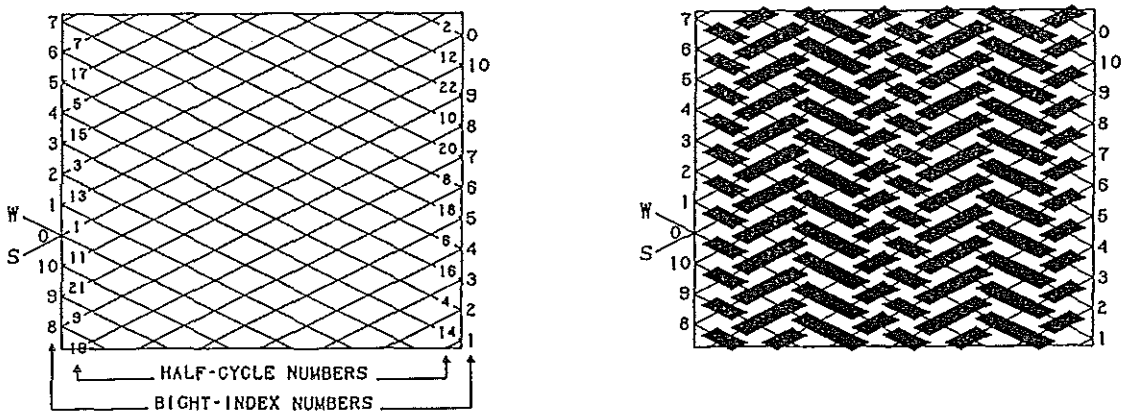


Fig. 100 — String-run diagram and grid-diagram.

This knot has 13-parts and 11-bights. Hence $|-p|_b = |-13|_{11} = 9$ $(\Delta^* = 5)$.
The complementary bight-number scheme of this knot is thus as follows:

0	5	10	4	9	3	8	2	7	1	6
·	·	·	·	·	·	·	·	·	·	·

This complementary bight-number scheme supplies us subsequently with the following algorithm diagram:

0	5	10	4	9	3	8	2	7	1	6	0	5	
·	\	/	/	\	\	/	\	/	/	\	\	/	·
	5	0	6	1	7	2	8	3	9	4	10	5	0

For half-cycle 6, hence $i = \dfrac{6-2}{2} = 2$, we read from the algorithm diagram (bottom line from right to left) the codings associated with the i-values in bold:

0	5	10	4	9	3	8	2	7	1	6	0	5	
·	\	/	/	\	\	/	\	/	/	\	\	/	·
	5	**0**	6	1	7	**2**	8	3	9	4	10	5	0
						↑							

This crossing is with the Standing End.

For half-cycle 12, hence $i = \dfrac{12-2}{2} = 5$, we read from the algorithm diagram (bottom line from right to left) the codings associated with the i-values in bold:

0	5	10	4	9	3	8	2	7	1	6	0	5	
·	\	/	/	\	\	/	\	/	/	\	\	/	·
	5	0	6	1	7	2	8	**3**	9	4	10	**5**	0
	↑											↑	

Crossing with the Standing End. *Crossing with the Standing End.*

For half-cycle 18, hence $i = \dfrac{18-2}{2} = 8$, we read from the algorithm diagram (bottom line from right to left) the codings associated with the i-values in bold:

0	5	10	4	9	3	8	2	7	1	6	0	5	
·	\	/	/	\	\	/	\	/	/	\	\	/	·
	5	0	6	1	7	2	**8**	3	9	4	10	5	0
							↑						

This crossing is with the Standing End.

One may wonder why we entered in the algorithm diagram the coding of each intersection column rather than two lines, one for the odd-numbered half-cycles and one for the even-numbered half-cycles, with **o**'s for **overs**, and **u**'s for **unders**. The reason being that the algorithm diagram does not only apply to half-cycles which are laid down from lower-left to upper-right and from lower-right to upper-left. In fact, the algorithm diagram is independent of the way in which the half-cycles are laid down. It only requires its upper-line of index-numbers to be read from left to right and to be associated with half-cycles which run from left to right (whether from lower-left to upper-right or from upper-left to lower-right), while its lower-line of index-numbers has to be read from right to left and is associated with half-cycles which run from right to left (whether from lower-right to upper-left or from upper-right to lower-left). Obviously the bight-numbers and bight-index numbers are assigned in the grid-diagram in accordance with the particular braiding sequence envisaged.

Let's take the knot of our Example and say we place the Standing End at the left-hand bight-boundary. Say we run the first five *odd* half-cycles from lower-left to upper-right and the first five *even* half-cycles from lower-right to upper-left. Let's call this the first braiding-phase. Hence the last of the ten first half-cycles (the last half-cycle of the first braiding-phase) will be laid down from lower-right to upper-left; this is half-cycle number 10 associated with bight-number $i = 4$ of the first braiding-phase. Say we then continue braiding with the Standing End. Consequently the last half-cycle laid down in the first braiding-phase becomes now the new *Standing End*, hence the **first** half-cycle of the second braiding-phase, and runs from upper-left to lower-right. The first half-cycle of the first braiding-phase becomes half-cycle 10 of the second braiding-phase and runs from upper-right to lower-left; it is associated with bight-number $i = 4$ of the second braiding-phase. The next half-cycle to be laid down is half-cycle 11, which is the "Standing End" of the first braiding-phase. It is associated with bight-number $i = 4$. Note that nothing has changed as far as the algorithm diagram is concerned; it is just as if we continue with laying down consecutive half-cycles. What has however changed is the **slant** of the half-cycles, which affects the interpretation of the coding symbols (*over* or *under*) in the algorithm diagram. The intersections which an *even*-numbered half-cycle makes with the Standing End of the ruling braiding-phase remains indicated by its associated bight-number position in the algorithm diagram.

We could start the first braiding-phase at the left-hand bight-boundary with a half-cycle from lower-left to upper-right, and end it after a number of half-cycles at the right-hand bight-boundary, then continue with the Standing End. In this case again nothing changes in the algorithm diagram. The only thing which does change is that in the second braiding-phase the **even**-numbered half-cycles run from upper-left to lower-right and the **odd**-numbered half-cycles run from upper-right to lower-left, consequently the **even**-numbered half-cycles in the second braiding-phase are now associated with the top-line of bight-numbers in the algorithm diagram while the **odd**-numbered half-cycles in the second braiding-phase are associated with the bottom-line of bight-numbers in the algorithm diagram.

Instead of starting the first braiding-phase with a half-cycle at the left-hand bight-boundary (two possible ways) we could of course also start the first braiding-phase at the right-hand bight-boundary (two possible ways), and there is of course again no change to the algorithm diagram. All we have to do is ensure that we read the algorithm diagram in accordance with the braiding procedure employed at the time.

In order to become thoroughly familiar with the construction and use of the algorithm diagram, the reader is advised to study Pamphlet No. 7 and its Supplement, Pamphlet No. 9, and Pamphlet No. 13. Each of these pamphlets contains several fully worked Examples.

$\star\star$ *In the algorithm diagram of a Regular Knot with p-parts and b-bights an arbitrary intersection column carries $i = a^*$ for the half-cycles which run from left to right, and carries $i = b^*$ for the half-cycles which run from right to left:*

$$\begin{array}{ccccccccccccccc}
0 & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & a^* & \cdot & \cdots & \cdot & \cdot & \cdot & & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot & \\
 & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & b^* & \cdot & \cdots & \cdot & \cdot & \cdot & 0 &
\end{array}$$

Prove that $a^* + b^* = b - 1$. When are the parities for all entries of the top-line and bottom-line predetermined? What are these parities?

Type I Regular Four-Bight Fiador Knots

We shall assume that the reader is now in the possession of the simple fiador knot mandrel described in *The Braider* No. 5, pg. 103, and that he or she has successfully completed the Exercises 1, 2, and 3 on pp. 103-106. Furthermore, we shall also assume that the reader has constructed the fiador knot depicted in Fig. 88 in accordance with the instructions given.

The grid-diagram of this fiador knot can be presented in four ways as shown in Fig. 101.

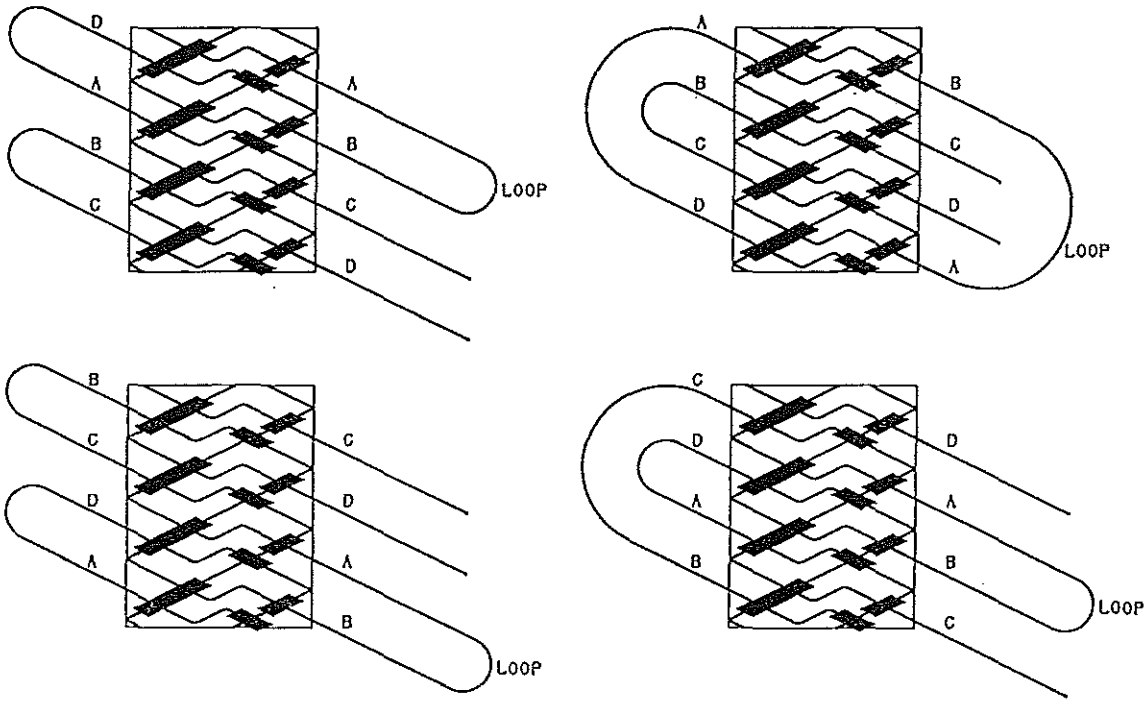


Fig. 101 — The four ways of depicting the grid-diagram.

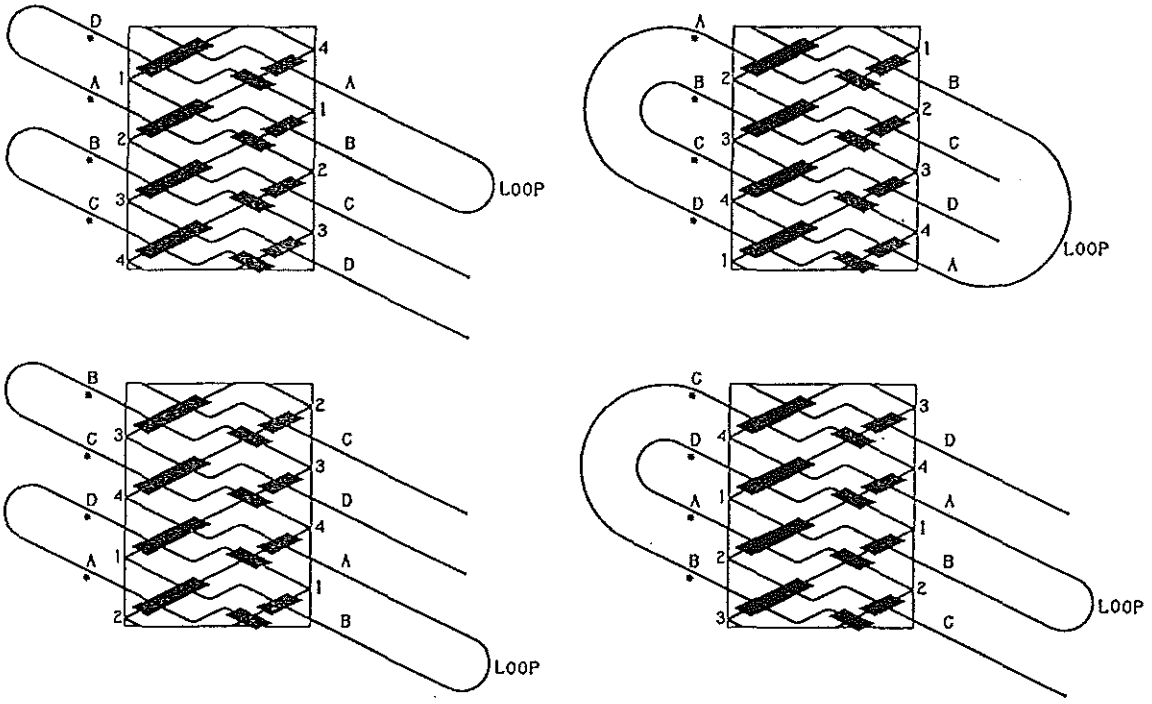


Fig. 102 — Pin-number allocation convention for type I regular four-bight fiador knots.

Fig. 102 shows the convention for the allocation of pin-numbers. The strings **A** and **B** are associated with the loop on the **right-hand** side. These strings are placed such that string **A** enters the knot from the *left* **below** pin-number **1** as an **asterisked half-cycle**, while string **B** enters the knot from the *left* **below** pin-number **2** as an **asterisked half-cycle**. At the left-hand side, string **A** goes over into string **D** which enters the knot from the *left* **below** pin-number **4** as an **asterisked half-cycle**, while string **B** goes over into string **C** which enters the knot from the *left* **below** pin-number **3** as an **asterisked half-cycle**. The strings **A**, **B**, **C** and **D** leave the knot on the right-hand side **above** their associated right-hand pin-numbers as **non asterisked half-cycles.**

The reader should now braid this fiador knot, and its 'rolled over' version, on the fiador knot mandrel in accordance with the instruction given in Fig. 103.

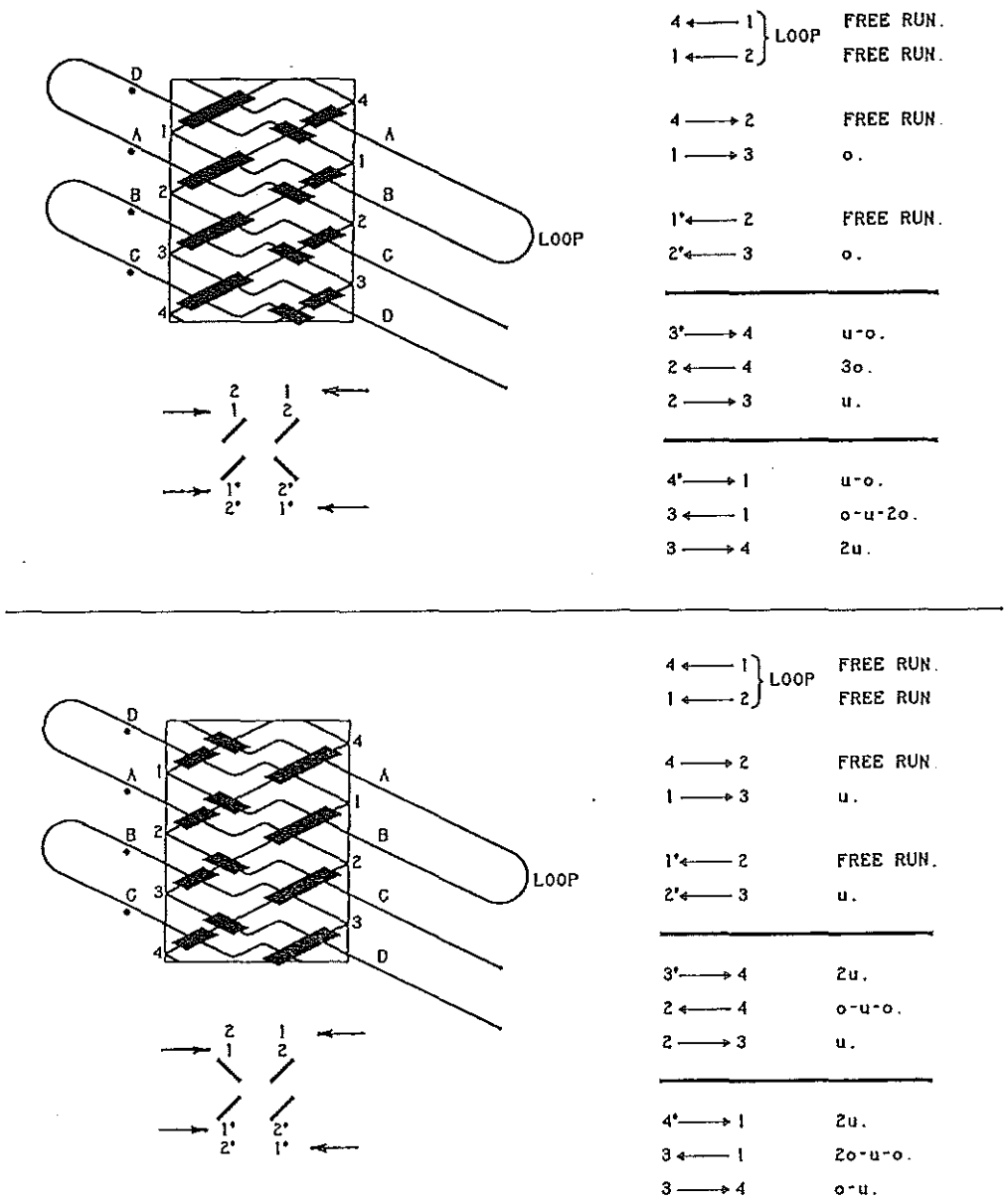

D
A
B
C
LOOP
4 ← 1 } LOOP FREE RUN.
1 ← 2 } LOOP FREE RUN.
4 → 2 FREE RUN.
1 → 3 o.
1* ← 2 FREE RUN.
2* ← 3 o.
3* → 4 u-o.
2 ← 4 3o.
2 → 3 u.
4* → 1 u-o.
3 ← 1 o-u-2o.
3 → 4 2u.
4 ← 1 } LOOP FREE RUN.
1 ← 2 } LOOP FREE RUN
4 → 2 FREE RUN.
1 → 3 u.
1* ← 2 FREE RUN.
2* ← 3 u.
3* → 4 2u.
2 ← 4 o-u-o.
2 → 3 u.
4* → 1 2u.
3 ← 1 2o-u-o.
3 → 4 o-u.

Fig. 103 — Instructions for braiding the fiador knot of Fig. 102, and its 'rolled over' version, on the fiador knot mandrel.

The order in which the strings A, B, C and D are laid down is the conventional method for braiding any Type I Regular Four-bight Fiador Knot on the fidor knot

mandrel. It consists of three stages: first **a preparation stage** which is followed by **two successive finishing stages.**

The **preparation stage** begins with **the loop** on the right-hand side of the knot. First we lay down half-cycle A_1 which runs from the right-hand bight-boundary to the left-hand bight-boundary, then we lay down half-cycle B_1 which runs from the right-hand bight-boundary to the left-hand bight-boundary. Next follows half-cycle A_2 which runs from the left-hand bight-boundary to the right-hand bight-boundary, then we lay down half-cycle B_2 which runs from the left-hand bight-boundary to the right-hand bight-boundary. Next follows half-cycle A_3 which runs from the right-hand bight-boundary to the left-hand bight-boundary and exits the knot **below** pin number 1, then we lay down half-cycle B_3 which runs from the right-hand bight-boundary to the left-hand bight-boundary and exits the knot **below** pin number 2. This finishes the preparation stage.

Now follows the first finishing stage, which consists of the three consecutive half-cycles of string **C**. On the left-hand side, string **B** goes over into string **C**. Its first half-cycle C_1 runs from **below** pin number 3 on the left-hand bight-boundary to the right-hand bight-boundary. Then follows half-cycle C_2, which runs from the right-hand bight-boundary to the left-hand bight-boundary. Next follows half-cycle C_3, which runs from the left-hand bight-boundary to the right-hand bight-boundary and exits the knot.

Then follows the second finishing stage, which consists of the three consecutive half-cycles of string **D**. On the left-hand side, string **A** goes over into string **D**. Its first half-cycle D_1 runs from **below** pin number 4 on the left-hand bight-boundary to the right-hand bight-boundary. Then follows half-cycle D_2, which runs from the right-hand bight-boundary to the left-hand bight-boundary. Next follows half-cycle D_3, which runs from the left-hand bight-boundary to the right-hand bight-boundary and exits the knot.

So far we have only indicated how the strings **A** and **B** are leaving the knot on the left-hand side, and how the strings **C** and **D**, entering the knot on the left-hand side, are positioned with respect to the left-hand pin numbers. We will of course have to know how each half-cycle has to be positioned in relation to the left-hand and right-hand pin numbers. It is convenient to refer to the half-cycles which either leave or enter the knot on the left-hand side as strings which **enter** the knot from the left, and to the half-cycles which either leave or enter the knot on the right-hand side as strings which **leave** the knot to the right.

In Type I Regular Four-bight Fiador Knots the strings A, B, C, D enter the knot from the left BELOW the respective pin numbers 1, 2, 3, 4. The strings A, B, C, D leave the knot to the right ABOVE their respective pin numbers which will have to be calculated.

The diagrams in Fig. 105 show the half-cycles rolled out and give the necessary formulae which enable us to calculate the pin numbers associated with the various half-cycles for Type I Regular Four-bight Fiador Knots.

The number of parts of a Regular Fiador Knot is the number of parts of the Regular Cylindrical Braid on which it is based.

Hence the number of parts of the fiador knots in Fig. 103 is equal to 3 since the respective Regular Cylindrical Braids they are based on are the Casa-coded Regular Cylindrical Braids in Fig. 104. Note that these braids are identical (one is the other turned upside down).



Fig. 104 — The Casa-coded Regular Cylindrical Braids on which the fiador knots of Fig. 103 are based.

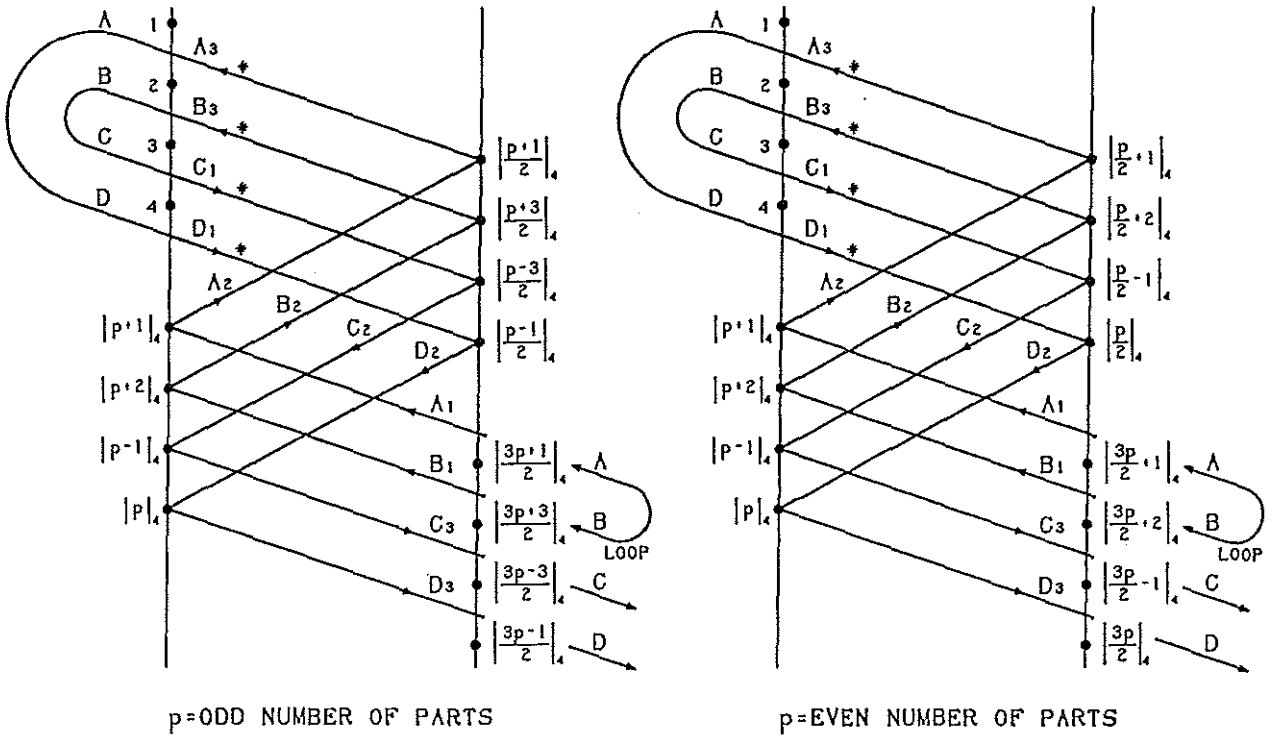


Fig. 105 — Half-cycles and their associated pin numbers.

In Fig. 106 we have depicted the development of the fiador knot mandrel.

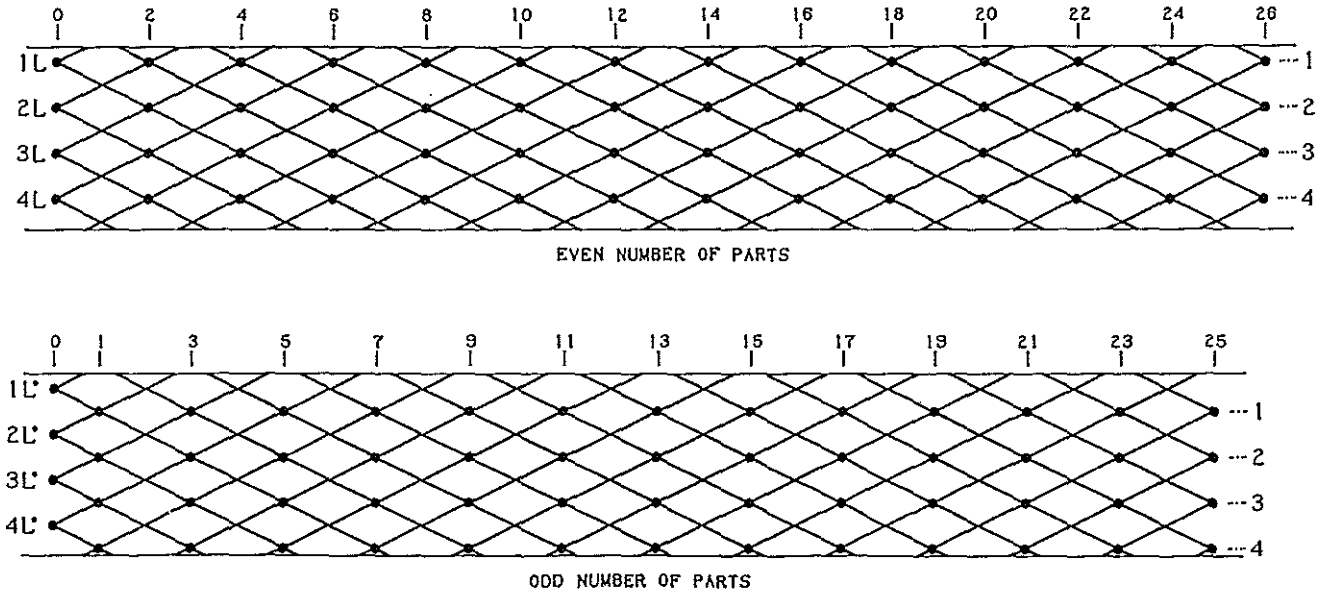


Fig. 106 — The fiador knot mandrel developed.

Note that for an **even** number of parts the left-hand pins $1L$-$3L$, $2L$-$4L$ and the right-hand pins $1R$-$3R$, $2R$-$4R$ are used (the left-hand pins $1L^*$-$3L^*$, $2L^*$-$4L^*$ are not used). For an **odd** number of parts the left-hand pins $1L^*$-$3L^*$, $2L^*$-$4L^*$ and the right-hand pins $1R$-$3R$, $2R$-$4R$ are used (the left-hand pins $1L$-$3L$, $2L$-$4L$ are not used).

Note also that for Type I Regular Four-bight Fiador Knots the calculation procedures for the pin numbers is independent of their coding.

Although we are now able to determine the pin numbers associated with the various half-cycles for Type I Regular Four-bight Fiador Knots, but in order to braid a fiador knot on the fiador knot mandrel, we also need to know its half-cycle algorithms. Hence let's have a look at Type I **Column-coded** Regular Four-bight Fiador Knots and develop a general method with which we can determine the half-cycle algorithms.

The string-run of Type I Regular Four-bight Fiador Knots consists of three types of half-cycles, which we indicate as type **x** (the asterisked half-cycles), type **y** and type **z** (the non asterisked half-cycles); see Fig. 107.

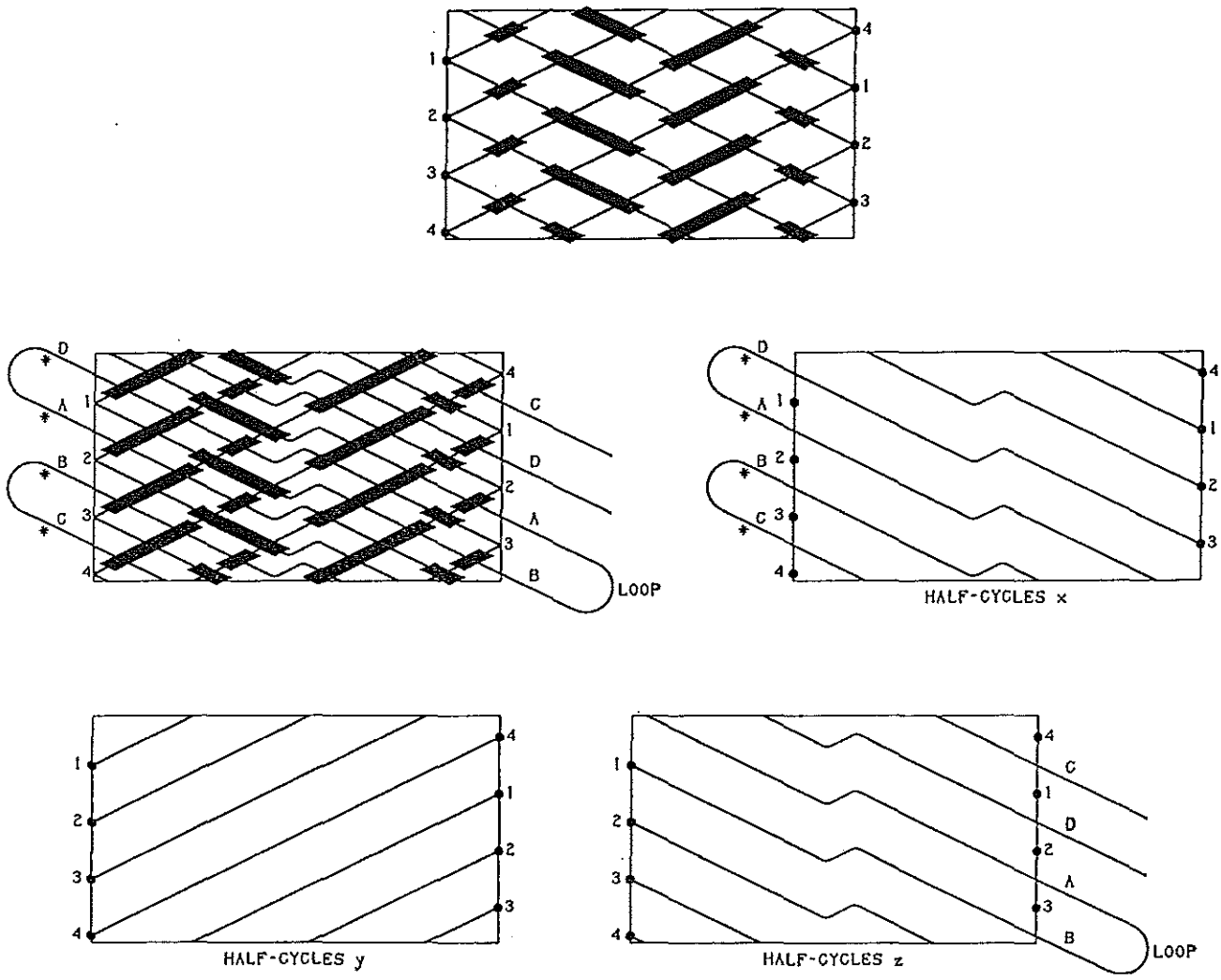


Fig. 107 — The half-cycles **x**, **y** and **z**.

In Fig. 108 are depicted the string-run half-cycles which have been and are being laid down in the three braiding stages. For each braiding stage the string-run half-cycles which have been and are being laid down can be depicted in three string-run diagrams.

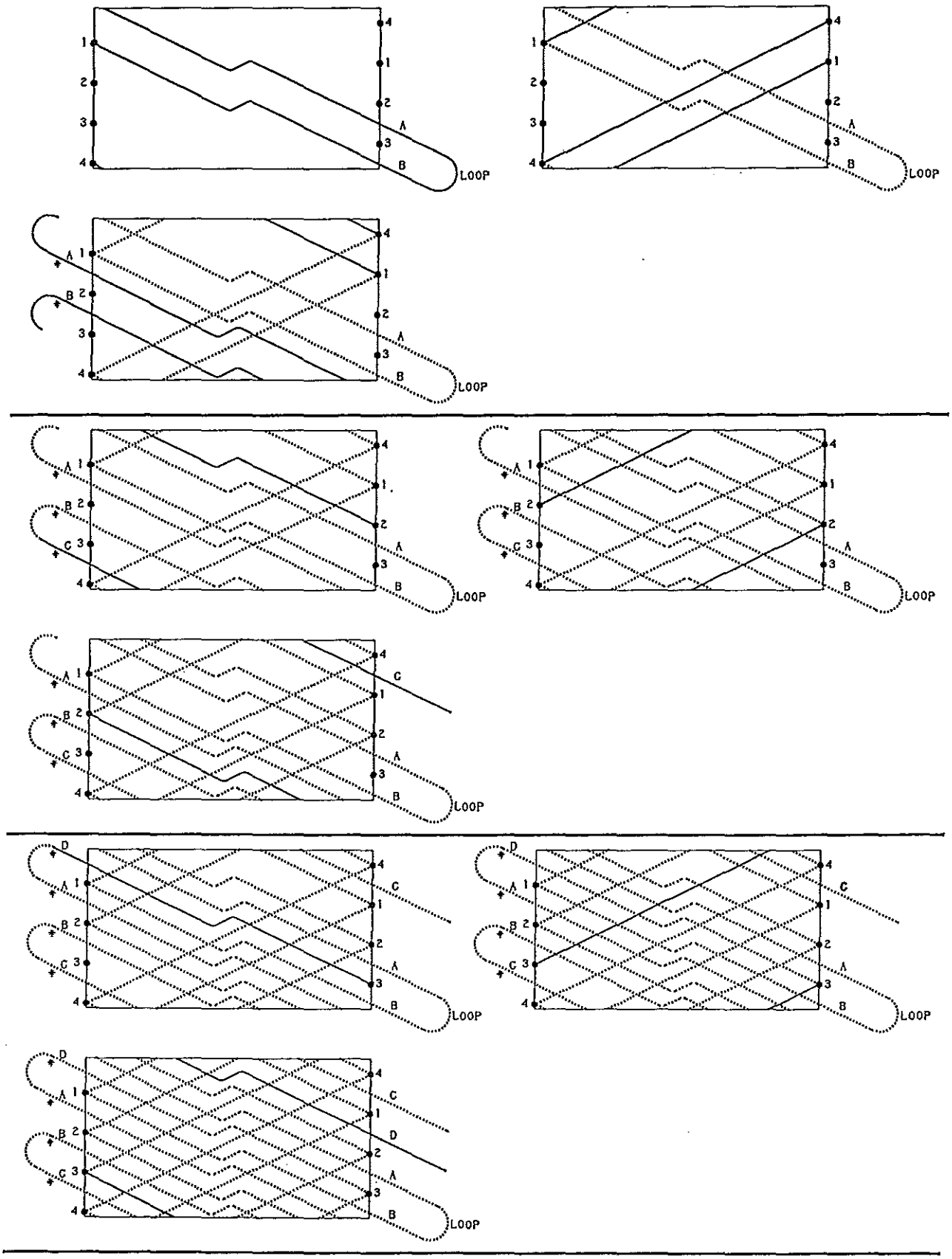


Fig. 108 — The string-run half-cycles in the successive braiding stages.

In the first stage (the preparation stage, associated with the strings **A** and **B**) the first diagram shows that the half-cycles A_1 and B_1 (this are non asterisked half-cycles; half-cycles **z**) are free-run half-cycles (no crossings). The second diagram in the first stage shows that the half-cycles A_2 and B_2 (half-cycles **y**) (may) intersect (cross) the non asterisked half-cycles which have already been laid down (the half-cycles A_1 and B_1 ; half-cycles **z**). The third diagram in the first stage shows that the half-cycles A_3 and B_3 (this are asterisked half-cycles; half-cycles **x**) (may) intersect (cross) the half-cycles A_2 and B_2 (half-cycles **y**) which have already been laid down.

In the second stage (the first finishing stage, associated with string **C**) the first diagram shows that half-cycle C_1 (an asterisked half-cycle; a half-cycle **x**) intersects half-cycles **y**. The second diagram shows that half-cycle C_2 (a half-cycle **y**) intersects half-cycles **x** and **z**. The third diagram shows that half-cycle C_3 (a half-cycle **z**) intersects half-cycles **y**.

In the third stage (the second finishing stage, associated with string **D**) the first diagram shows that half-cycle D_1 (an asterisked half-cycle; a half-cycle **x**) intersects half-cycles **y**. The second diagram shows that half-cycle D_2 (a half-cycle **y**) intersects half-cycles **x** and **z**. The third diagram shows that half-cycle D_3 (a half-cycle **z**) intersects half-cycles **y**.

Note that only half-cycles **y**, when being laid down in the two finishing stages (hence involving strings **C** and **D**) intersect two different types of half-cycles (the asterisked half-cycles **x** and the non asterisked half-cycles **z**).

Let's now number the intersection columns for the asterisked half-cycles (the half-cycles **x**) and for the non asterisked half-cycles (the half-cycles **z**) separately as shown in Fig. 109.

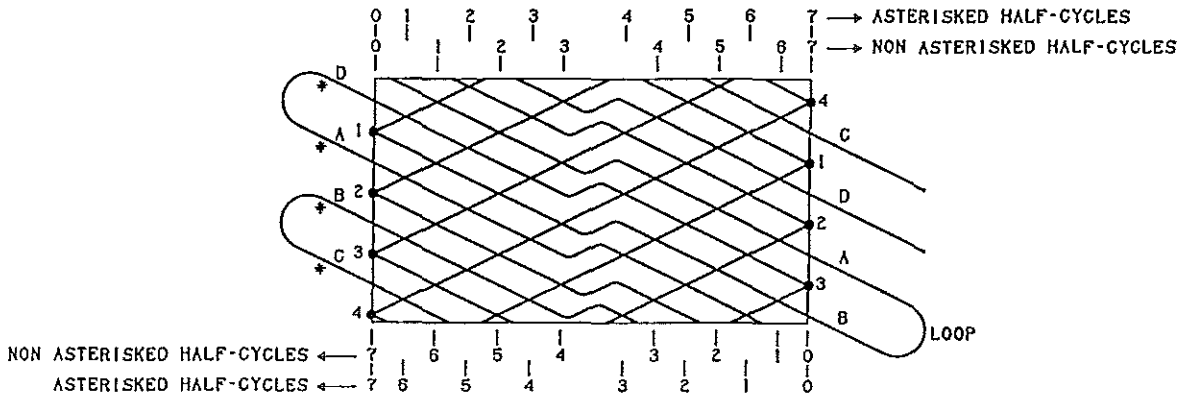


Fig. 109 — The numbering of the intersection columns.

Since the half-cycles C_2 and D_2 are being laid down from upper-right to lower-left, they will intersect on a non asterisked intersection column **before** they intersect on an asterisked intersection column of the same value.

Let's indicate the non asterisked intersection columns by C, and the asterisked intersection columns by C^*. We are dealing with Regular Fiador Knots which have **four** bights, and hence if an intersection column $C = a$ is an intersection column of a half-cycle, then the intersection columns $C = a + 4n$ (where $n = 0, 1, 2, \cdots$ and $0 < a + 4n < p$) are intersection columns of this half-cycle; similarly, if an intersection column $C^* = a^*$ is an intersection column of a half-cycle, then the intersection columns $C^* = a^* + 4n$ (where $n = 0, 1, 2, \cdots$ and $0 < a^* + 4n < p$) are intersection columns

of this half-cycle. Hence instead of numbering the intersection columns in a successive ascending order, it will be more convenient for Column-coded Regular Four-bight Fiador Knots to use their modular values $|C|_4$ and $|C^*|_4$ instead.

The modular intersection column-numbers associated with the half-cycles of Type I Column-coded Regular Four-bight Fiador Knots are as follows:

Preparation Stage:

$$\left\{\begin{matrix} L \longleftarrow R & : & A_1 \\ L \longleftarrow R & : & B_1 \end{matrix}\right]_{\text{loop}} \quad : \quad \text{free run.} \quad : \quad \text{free run.}$$

$$\left\{\begin{matrix} L \longrightarrow R & : & A_2 \\ L \longrightarrow R & : & B_2 \end{matrix}\right. \quad : \quad |C|_4 = 3\,,\,0\,. \quad : \quad |C|_4 = 1\,,\,0\,.$$

$$\left\{\begin{matrix} L \longleftarrow R & : & A_3 \\ L \longleftarrow R & : & B_3 \end{matrix}\right. \quad : \quad |C^*|_4 = 3\,,\,0\,. \quad : \quad |C^*|_4 = 1\,,\,0\,.$$

Finishing Stage Pt. 1:

$$L \longrightarrow R \quad : \quad C_1 \quad : \quad |C^*|_4 = |p-2|_4\,,\,|p-1|_4\,.$$

$$L \longleftarrow R \quad : \quad C_2 \quad : \quad \left\{\begin{matrix} |C|_4 = |p-2|_4\,,\,|p-1|_4\,. \\ |C^*|_4 = 2\,,\,3\,,\,0\,. \end{matrix}\right.$$

$$L \longrightarrow R \quad : \quad C_3 \quad : \quad |C|_4 = 2\,,\,3\,,\,0\,.$$

Finishing Stage Pt. 2:

$$L \longrightarrow R \quad : \quad D_1 \quad : \quad |C^*|_4 = |p-3|_4\,,\,|p-2|_4\,,\,|p-1|_4\,.$$

$$L \longleftarrow R \quad : \quad D_2 \quad : \quad \left\{\begin{matrix} |C|_4 = |p-3|_4\,,\,|p-2|_4\,,\,|p-1|_4\,. \\ |C^*|_4 = 1\,,\,2\,,\,3\,,\,0\,. \end{matrix}\right.$$

$$L \longrightarrow R \quad : \quad D_3 \quad : \quad |C|_4 = 1\,,\,2\,,\,3\,,\,0\,.$$

We have got now all the necessary tools for assembling the complete half-cycle algorithms associated with a Type I Column-coded Regular Four-bight Fiador Knot.

Example:

The coding and modular intersection column-number assignment for the fiador knot in Fig. 107 is as follows:

$$\begin{array}{cccccccc} \longrightarrow & 1 & 2 & 3 & 0 & 1 & 2 & \\ & 2 & 1 & 0 & 3 & 2 & 1 & \longleftarrow \\ & / & \backslash & \backslash & / & / & / & \\ & / & / & / & / & / & \backslash & \\ \longrightarrow & 1^* & 2^* & 3^* & 0^* & 1^* & 2^* & \\ & 2^* & 1^* & 0^* & 3^* & 2^* & 1^* & \longleftarrow \end{array}$$

In order to explain the use of this schematic layout let's number the rows from 1 to 6, top to bottom. The rows 1 to 3 (the uppermost three rows) are associated with the intersection column-numbers and codings of the half-cycles z. In the first

row the intersection columns are numbered in modular value from the left-hand bight-boundary, while in the second row they are numbered in modular value from the right-hand bight-boundary. The rows 4 to 6 (the lowermost three rows) are associated with the intersection column-numbers and codings of the half-cycles **x**. In the fifth row the intersection columns are numbered in modular value from the left-hand bight-boundary, while in the sixth row they are numbered in modular value from the right-hand bight-boundary. The rows 2 & 3 are used for the half-cycles A_1 and B_1. The rows 1 & 3 are used for the half-cycles A_2 and B_2. The rows 4 & 6 are used for the half-cycles A_3 and B_3. The rows 4 & 5 are used for half-cycle C_1. The rows 2 & 3 and 4 & 6 are used for half-cycle C_2. The rows 1 & 3 are used for half-cycle C_3. The rows 4 & 5 are used for half-cycle D_1. The rows 2 & 3 and 4 & 6 are used for half-cycle D_2. The rows 1 & 3 are used for half-cycle D_3.

Hence we obtain the following algorithm-table which contains all the information which enables us to braid this fiador knot.

Preparation Stage:

$\left\{\begin{matrix}4 \longleftarrow 3 \\ 1 \longleftarrow 4\end{matrix}\right.$	: $\left.\begin{matrix}A_1 \\ B_1\end{matrix}\right]$ loop	:	free run. free run.
$\left\{\begin{matrix}4 \longrightarrow 4 \\ 1 \longrightarrow 1\end{matrix}\right.$	: $\begin{matrix}A_2 \\ B_2\end{matrix}$	:	$u - o.$ $3o.$
$\left\{\begin{matrix}1^* \longleftarrow 4 \\ 2^* \longleftarrow 1\end{matrix}\right.$	: $\begin{matrix}A_3 \\ B_3\end{matrix}$	:	$2u.$ $o - 2u.$

Finishing Stage Pt. 1:

$3^* \longrightarrow 2$	: C_1	:	$3u - o.$
$2 \longleftarrow 2$	: C_2	:	$5o - u - 2o.$
$2 \longrightarrow 1$	: C_3	:	$2o - 2u.$

Finishing Stage Pt. 2:

$4^* \longrightarrow 3$	: D_1	:	$4u - o.$
$3 \longleftarrow 3$	: D_2	:	$o - u - 3o - u - o - u - 3o.$
$3 \longrightarrow 2$	: D_3	:	$u - 2o - 3u.$

The most simple fiador knots are the Type I Column-coded Regular Four-bight Fiador Knots. They are one of the the types discussed in Book 2/1 *"BRAIDING — Regular Fiador Knots"*. In order to become fully conversant with the process for obtaining the complete half-cycle algorithm-tables for these fiador knots, as well as braiding them with the aid of the fiador knot mandrel, the reader should exercise with the many examples which may be found in this book on pp. 29-60. Many of the presented examples are accompanied by a photographic illustration. It should be noted that this book discusses the more conventional calculation method in which the intersection columns are numbered in the consecutive ascending order.

Orientation and Similarity

On page 41 of *The Braider* No. 3, Fig. 37 depicts two examples of lopsided coding arrangements. As the text indicates these lopsided weaving-patterns are very seldom of any use. In regards to the left example of Fig. 37 this is especially true when used as a narrow flat braid, hence a braid with a relative small number of parts.

However, this lopsided type of weaving-pattern is not to be dismissed entirely, particularly when tied as as a relatively long woven knot as in Fig. 110.

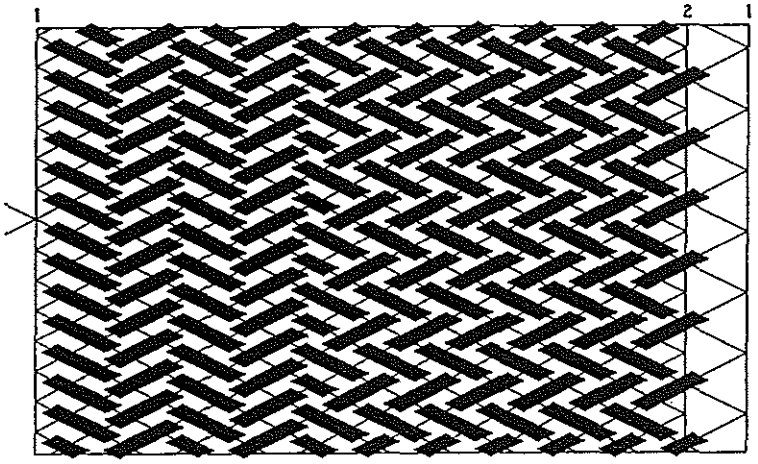


Fig. 110 — A single-colour, 2-pass Gaucho/Herringbone coding.

A general make-up of these knots is depicted in Fig. 111 whereby a section of **Gaucho**-coded braid is woven on one end, a section of **Herringbone**-coded braid is woven on the other end, and these two sections being joined by a *transition* weave†. Shown are a general 2-pass Gaucho/Herringbone mix and its common transition, and two general 3-pass Gaucho/Herringbone mixes containing two of the several possible transitions for them.

When woven in a single colour these lopsided **weaving-patterns** are lacking the aspects of **balance** and **symmetry**. However, they can still have some merit when a sufficiently large number of parts are used in each of the *Gaucho* and the *Herringbone* section. Then some observer, be them a braider or a non-braider can readily distinguish between these two different weaves and know that this was done by design and not by accident. Fig. 112 and Fig. 113 depict a 2-pass respectively a 3-pass colour pattern coding. It is evident that the aspects of **balance** and **symmetry** are also lacking in the overall **colour-pattern** produced here.

We can, however, consider two additional aspects which, do indeed, make especially these lopsided colour-patterns useful and attractive. These aspects are **orientation** and **similarity**.

Orientation of a lopsided *colour-pattern* as well as of a lopsided *weaving-pattern* has essentially to do with identifying a *top* and a *bottom* to the woven knot.

As a general rule, the desirable orientation for a single-colour knot is governed by the weaving-patterns, and the desirable orientation for a multi-colour knot is governed by the colour-patterns.

† Lopsided weaves would include any mix of different weaves. A Gaucho/Herringbone mix would be only one of many.

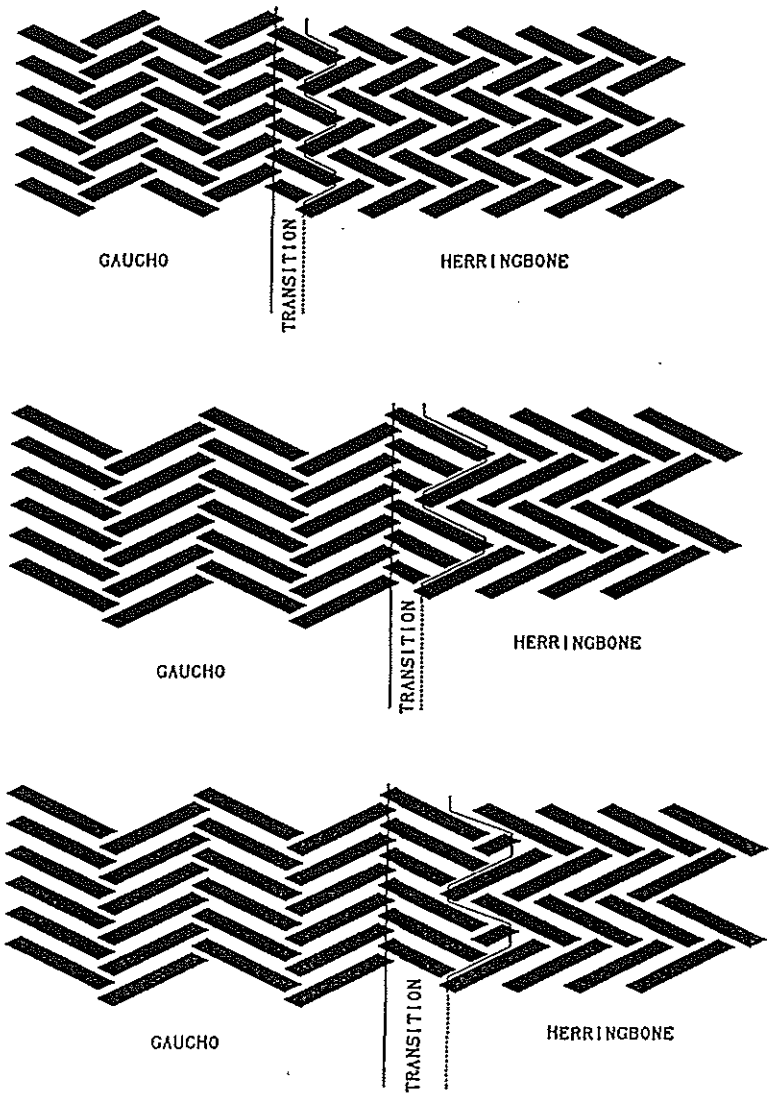


Fig. 111 — Gaucho/Herringbone codings.

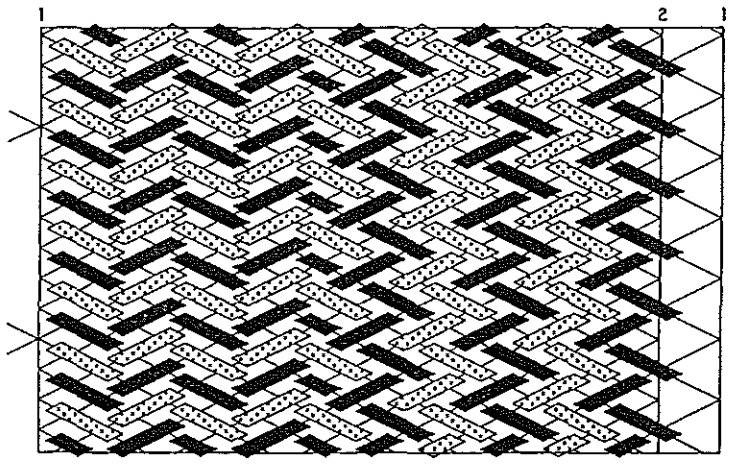


Fig. 112 — 2-colour, 2-pass Gaucho/Herringbone coding.

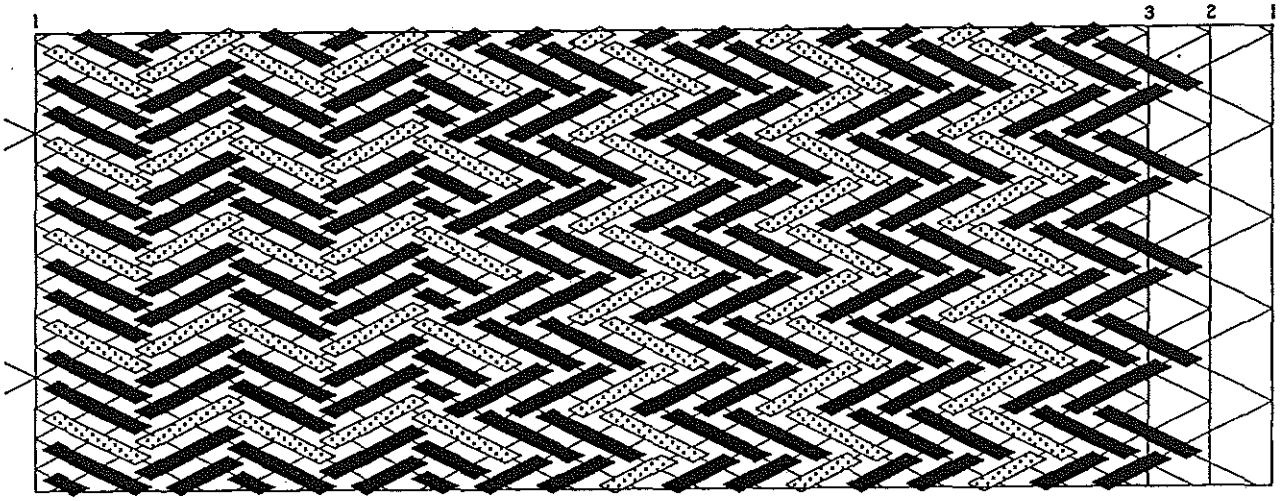


Fig. 113 — 2-colour, 3-pass Gaucho/Herringbone coding.

For the purpose of an Example we can consider such knots being used to make a handle knot for a quirt:

(1). The Herringbone-coding is placed towards the wrist-loop end of the quirt-handle, and the Gaucho-coding is placed towards the lash end of the quirt-handle.

(2). The Herringbone-coding is placed towards the lash end of the quirt-handle, and the Gaucho-coding is placed towards the wrist-loop end of the quirt-handle.

The much superior orientation for this particular application in single-colour is that given in **(2)** above. The Herringbone-coded section should preferably be much longer than the Gaucho-coded section. The orientation in **(1)** for this particular single-colour application has the appearance of being upside-down on the finished product.

The much superior orientation for this particular application in multi-colour is that given in **(1)** above. The Herringbone-coded section should preferably be shorter than the Gaucho-coded section. Especially the 2-clour application gives a very attractive result. The orientation in **(2)** for this particular multi-colour application has the appearance of being upside-down on the finished product.

When considering the orientation of lopsided weaving-patterns and colour-patterns in the design of a project, there will most likely be one orientation that gives a superior and more attractive finished result.

The colour-pattern in Fig. 113 can be derived at by using a **primary** colour to make two passes of the knot, then interweaving a *secondary* colour to make the third pass. There is a measure of **similarity** between the clour-patterns resulting from the *Herringbone*-weave and that of the *Gaucho*-weave. In each instance, connected weaves of the *secondary* colour produce a number of separate and distinct zig-zag patterns. In the case of the Herringbone section the zig-zags go around the circumference of the knot, while in the Gaucho section the zig-zags go from the left bight-boundary towards the right bight-boundary.

Fig. 114 depicts another 3-pass knot with a Gaucho/Herringbone mix of weaves. However, here we use an alternative transition to produce a different colour-pattern in

the Herringbone section while leaving the colour-pattern in the Gaucho section as it was before. The colour-pattern of the Herringbone section now consists of disconnected weaves of the *secondary* colour, producing in effect a broken pattern. This pattern by itself is quite attractive, but is dissimilar to the connected weave colour-pattern of the Gaucho section. This combination of colour-patterns is at least somewhat less desirable because of its lack of *similarity*. It is also the result of the colour-patterns being dissimilar that it is more difficult to orient such a combination in an attractive fashion.

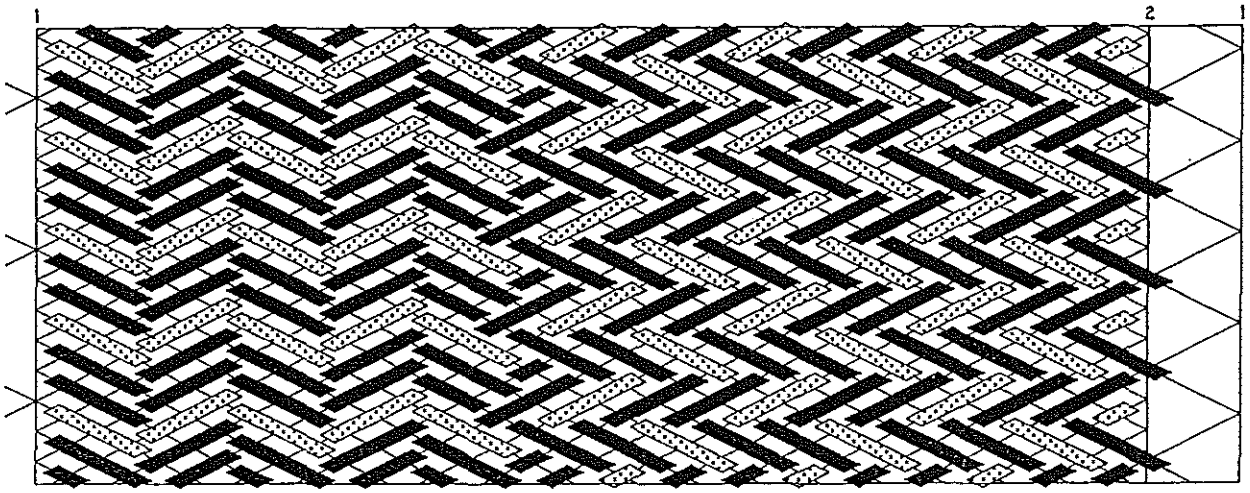


Fig. 114 — An alternative 2-colour, 3-pass Gaucho/Herringbone coding.

The values of the parameters which govern the type of knots depicted in Figs. 110, 112, 113, 114 can readily be calculated as follows:

Let x be the column-distance between the left-hand bight-boundary 1 and the innermost right-hand bight-boundary (2 in the Figs. 110, 112, 114 and 3 in Fig. 113). Let B be the total number of bights on the left-hand bight-boundary, and let P be the total number of parts.

For the single-string type of knot depicted in Fig. 110:

$$\begin{aligned} B &= \text{even (condition imposed by Herringbone-coding)}, \\ x &= \text{odd (condition imposed by } B = \text{even)}, \\ |x+1|_{\frac{B}{2}} &= \text{coprime with } \frac{B}{2}, \\ P &= x+1. \end{aligned}$$

For the 2-string type of knot depicted in Fig. 112:

$$\begin{aligned} B &= \text{even (condition imposed by Herringbone-coding)}, \\ x &= \text{even (condition imposed by } B = \text{even)}, \\ \left|\frac{x}{2}+1\right|_{\frac{B}{2}} &= \text{coprime with } \frac{B}{2}, \\ \left|\frac{x}{2}\right|_{\frac{B}{2}} &= \text{coprime with } \frac{B}{2}, \\ P &= x+1. \end{aligned}$$

The overall weave consists of two interwoven knots. One with a string-run which runs between the left-hand bight-boundary 1 and the right-hand bight-boundary 1; this knot has a number of parts $p_1 = \frac{x}{2} + 1$, and a number of bights $b_1 = \frac{B}{2}$. The other, a Casa Knot, with a string-run which runs between the left-hand bight-boundary 1 and the right-hand bight-boundary 2; this knot has a number of parts $p_2 = \frac{x}{2}$, and a number of bights $b_2 = \frac{B}{2}$.

For the 2-string type of knot depicted in Fig. 113:

B = a multiple of 3 (condition imposed by Herringbone-coding),

x = a multiple of 3 (condition imposed by B = a multiple of 3),

$\left|\frac{2x}{3} + 2\right|_{\frac{B}{3}}$ = coprime with $\frac{B}{3}$,

$\left|\frac{x}{3}\right|_{\frac{B}{3}}$ = coprime with $\frac{B}{3}$,

$P = x + 2$.

The overall weave consists of two interwoven knots. One with a string-run which runs between the left-hand bight-boundary 1 and the right-hand bight-boundaries 1 & 2; this knot has a number of parts $p_1 = \frac{2x}{3} + 2$, and a number of bights $b_1 = \frac{2B}{3}$. The other, a Casa Knot, with a string-run which runs between the left-hand bight-boundary 1 and the right-hand bight-boundary 3; this knot has a number of parts $p_2 = \frac{x}{3}$, and a number of bights $b_2 = \frac{B}{3}$.

For the 3-string type of knot depicted in Fig. 114:

B = a multiple of 3 (condition imposed by Herringbone-coding),

x = a multiple of 3 (condition imposed by B = a multiple of 3),

$\left|\frac{x}{3} + 1\right|_{\frac{B}{3}}$ = coprime with $\frac{B}{3}$,

$\left|\frac{x}{3}\right|_{\frac{B}{3}}$ = coprime with $\frac{B}{3}$,

$P = x + 1$.

The overall weave consists of three interwoven knots. One with a string-run which runs between the left-hand bight-boundary 1 and the right-hand bight-boundary 1; this knot has a number of parts $p_1 = \frac{x}{3} + 1$, and a number of bights $b_1 = \frac{B}{3}$. Two other, one of which is a Casa Knot, with string-runs which run between the left-hand bight-boundary 1 and the right-hand bight-boundary 2; these two knots have respectively a number of parts $p_2 = p_3 = \frac{x}{3}$, and a number of bights $b_2 = b_3 = \frac{B}{3}$.

It will be instructive for the reader to derive the above relationships, which may readily be obtained from their general string-runs and coding-patterns.

The uppermost grid-diagram in Fig. 115 depicts a weave as in Fig. 112, but which requires three strings in its construction (B = a multiple of 4 ; x = a multiple of 4 ; $\left|\frac{x}{2}+1\right|_{\frac{B}{2}}$ = coprime with $\frac{B}{2}$; $\left|\frac{x}{4}\right|_{\frac{B}{4}}$ = coprime with $\frac{B}{4}$; $P = x+1$). The 'black' string should here be the strong contrasting colour, while the 'light' strings should be more neutral and much less contrasting in colour.

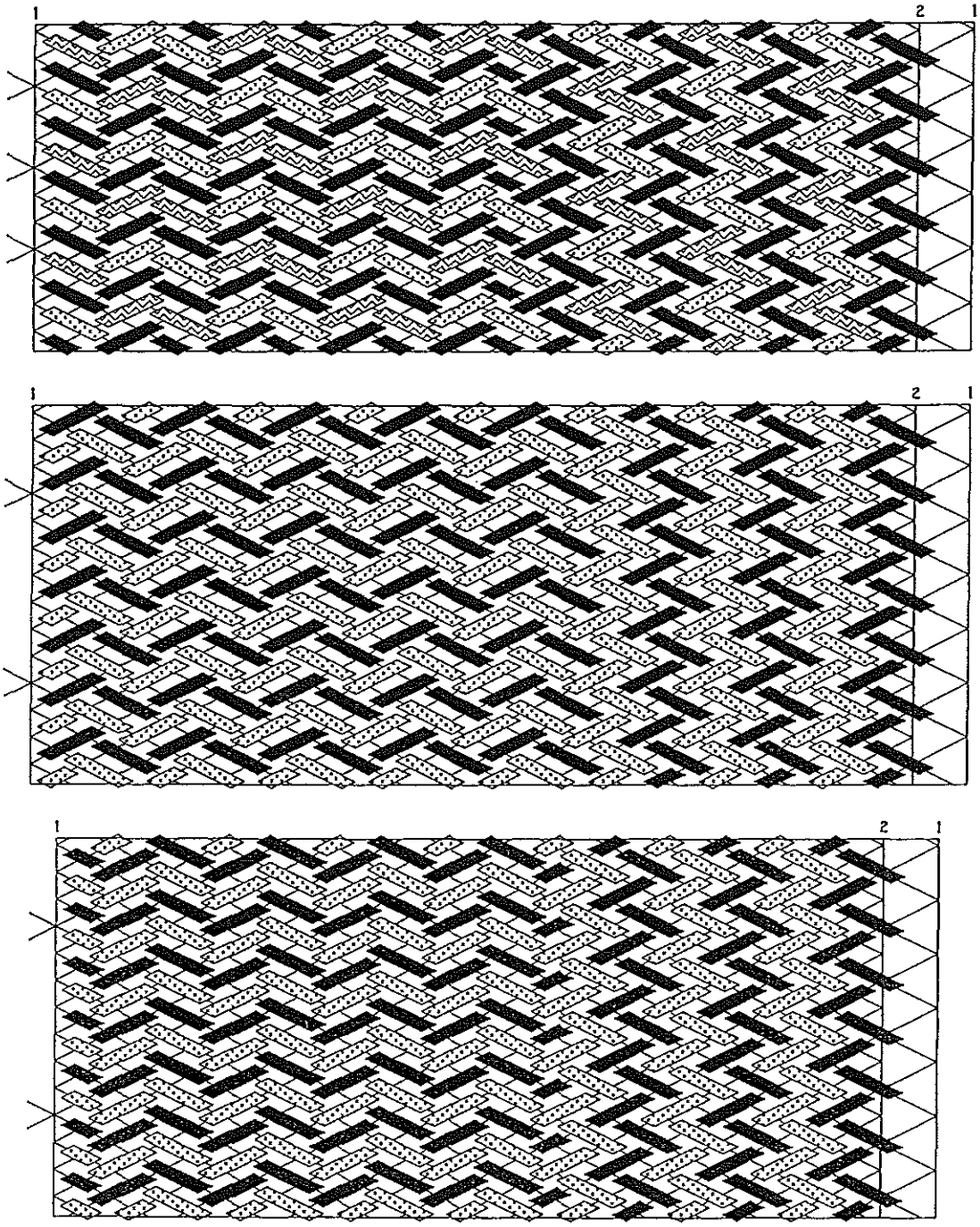


Fig. 115 — Some further examples.

Not only can similar colour-patterns often be obtained with different weave combinations, but transitions between the weave-patterns can often be absorbed by one of these weave-patterns as shown by the two lowermost grid-diagrams in Fig. 115.

THE BRAIDER

No.7 **AUGUST 1996.**

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Solutions to the Questions in Issue No. 6

Question on pg. 115.

The leftmost point in the *complementary cyclic bight-number scheme* receives the number 0. The point immediately to the right of this leftmost point receives the number Δ^*. In order to reach this point we have counted in cyclic fashion, from the leftmost point onwards, Δ^* times $|-p|_b$ points to the right. Hence $\left|\Delta^* \cdot |-p|_b\right|_b = 1$.

Since $|-p|_b = b - |p|_b$, we obtain $\left|\Delta^* \cdot |-p|_b\right|_b = \left|\Delta^* \cdot (b - |p|_b)\right|_b = \left|-\Delta^* \cdot |p|_b\right|_b = 1$.

Hence $\left|\Delta^* \cdot |p|_b\right|_b = b - 1$, and thus:

$$\Delta^* = \frac{n^* \cdot b - 1}{|p|_b}$$ where n^* is the smallest positive integer such that Δ^* is an integer.

The reader should note that $\Delta + \Delta^* = b$ (this follows from the fact that b and $|p|_b$ are coprime (since p and b are coprime)); both Δ and Δ^* are coprime with b, hence Δ and Δ^* are coprime.

Question on pg. 117.

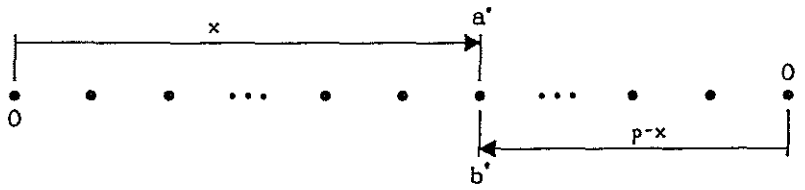


Fig. 116 — The schematic layout of the algorithm diagram.

Let the arbitrary intersection column which carries $i = a^*$ for the half-cycles which run from left to right be at a distance of x columns from the left-hand bight-boundary, then this intersection column is at a distance of $(p - x)$ columns from the right-hand bight-boundary. Hence:

$$a^* = |x \cdot \Delta^*|_b ,$$
$$b^* = \left|(p - x) \cdot \Delta^*\right|_b .$$

Furthermore:

$$\left.\begin{array}{l} |p \cdot \Delta^*|_b = \left||p|_b \cdot \Delta^*\right|_b = b - 1 \\ |x \cdot \Delta^*|_b \leq b - 1 \end{array}\right\} \text{hence:} \quad \left||p \cdot \Delta^*|_b - |x \cdot \Delta^*|_b\right|_b = |p \cdot \Delta^*|_b - |x \cdot \Delta^*|_b .$$

Thus:

$$a^* + b^* = |x \cdot \Delta^*|_b + \left|(p - x) \cdot \Delta^*\right|_b = |x \cdot \Delta^*|_b + \left||p \cdot \Delta^*|_b - |x \cdot \Delta^*|_b\right|_b = |p \cdot \Delta^*|_b = b - 1 .$$

When $b =$ **odd**, then $b - 1 =$ **even**; p may be **odd** or **even**. Hence a^* and b^* both **odd** or both **even**. Δ^* may be **odd** or **even**. Consequently when b is **odd**, we only know that the parities of a^* and b^* are the same, but do not a priori know what they are.

When $b =$ **even**, then $b - 1 =$ **odd**; p must be **odd** (p and b are coprime). Hence a^* and b^* are of different parity. Furthermore:

$$\left.\begin{array}{l} b = \text{even} \\ p = \text{odd} \\ b = \text{even} \end{array}\right\} \text{ hence: } \left.\begin{array}{l} nb+1 = \text{odd} \\ |-p|_b = \text{odd} \end{array}\right\} \text{ hence: } \Delta^* = \frac{nb+1}{|-p|_b} = \frac{\text{odd}}{\text{odd}} = \text{odd}.$$

Hence when b is **even**, we know that the parities of the entries in the top-line and in the bottom-line of the algorithm diagram are as follows:

$$\begin{array}{ccccccccccccccccc} 0 & o & e & o & e & o & e & \cdots & o & e & o & e & o & e \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & e & o & e & o & e & o & \cdots & e & o & e & o & e & o & 0 \end{array}$$

The Regular Knots

In *The Braider* No. 5, pg. 90, we showed that Regular Knots were essentially made incarnate by their string-run only; hence strictly speaking, their coding (their weaving pattern) does not play an essential role. In the previous issue (*The Braider* No. 6) we have seen that **all** the Regular Knots are uniquely presented in the **RKT**, and that any specific arbitrary Regular Knot has evolved from the 1-part/1-bight Regular Knot via an unique sequence of uniquely distinct Regular Knots.

These facts expose one of the fundamental misconceptions in the classical knot theory, and renders that theory useless for real (physical) knots.†

We have seen that the RKT was the direct consequence of the two distinct enlargement processes I and II, which were respectively represented by the Diophantine equations $p \cdot \Delta b = b \cdot \Delta p + 1$ and $p \cdot \Delta b = b \cdot \Delta p - 1$.

⋆⋆ *What is the essential difference between the geometrical (graphical) and the analytical derivations, apart from one being geometrical and the other analytical, of these Diophantine equations on pg. 109? Give for each Diophantine equation a geometrical derivation based on the principal parameters used in the analytical approach on pg. 109, and an analytical derivation based on the principal parameters used in the geometrical approach on pg. 109.*‡

We have seen that going from the trunk-node 1/1 up the tree to the node p/b is a simple matter when we know the path-formula associated with the Regular Knot p/b. Hence the question: how do we obtain this path-formula?

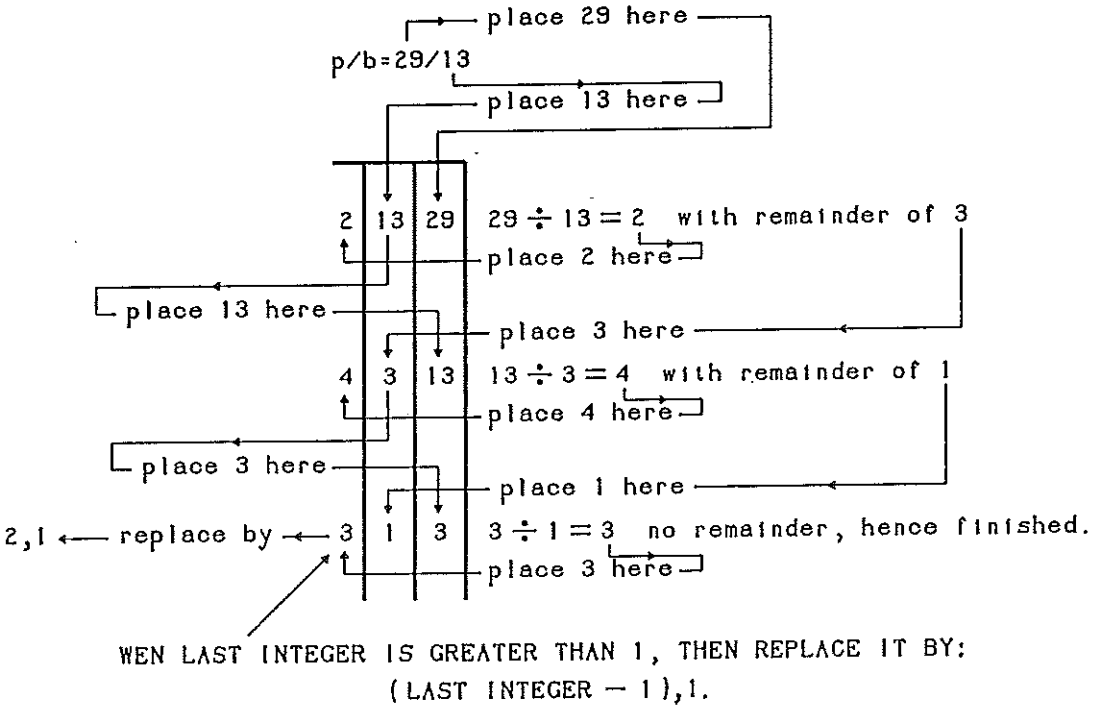
The path-formula is obtained with the aid of **Euclid's algorithm**. This algorithm is set out on pg. 135 using two examples. The reader will note that the path-formula is obtained from the left-hand column of numbers.

We have now all the necessary tools which enable us to navigate in the RKT, a process of some great practical importance. So let's have a look at a very basic application, namely the braiding of Casa-coded Regular Knots.

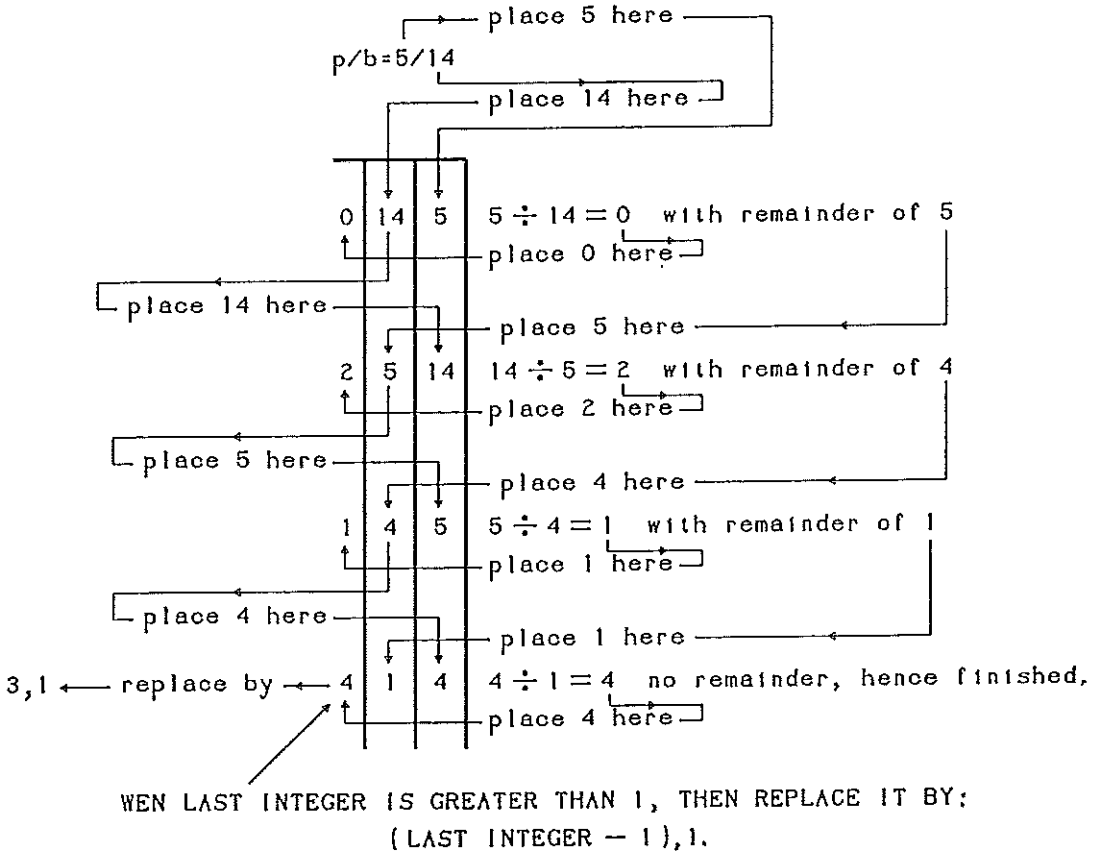
---

† Since the classical knot theory fails to recognize the independent nature of the string-run and the coding, it erroneously finds many uniquely different knots to be equivalent.

‡ In order to become fully acquainted with these enlargement methods, it is of great importance to fully understand the mechanics of these derivations.


place 29 here
p/b=29/13
place 13 here
2 13 29 29 ÷ 13 = 2 with remainder of 3
place 2 here
place 13 here
place 3 here
4 3 13 13 ÷ 3 = 4 with remainder of 1
place 4 here
place 3 here
place 1 here
2,1 ← replace by ← 3 1 3 3 ÷ 1 = 3 no remainder, hence finished.
place 3 here
WEN LAST INTEGER IS GREATER THAN 1, THEN REPLACE IT BY:
(LAST INTEGER − 1),1.

The path-formula from the trunk-node 1/1 to the node 29/13 is [2;4,2,1].


place 5 here
p/b=5/14
place 14 here
0 14 5 5 ÷ 14 = 0 with remainder of 5
place 0 here
place 14 here
place 5 here
2 5 14 14 ÷ 5 = 2 with remainder of 4
place 2 here
place 5 here
place 4 here
1 4 5 5 ÷ 4 = 1 with remainder of 1
place 1 here
place 4 here
place 1 here
3,1 ← replace by ← 4 1 4 4 ÷ 1 = 4 no remainder, hence finished.
place 4 here
WEN LAST INTEGER IS GREATER THAN 1, THEN REPLACE IT BY:
(LAST INTEGER − 1),1.

The path-formula from the trunk-node 1/1 to the node 5/14 is [0;2,1,3,1].

Since Casa-coded Regular Knots have an alternating *over–under* coding, any added half-cycles, due to the enlargement of such a knot to a Regular Knot which is also Casa-coded, must be in pairs. Hence we must always apply two consecutive enlargements of which the second one lays down half-cycles which are adjacent to the half-cycles laid down by the first one.

Let the Casa-coded Regular Knot which has to be enlarged have p-parts and b-bights. Let the **fundamental enlargement** of this knot produce a Regular Knot with $(p + p_0)$-parts and $(b + b_0)$-bights. Say we give the Casa Knot an order n enlargement for the first enlargement. The resulting enlarged Regular Knot has then $(p_0 + np)$-parts and $(b_0 + nb)$-bights. Hence the first enlargement produces $(p_0 + [n-1]p)$ additional parts and $(b_0 + [n-1]b)$ additional bights.

The Regular knot represented in the RKT by the node immediately preceeding the node of this enlarged knot has $(p_0 + [n-1]p)$-parts and $(b_0 + [n-1]b)$-bights. Say that for the second enlargement we apply an order m enlargement, which is then applied on the knot with $(p_0 + np)$-parts and $(b_0 + nb)$-bights. The resulting enlarged knot has then $([m+1]p_0 + [n(m+1)-1]p)$-parts and $([m+1]b_0 + [n(m+1)-1]b)$-bights. Hence the second enlargement produces $(mp_0 + [mn-1]p)$ additional parts and $(mb_0 + [mn-1]b)$ additional bights.

In order to enable the final Regular Knot to have a Casa-coding, we must ensure that the increase in parts resulting from the first enlargement is equal to the increase in parts resulting from the second enlargement, and that the increase in bights resulting from the first enlargement is equal to the increase in bights resulting from the second enlargement. Hence it will be obvious that the value of m must be 1, and that the two enlargements must be of the same method, hence both of Method I or both of Method II. Note that n may have any value ≥ 1. Hence we obtain the following Rule for enlarging a Casa-coded Regular Knot to a larger Casa-coded Regular Knot:

To enlarge a Casa-coded Regular Knot to a larger Casa-coded Regular Knot, the original Casa-coded knot must be given two successive enlargements of the same Method; the first enlargement may be of any order, while the second enlargement must be of the first order only.

The two general enlargement paths in the RKT, one for Method I and one for Method II, from one Casa-coded Regular Knot to the next larger Casa-coded Regular Knot are depicted in Fig. 117.

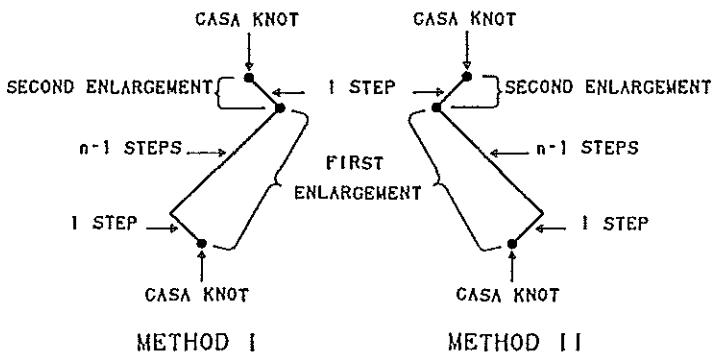


Fig. 117 — The general enlargement paths for Casa-coded Regular Knots.

For a Casa-coded Regular Knot the Figs. 118–120 depict a Method I order 3 enlargement, and the Figs. 121–123 depict a Method II order 3 enlargement.

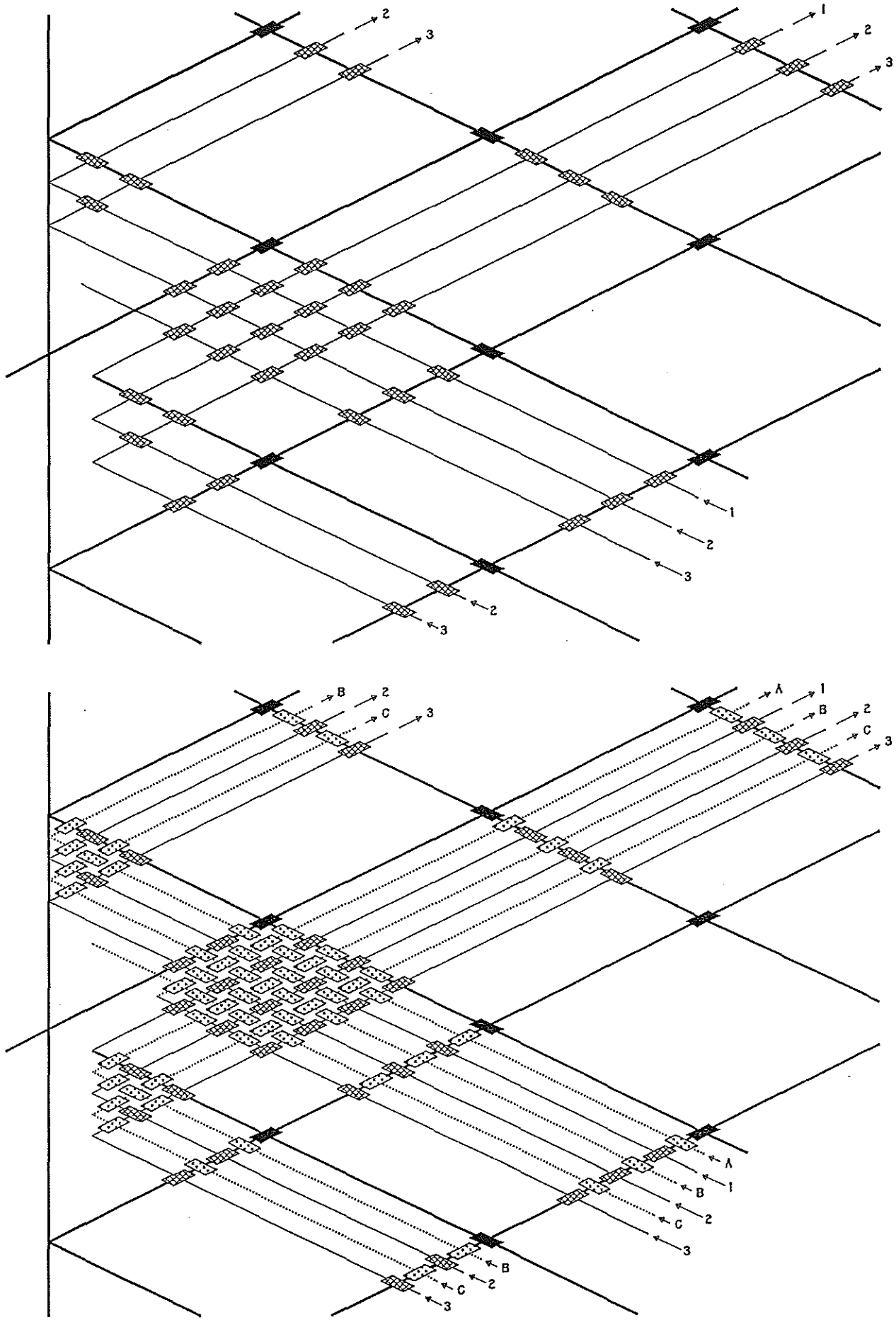


Fig. 118 — A Casa-coding enlargement Method I, order 3. First and second enlargement.

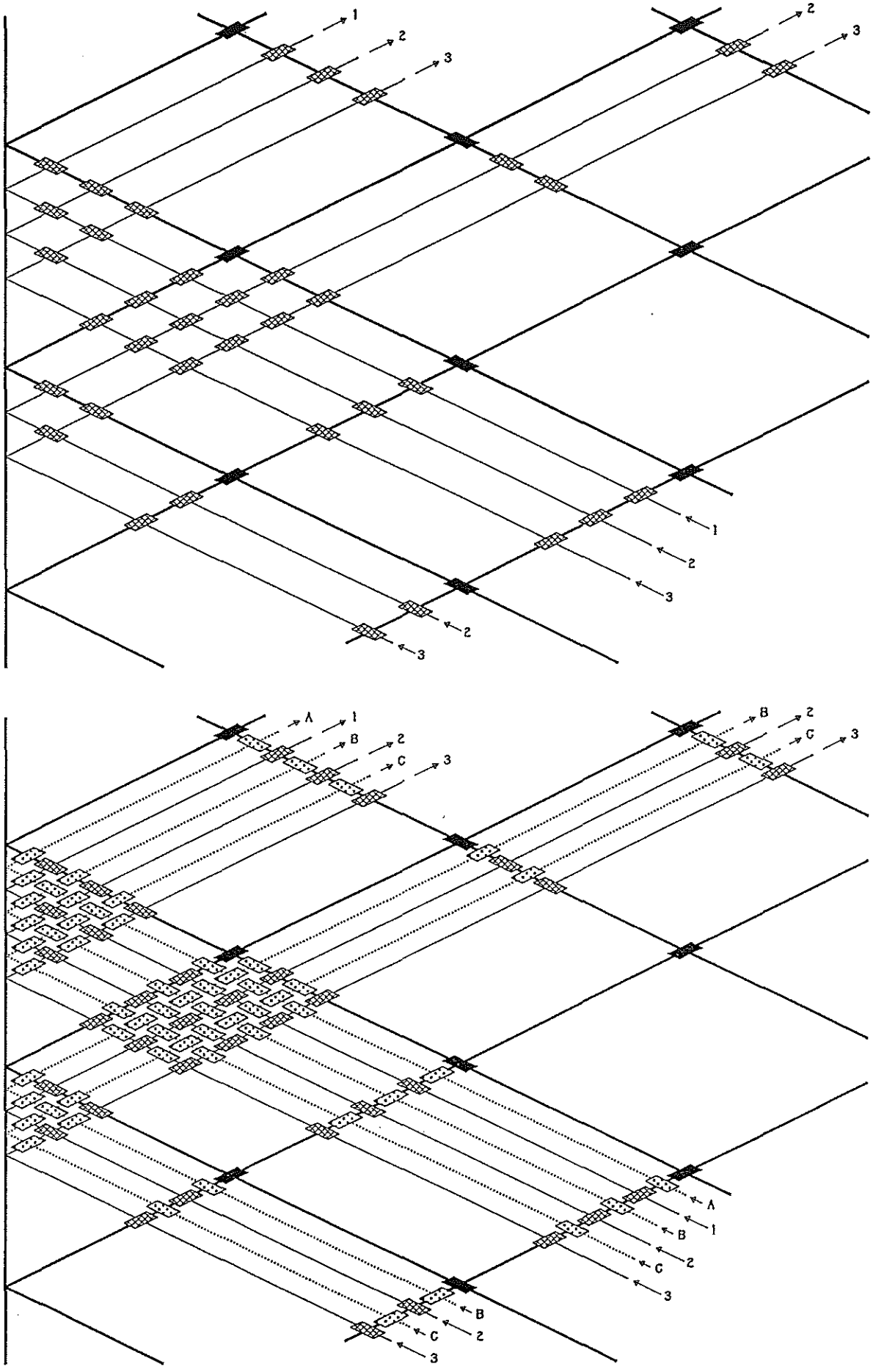


Fig. 119 — A Casa-coding enlargement Method I, order 3. First and second enlargement.

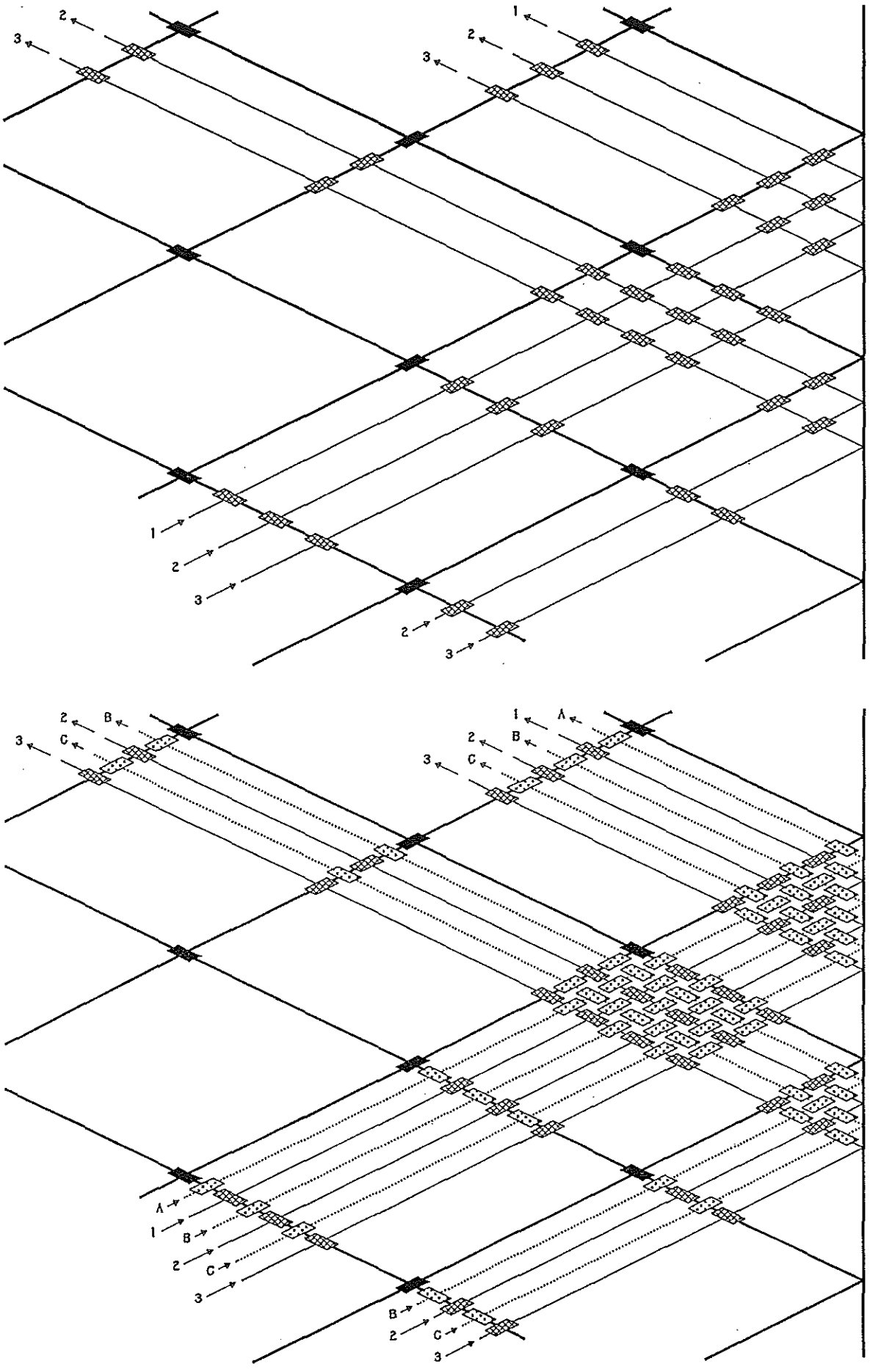


Fig. 120 — A Casa-coding enlargement Method I, order 3. First and second enlargement.

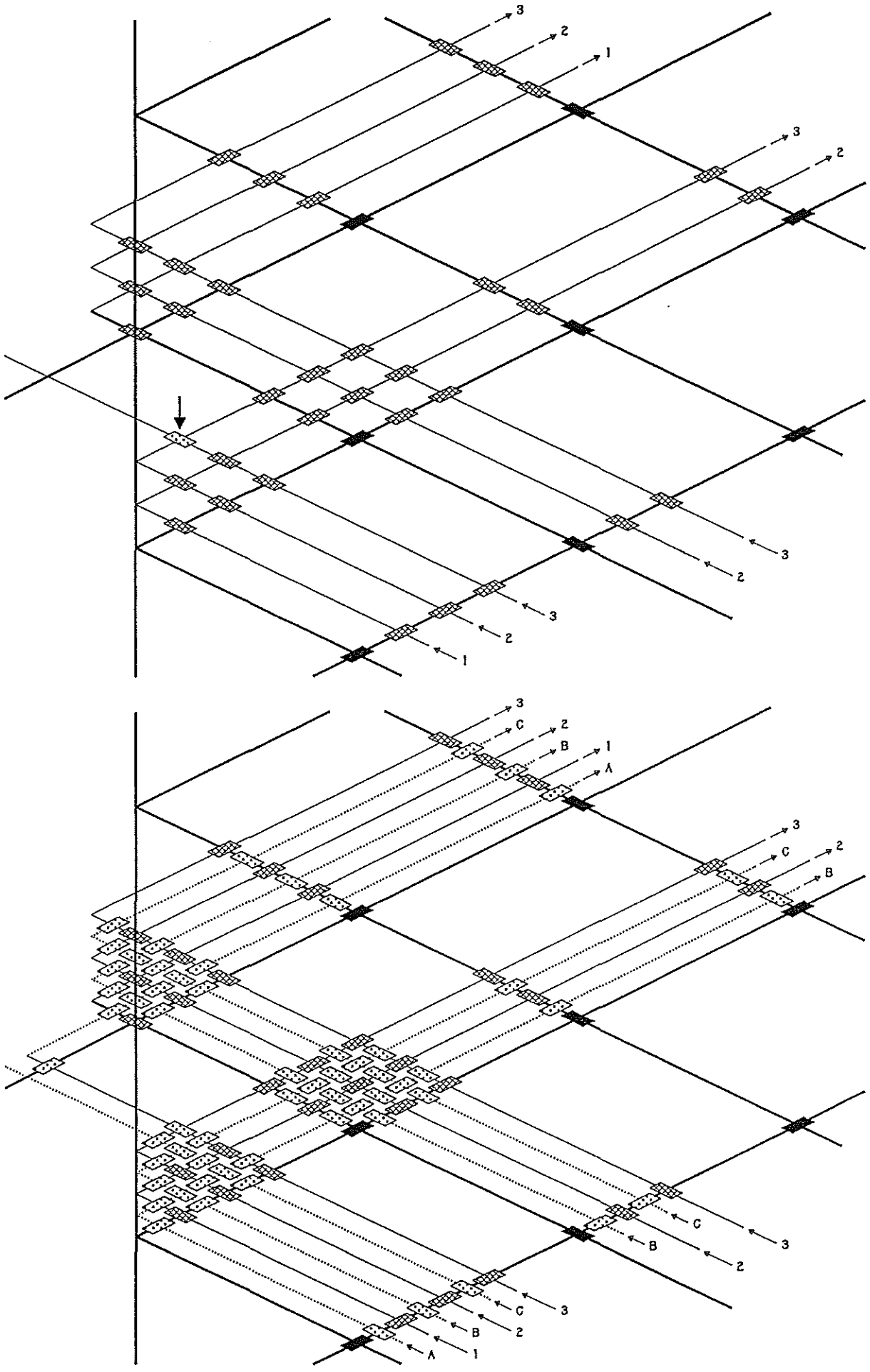


Fig. 121 — A Casa-coding enlargement Method II, order 3. First and second enlargement.

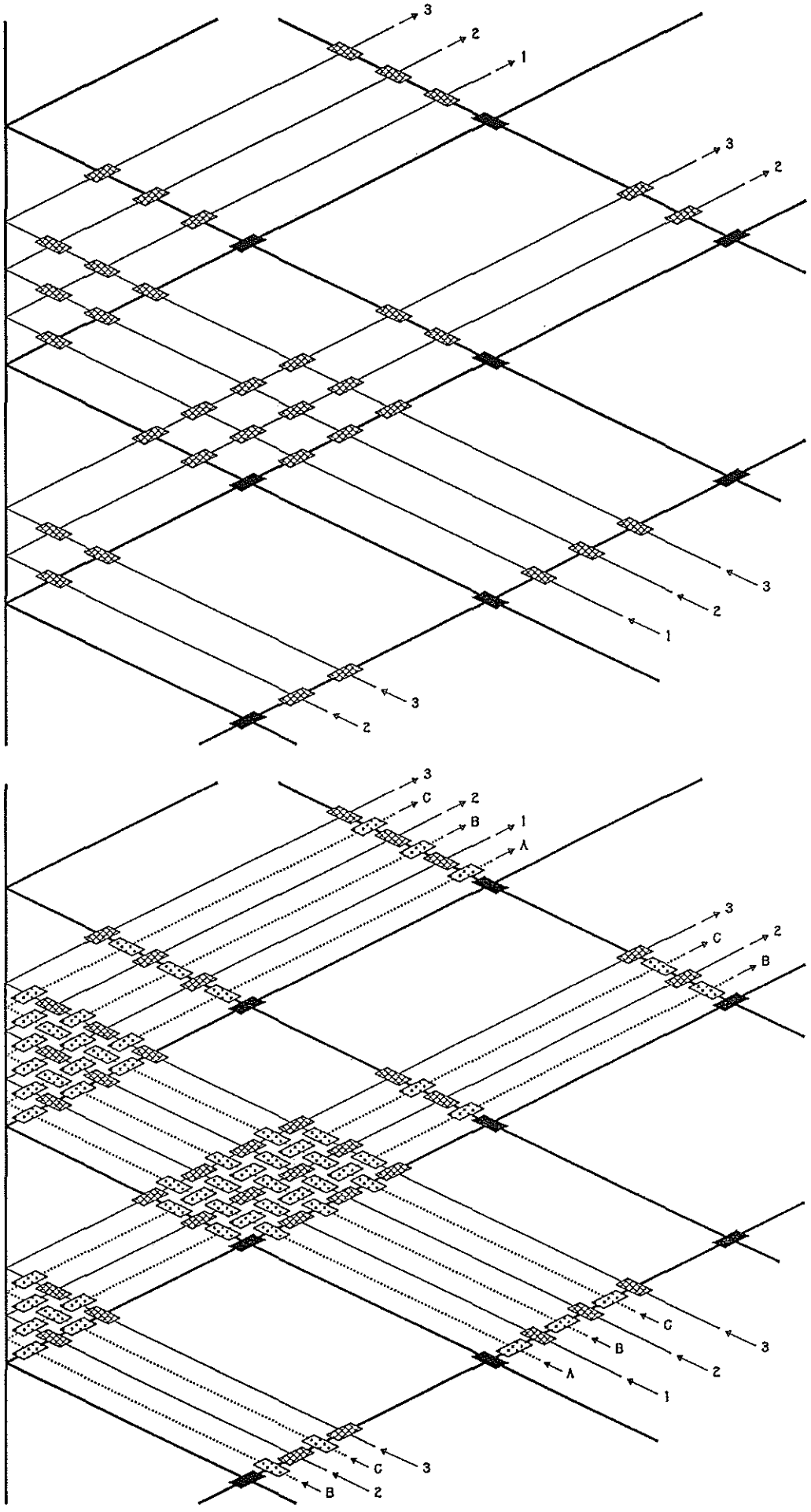


Fig. 122 — A Casa-coding enlargement Method II, order 3. First and second enlargement.

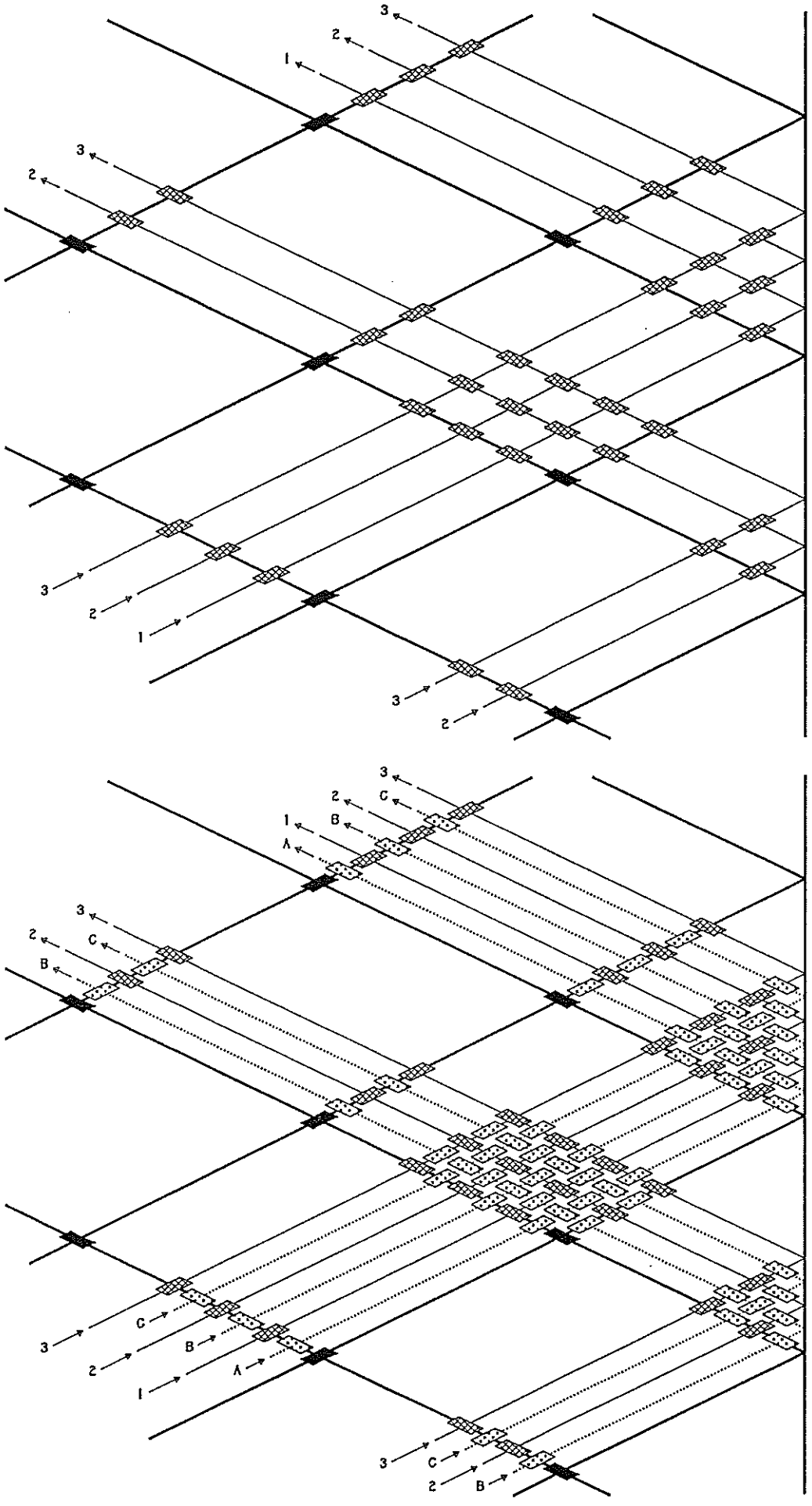


Fig. 123 — A Casa-coding enlargement Method II, order 3. First and second enlargement.

We have seen that in order to enlarge a Casa-coded Regular Knot to a larger Casa-coded Regular Knot, we must give the original Casa-coded knot two successive enlargements of the same Method; the first enlargement may be of any order, while the second enlargement must be of the first order only. The first enlargement we shall denote by '**laying the track**' and the second enlargement by '**splitting the tracks**'. During the *track-laying phase*, we follow an already existing half-cycle, which we shall denote by '**the standing-track**', while the track half-cycle we are laying down is denoted by '**the working-track**'. From the Figs. 118–123 we derive the following Rules:

In an Enlargement Method I raising process, the working-track lies immediately below the standing-track.

In an Enlargement Method II raising process, the working-track lies immediately above the standing-track.

For Enlargement Method I, the first crossing-movement of a working-track half-cycle is the same as the last crossing-movement of the immediately preceding working-track half-cycle. Any other crossing-movement of a working-track half-cycle over a half-cycle is the same as that of the standing-track over the same half-cycle.

For Enlargement Method II, when $p > 3$ (the number of parts of the Casa-coded Regular Knot obtained through the enlargement processes), the last crossing-movement of a working-track half-cycle is the same as the first crossing-movement of the immediately succeeding working-track half-cycle. Any other crossing-movement of a working-track half-cycle over a half-cycle is the same as that of the standing-track over the same half-cycle.

For Enlargement Method I, 'splitting the tracks' begins with the first crossing of the first 'track-splitting' half-cycle.

For Enlargement Method II, 'splitting the tracks' begins with the penultimate crossing of the last 'track-laying' half-cycle when $p > 3$ (the number of parts of the Casa-coded Regular Knot obtained through the enlargement processes), and with the final crossing of the last 'track-laying' half-cycle when $p = 3$.

The minimum number of parts for a fully fledged Casa-coded Regular Knot is two; although the 2-parts/1-bight Regular Knot could be regarded as an exception, we shall however regard it as a Casa-coded Regular Knot. Furthermore in order to simplify the rules for the raising of Casa-coded Regular Knots, we shall include under the Casa-coded Regular Knots the Regular Knots which have 1-part. The 1-part Regular Knots may be seen as embryonic Casa knots.

When such embryonic Casa Knots are enlarged by Method I or II, the first crossing in the track-laying phase (indicated by F in Fig. 124) may be chosen freely. In the case of Method I, this first crossing occurs at the beginning of the 2nd-order track-laying process, while in the case of Method II this first crossing occurs at the beginning of the 1st-order track-laying process. It will be observed from Fig. 124 that for Method II during the 1st-order track-laying process, we can say that the first crossing-movement of a working-track half-cycle is the same as the last crossing-movement of the immediately preceding working-track half-cycle, hence a process according to a Rule identical to that for Method I. The rightmost diagram in Fig. 124 shows that during the 2nd-order track-laying process we have to follow the normal Rules for Method II.† Thus for a

† Note from the rightmost diagram in Fig. 124 that for the first half-cycle of the

Method II enlargement process when the number of parts of the finally obtained Casa-coded Regular Knot is greater than 3 ($p > 3$), we must use for track-laying the Rules associated with this Enlargement process.

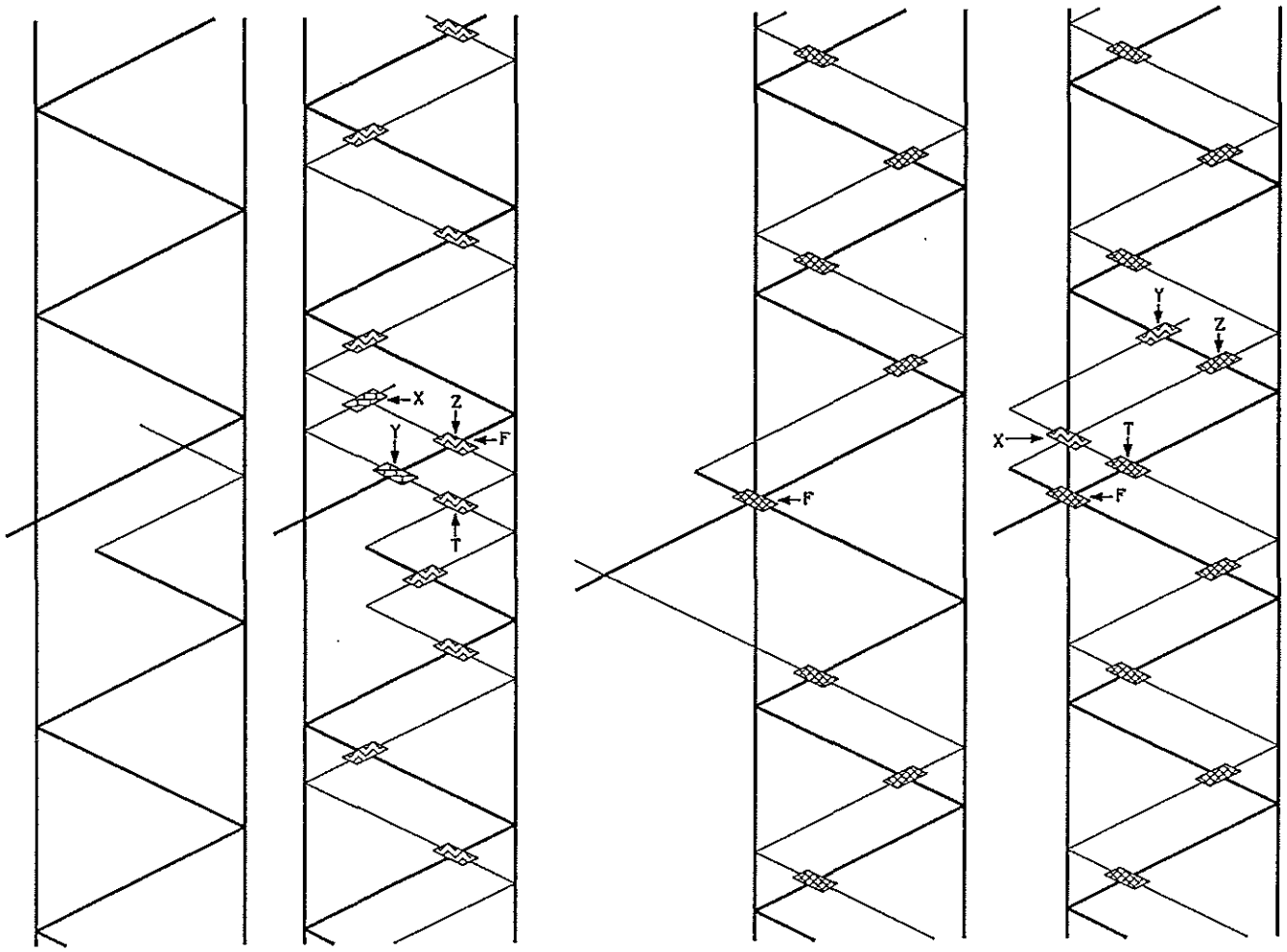


Fig. 124 — Track-laying in the Casa-coding enlargement of 1-part Regular Knots. Left two diagrams Method I. Right two diagrams Method II.

By selecting the appropriate enlargement order for the track-laying phase (recall that the track-splitting phase always has to be of the 1st-order), we are able to obtain any size Casa-coded Regular Knot from the following **four** basic knot types :

2nd-order track-laying we can still use the Rule for Method I, and that for the second half-cycle of the 2nd-order track-laying we can use for the first crossing-movement the Rule for Method I while for the last crossing-movement the Rule for Method II must be used (the crossing-movement 'X' must be the same as the crossing-movement 'Y' which in turn is determined by 'Z' (*over*-movements due to our initial free choice of crossing-movement); however, we can for these first two half-cycles also use the rules for Method II only (the crossing-movement 'T' must be the same as the crossing-movement 'F', and the (last) crossing-movement of the first half-cycle of the 2nd-order track-laying must be the same as the crossing-movement 'T' (*over*-movements due to our initial free choice of crossing-movement).

Any Casa-coded Regular Knot of which the number of parts is smaller than the number of bights can be obtained from either a 1-part or 2-parts Casa-coded Regular Knot.

Any Casa-coded Regular Knot of which the number of parts is greater than the number of bights can be obtained from either a 1-bight or 2-bights Casa-coded Regular Knot.

Note that 1-part Casa-coded Regular Knots have in fact no coding since they are embryonic Casa Knots (refer to our earlier discussion of these Regular Knots).

A braider can readily braid on his or her fingers **any** of the small Casa-coded Regular Knots especially. In fact every braider should be able to do so without remembering the appropriate braiding recipe for each of the many small Casa Knots. All that is required, is to know the above easy to remember enlargement Rules, and to be able to work in the RKT. We have to be able to work in the RKT in order to know what enlargement-order must be used for the track-laying enlargement phase. Let's illustrate this with three Examples:

Example 1:

Say we want to braid a 5-parts/8-bights Casa-coded regular Knot.

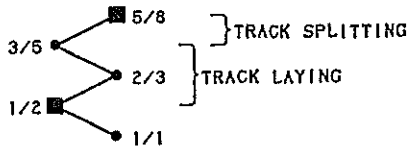


Fig. 125 — Example 1.

From its path-formula ([0; 1, 1, 1, 1, 1]) we know that the enlargement steps are [II; I, II, I, II,], consequently the last step is a 1st-order Enlargement Method II ([II, I, II, I, **II**,]) since this **II** is equal to one elementary step (1). This will be the track-splitting phase. The track-laying phase must therefore also be an Enlargement Method II. The track-laying phase is therefore indicated by the **II, I** ([II, I, **II, I**, II,]) before the last II in the path-formula. This **II, I** is 1, 1, hence a 2nd-order Enlargement Method II. Thus we start with a 1-part/2-bights Regular Knot, then a track-laying phase according to Method II† till we have obtained 5 bights (the 3/5 Regular Knot), and finally followed by the 1st-order Enlargement Method II track-splitting phase.

Example 2:

Say we want to braid a 7-parts/5-bights Casa-coded Regular Knot.

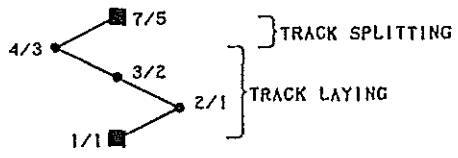


Fig. 126 — Example 2.

From its path-formula ([1; 2, 1, 1]) we know the enlargement steps are [II; I, II,]. The last step is one elementary 1st-order Enlargement Method II ([II, I, **II**,]). This 1st-order Enlargement Method II track-splitting phase must be preceded by an Enlargement Method II track-laying phase; it is contained in the steps **II, I** in [**II**; I, II,].

† The 1st-order part of this phase is according to Method I as explained on pp. 143–144.

Since **II,I** is 1,2, this track-laying phase is a 3rd-order Enlargement Method II, performed on the 1-part/1-bight Casa-coded Regular Knot. Thus we start with a 1-part/1-bight Regular Knot, then a track-laying phase according to Method II† till we have obtained 3 bights (the 4/3 Regular Knot), and finally followed by the 1st-order Enlargement Method II track-splitting phase.

Example 3:
Say we want to braid a 23-parts/9-bights Casa-coded Regular Knot.

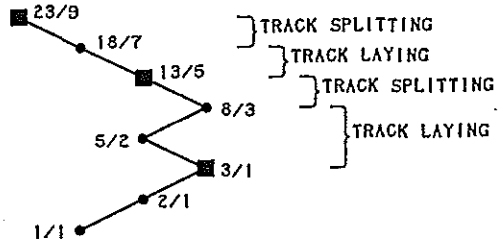


Fig. 127 — Example 3.

From its path-formula ([2;1,1,3,1,]) we know that the enlargement steps are [II;I,II,I,]. The last step consists of 3 elementary steps, which have to be assembled into 1 elementary step followed by 2 elementary steps. This 1 elementary step is a 1st-order track-splitting phase, Enlargement Method I, which results in a Casa-coded Regular Knot (the 13/5 Casa Knot). This 1st-order Enlargement Method I track-splitting phase must be preceded by an Enlargement Method I track-laying phase; it is contained in the steps **I,II** in [II;**I,II**,I,]. Since **I,II** is 1,1, this track-laying phase is a 2nd-order Enlargement Method I, performed on the 3-parts/1-bight Casa-coded Regular Knot till 3 bights have been obtained. Then follows a track-splitting phase which results in the 13/5 Casa-coded Regular Knot. Next, the 13/5 Casa-coded Regular Knot undergoes a 1st-order Enlargement Method I track-laying phase, followed by a 1st-order Enlargement Method I track-splitting phase. This will result in the 23-parts/9-bights Casa-coded Regular Knot.

The reader is advised to study the Supplement to Pamphlet No. 4, in which several examples are fully worked out.

Braiding Instructions
and their
Presentation

Round braids are braids in the form of a cylinder without ends, or to put it technically a little more precise: it are braids in the form of a cylinder which has one end at minus infinity and the other end at plus infinity (Euclidian geometry). The cylindrical braid is formed by two sets of strings: one set in which each string describes a right-hand helix while each string in the other set describes a left-hand helix‡. In the

† The 1st-order part of this phase is according to Method I as explained on pp. 143–144.
‡ A round braid in which the string-run consists solely out of either right-hand helixes

most elementary case, each set contains an equal number of strings; this are the round braids we normally encounter, and to which we shall in this article limit our discussion.

The 'standard' method for laying down the string-run sequence in round braids with an even number of strings is depicted in Fig. 128 for an 8-string round braid. Note that we are discussing the **string-run**, hence discard any interweaving due to a superimposed coding.

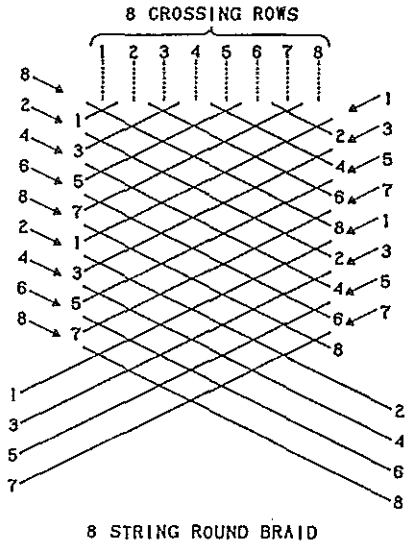


Fig. 128 — The 'standard' method for laying down the string-run in round braids.

The diagram clearly depicts the four strings which describe a right-hand helix (the strings $1, 3, 5, 7$) and the four strings which describe a left-hand helix (the strings $2, 4, 6, 8$). The free-ends of the strings which describe a right-hand helix point to the lower left, and the free-ends of the strings which describe a left-hand helix point to the lower right. The procedure for laying down the consecutively numbered strings is as follows:

- Take the free-end of string 1 and run it, from left to right, along the **back** of the braid; then, after giving it a full turn (right helix down), lay it down from upper-right to lower-left along the **front** of the braid, hence crossing the strings $2, 4, 6, 8$ in sequence.
- Take the free-end of string 2 and run it, from right to left, along the **back** of the braid; then, after giving it a full turn (left helix down), lay it down from upper-left to lower-right along the **front** of the braid, hence crossing the strings $3, 5, 7, 1$ in sequence.
- Take the free-end of string 3 and run it, from left to right, along the **back** of the braid; then, after giving it a full turn (right helix down), lay it down from upper-right to lower-left along the **front** of the braid, hence crossing the strings $4, 6, 8, 2$ in sequence.
- Take the free-end of string 4 and run it, from right to left, along the **back** of the braid; then, after giving it a full turn (left helix down), lay it down from upper-left to lower-right along the **front** of the braid, hence crossing the strings $5, 7, 1, 3$ in sequence.

---

or left-hand helixes is called an **embryonic round braid**. Embryonic round braids and embryonic cylindrical braids are some of the most elementary braidforms, and are hence of paramount importance to physical braids. It is, for example, with the treatment of these by the Classical Knot Theory that this theory is of little or no relevance to physical braids. In some later issue we will discuss the importance of these embryonic braidforms.

• Take the free-end of string 5 and run it, from left to right, along the **back** of the braid; then, after giving it a full turn (right helix down), lay it down from upper-right to lower-left along the **front** of the braid, hence crossing the strings $6, 8, 2, 4$ in sequence.
• Take the free-end of string 6 and run it, from right to left, along the **back** of the braid; then, after giving it a full turn (left helix down), lay it down from upper-left to lower-right along the **front** of the braid, hence crossing the strings $7, 1, 3, 5$ in sequence.
• Take the free-end of string 7 and run it, from left to right, along the **back** of the braid; then, after giving it a full turn (right helix down), lay it down from upper-right to lower-left along the **front** of the braid, hence crossing the strings $8, 2, 4, 6$ in sequence.
• Take the free-end of string 8 and run it, from right to left, along the **back** of the braid; then, after giving it a full turn (left helix down), lay it down from upper-left to lower-right along the **front** of the braid, hence crossing the strings $1, 3, 5, 7$ in sequence.
• Repeat the above eight steps as often as required in order to obtain the desired length of cylindrical braid.

Every time a string is laid down along the **front** of the braid, its crossing-movements should start with one or more **unders** if possible. This is not only important for an easier braiding operation, but it also facilitates in obtaining a neat braid.

In most cases each end of a round braid receives further braiding operations which range from integrated terminal knots to transitions which go over into other braid-forms.

It is therefore important to start a round braid in the correct way, the way which enables the braider, after having braided the round braid for a certain distance in one direction, to continue the braiding in the opposite direction without the hassle of finding the correct way for holding the two sets of strings.

Unfortunately, the existing literature, and indeed most braiders, do not pay due attention to the way in which a round braid should be started, consequently bad braiding habits are being passed on from one generation of braiders to the next.

We shall depict in the following Figs. the correct starting procedures for various round braids.

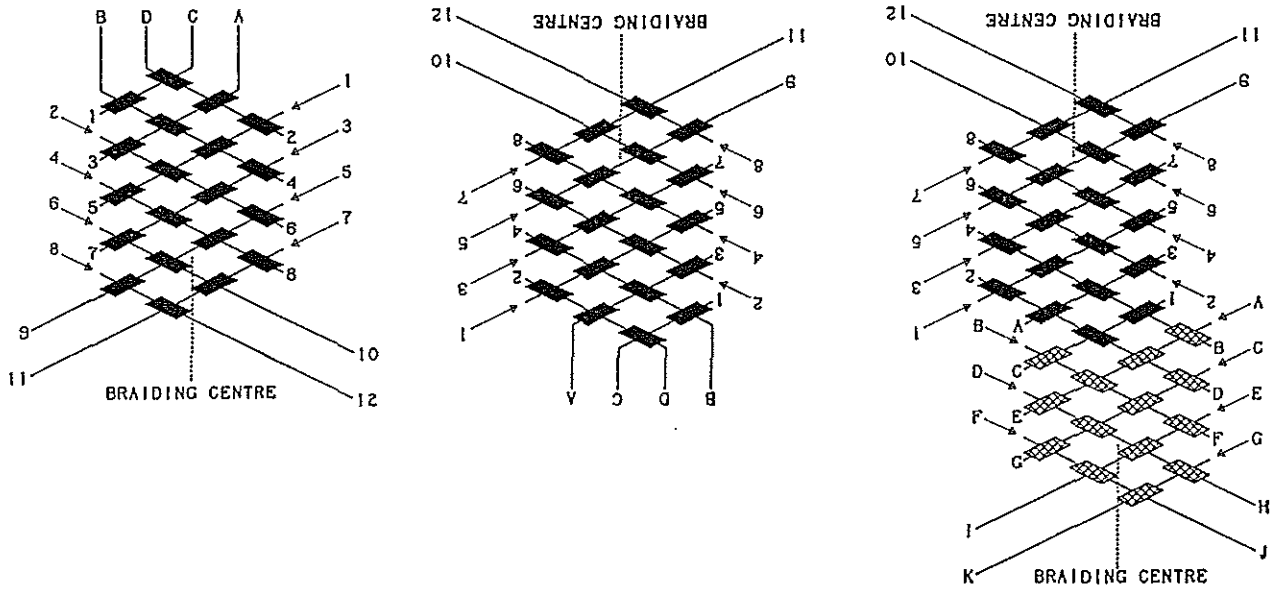


Fig. 129 — The 4-string round braid.

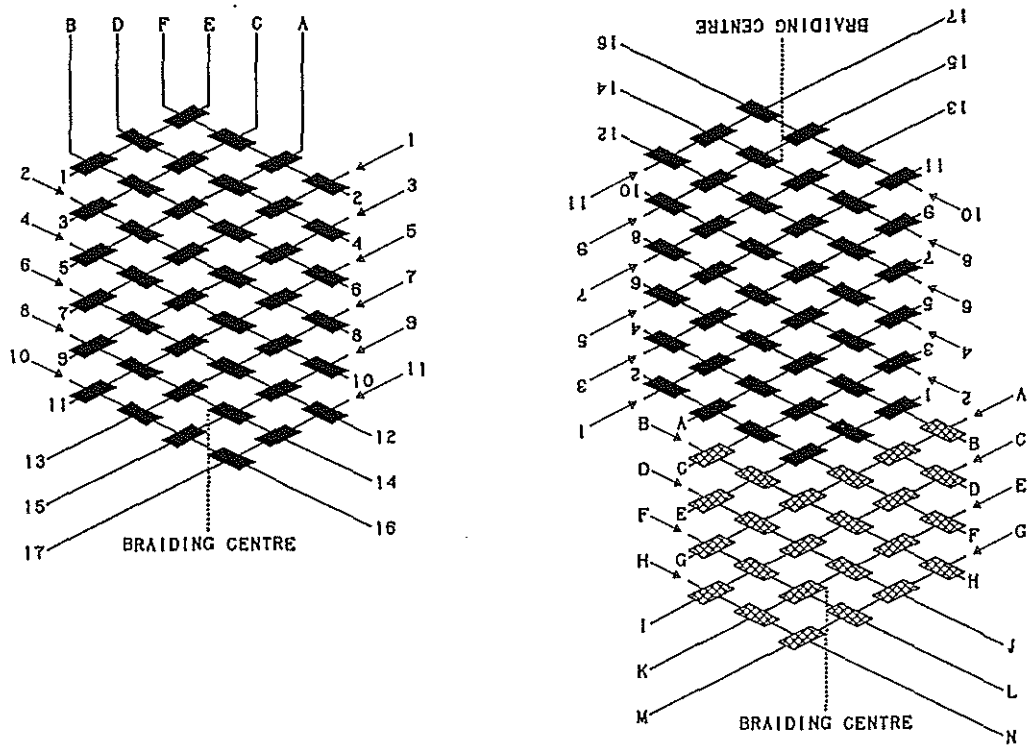


Fig. 130 — The 6-string Casa-coded round braid.

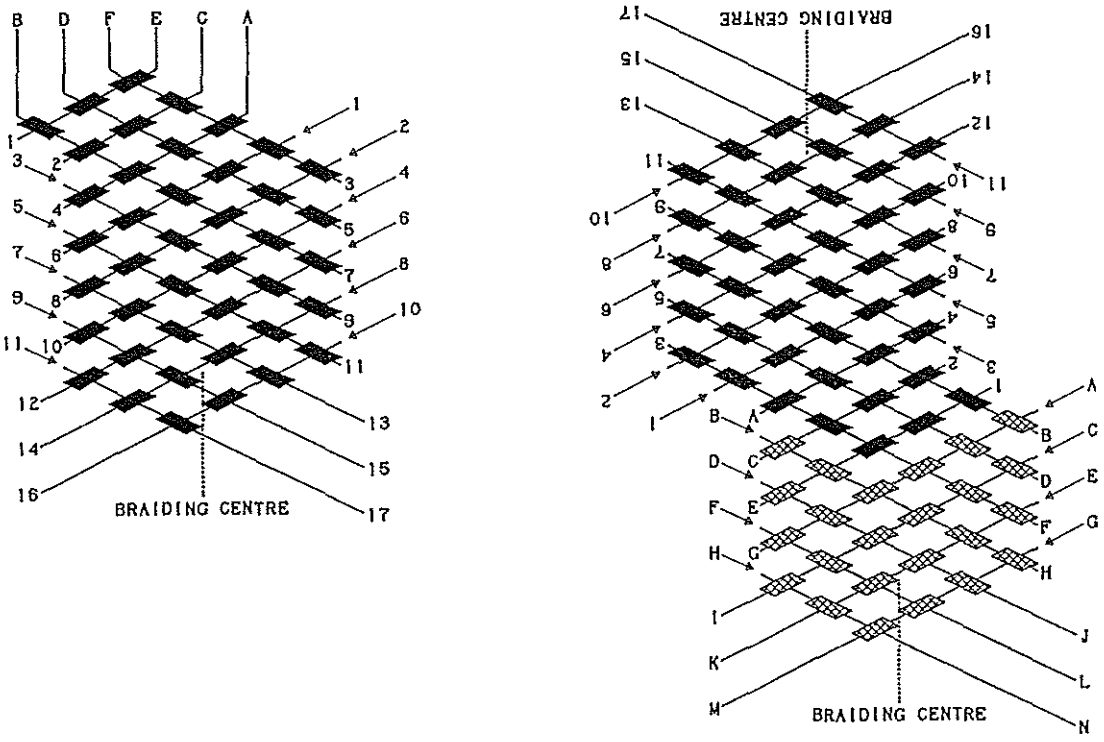


Fig. 131 — The 6-string half-round braid.

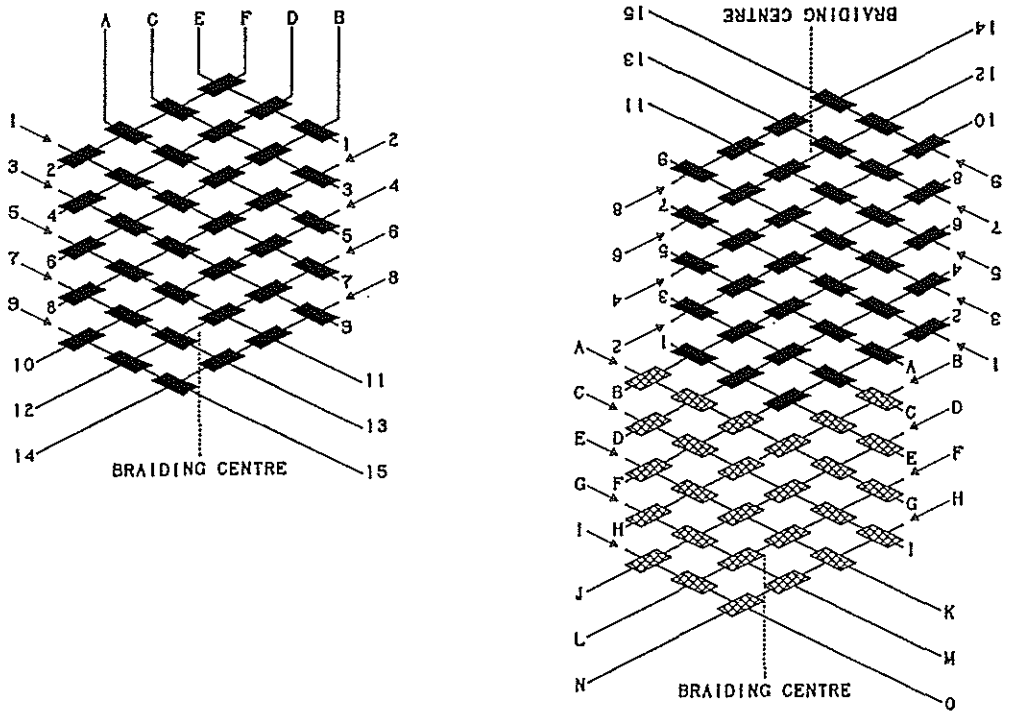


Fig. 132 — The 6-string half-round braid.

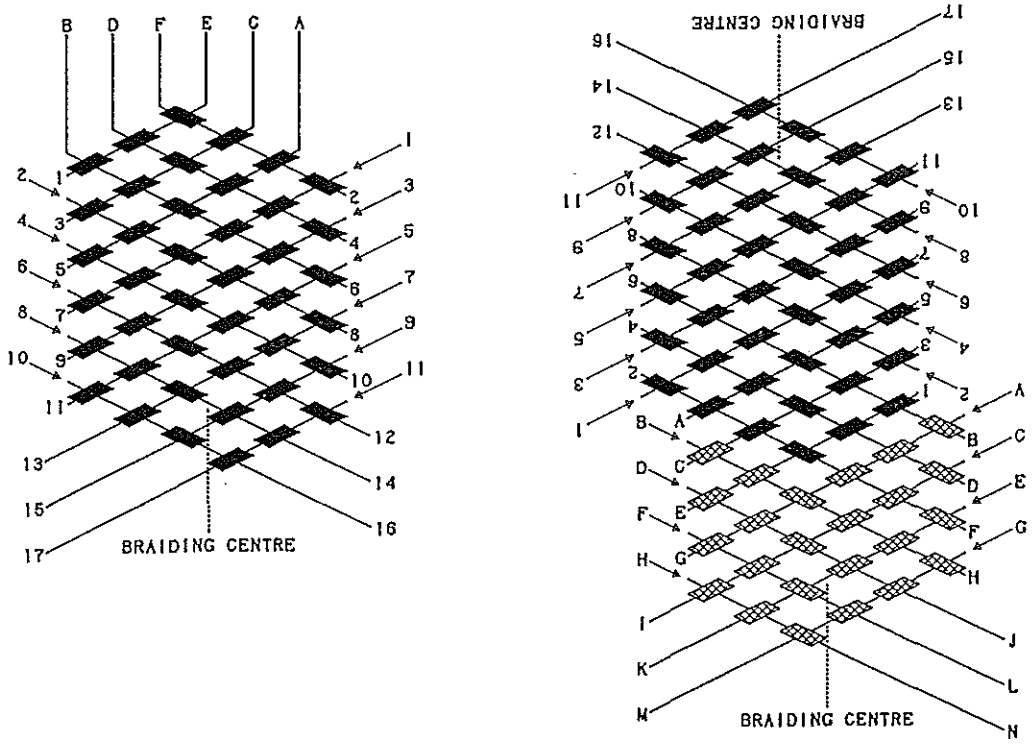


Fig. 133 — The 6-string round braid.

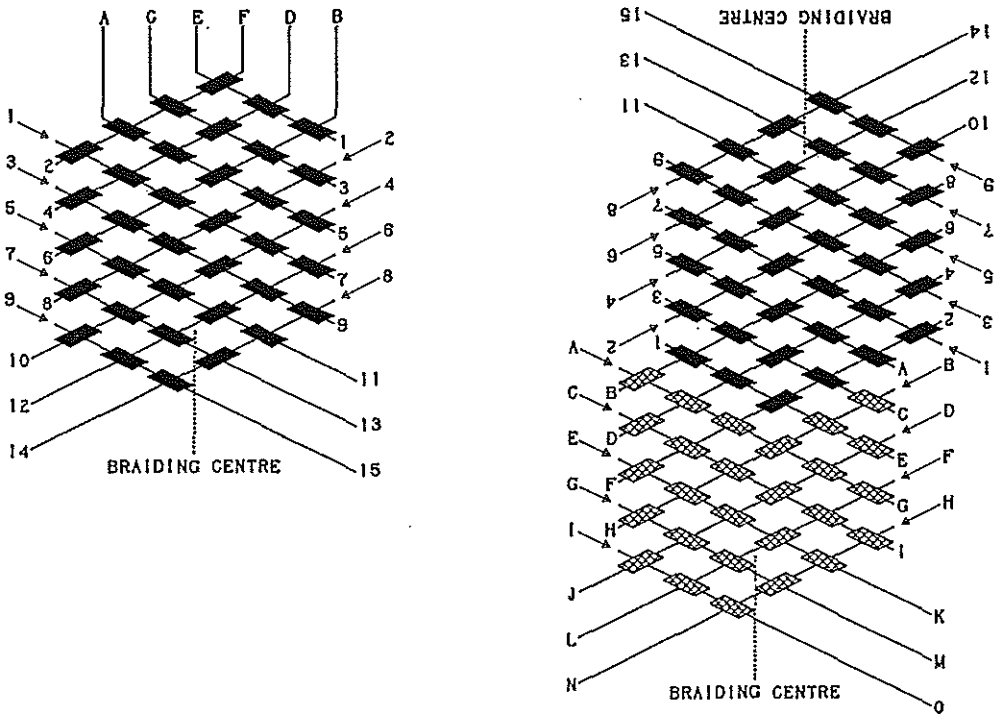


Fig. 134 — The 6-string round braid.

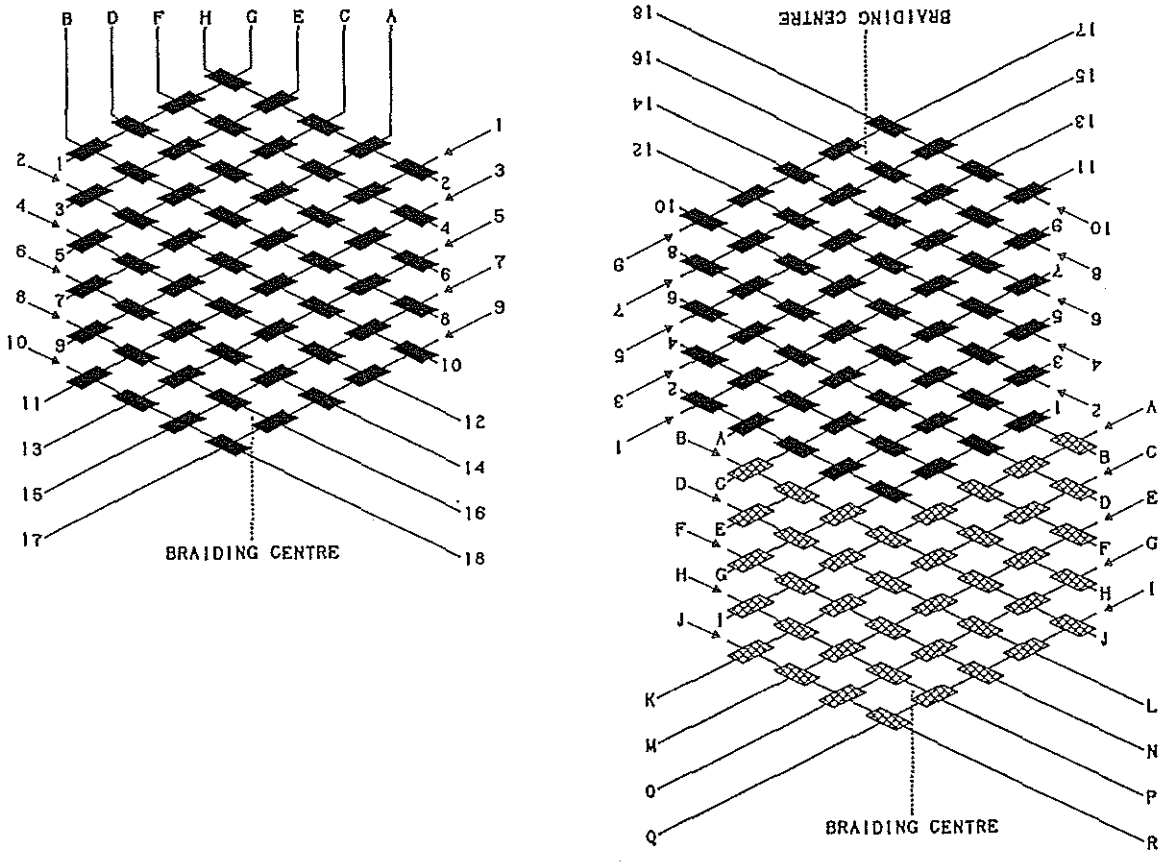


Fig. 135 — The 8-string Casa-coded round braid.

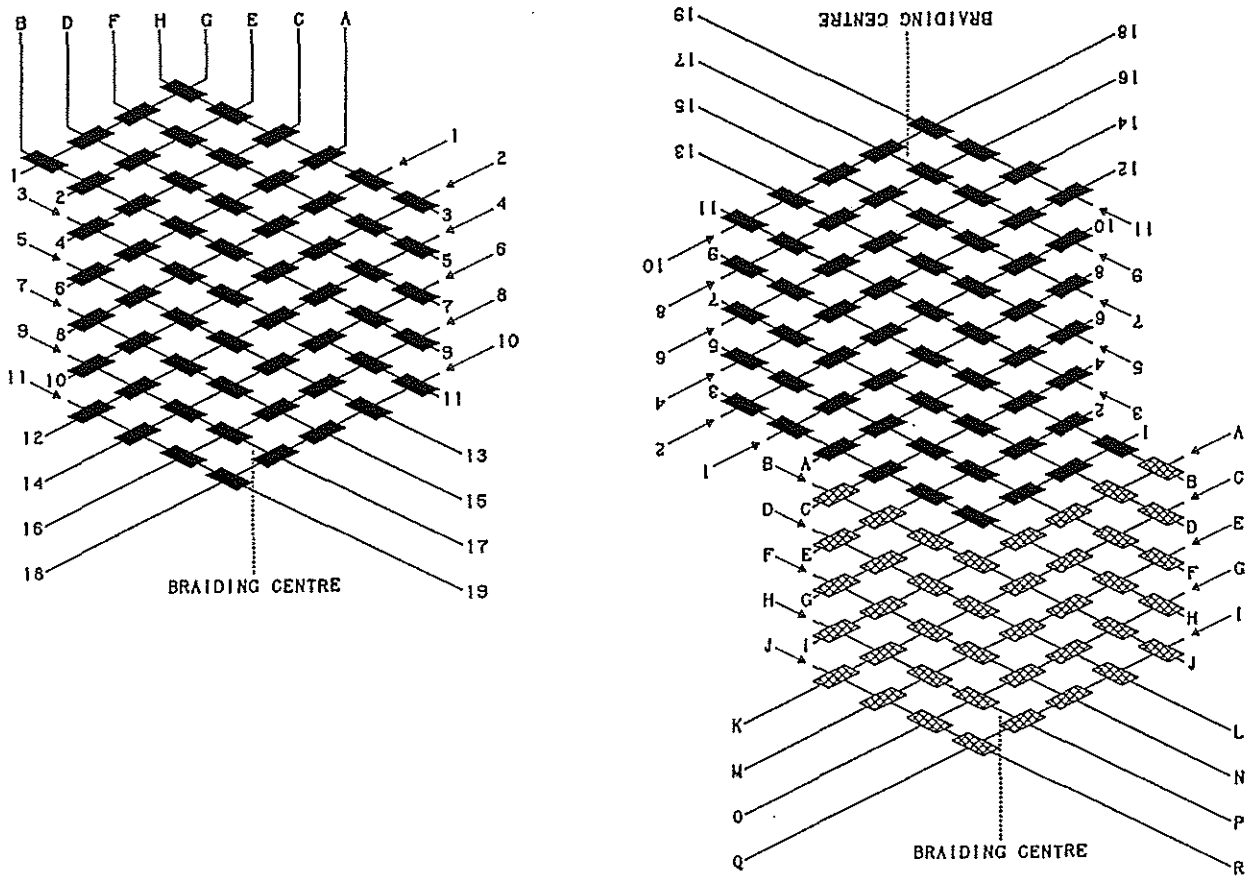


Fig. 136 — The 8-string round braid (square braid).

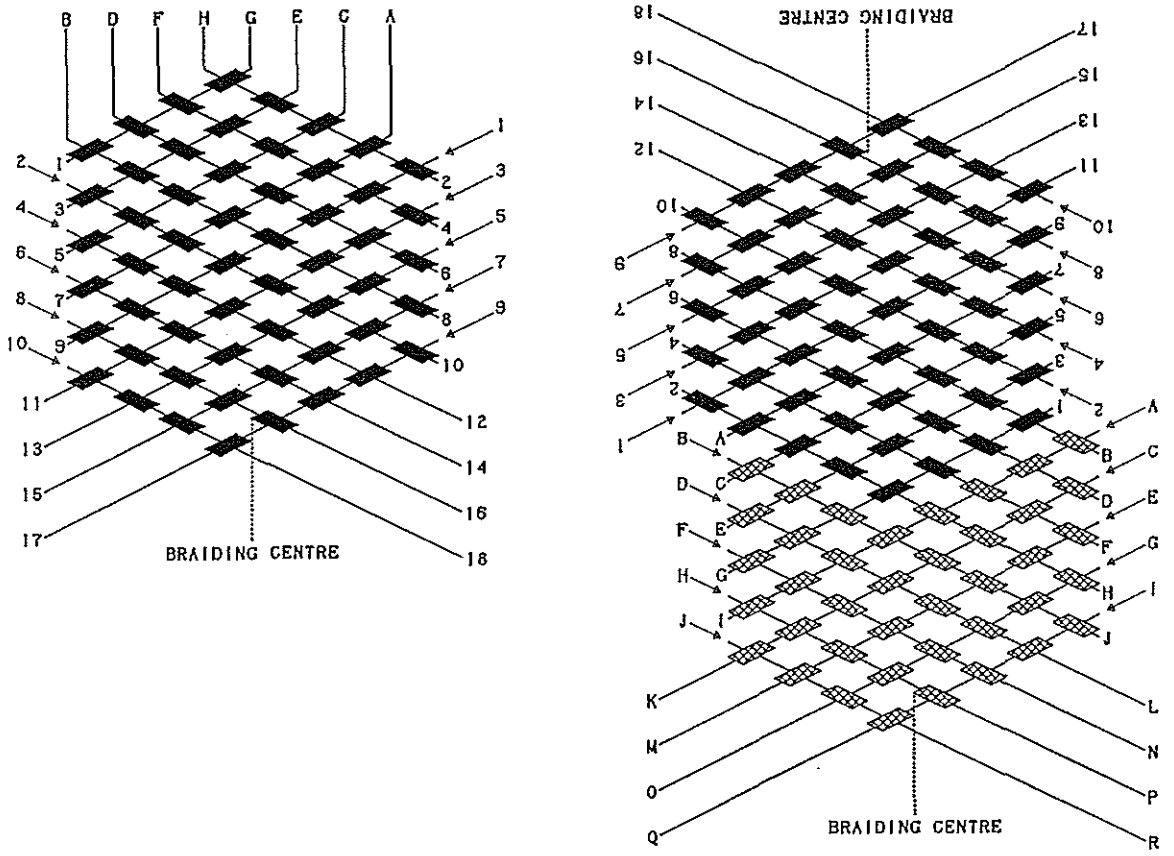


Fig. 137 — The 8-string oval braid (rectangular braid).

An overview of the correct starting procedures for the above depicted round braids is shown in sequential order below. The table below each diagram gives the braiding algorithm for the round braid concerned. The sequence of crossing-movements for a string which is being laid down from upper-right to lower-left along the **front** of the braid, is read from right to left as indicated by the right-hand arrow in the table, while the sequence of crossing-movements for a string which is being laid down from upper-left to lower-right along the **front** of the braid, is read from left to right as indicated by the left-hand arrow in the table.

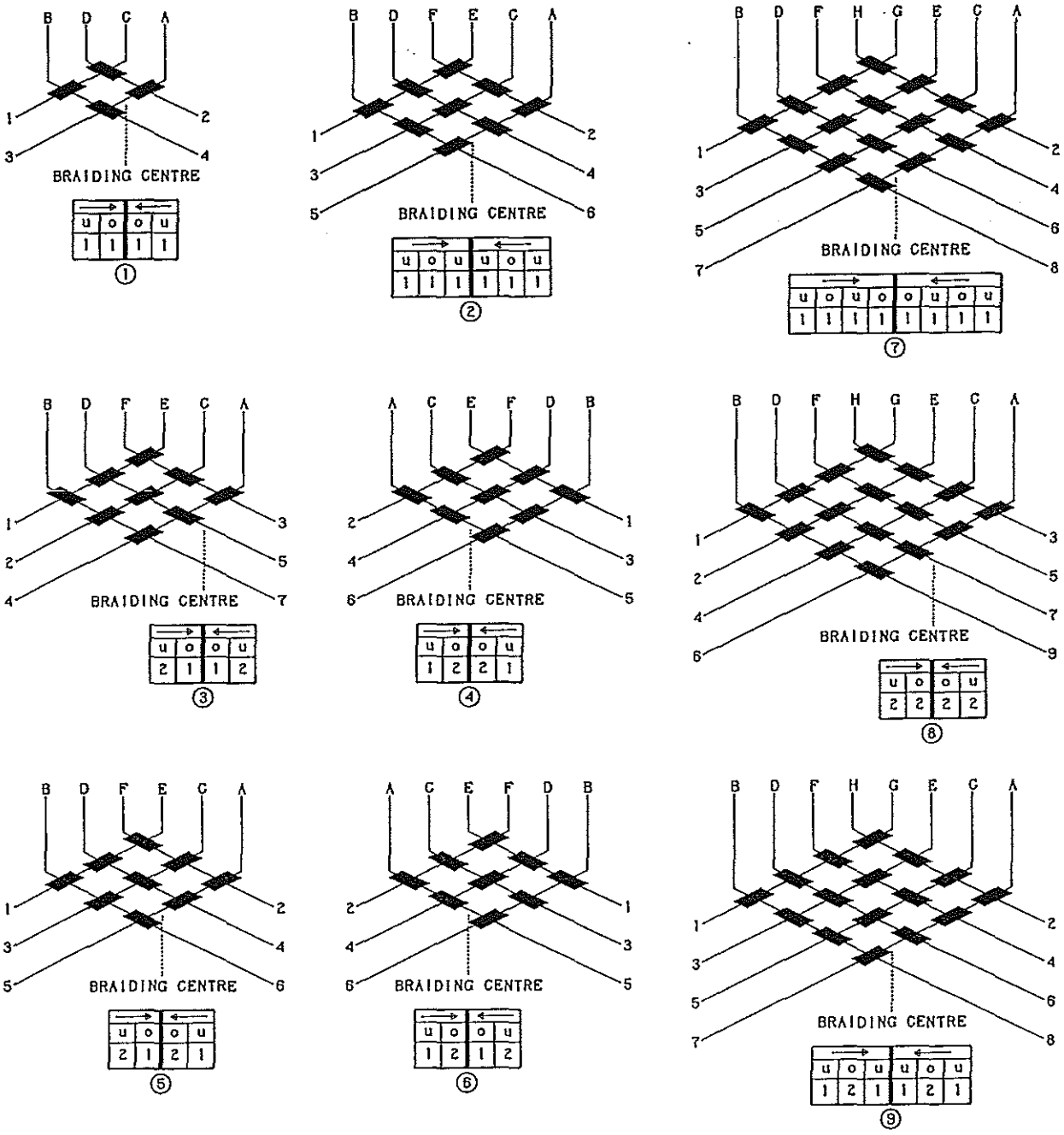


Fig. 138 — An overview of the correct starting procedures for round braids.

Reviews

The Bushman's Handcrafts ISBN 0 9599299 0 8 221 pages 220x145mm., hard cover, sewn spine.

Author: R.M. Williams.

Publisher: R.M. WILLIAMS PTY LTD; 5 Percy Street; Prospect; P.O. Box 55; S.A. 5082; Australia.

This book deals with much more than just braiding. It contains a good section on tanning and fur dressing. Unfortunately, the most simple "tanning" process, suitable for hides and skins whether or not the hair is removed, has not been mentioned. This process consists of painting the hide or skin with a slurry made by mixing kerosene with baking soda (sodium bicarbonate). The painting process is repeated about four times depending on the thickness of the hide. It produces a beautiful white leather of an excellent lasting quality.

The book has furthermore sections on saddlery, the making of elasic side boots (e.s. boots), leather carving, and even on how to shoe a horse.

There are four sections which are associated with braiding: one entitled "plaiting" (48 pages), one entitled "rawhide handcraft" (16 pages), one entitled "horsehair handcraft" (16 pages), and one entitled "additional plaiting" (44 pages).

The many illustrations in each of these sections are superb, and undoubtedly the best one will have ever seen.

Full instructions for some projects are given, such as for braided belts, whips, a rawhide bed frame, rawhide rope, and rawhide hobbles.

The instructions for the "Whip Handle Turks Head", page 191, contain on page 192 under 3. an error. It should have been mentioned that in the fourth move, the continuation of No. 3 passes at the base of the knob also **under** No. 1. The complete knot as under 4., page 193, is a 3-parts/4-bights Casa-coded Regular Knot. It should be noted that the "Bush Knot For Whip Handle Top" on pp. 82–83 has as basic knot a 5-parts/4-bights Casa-coded Regular Knot (fig. 10, pg. 83). We like here to refer the reader to our comments in *The Braider* issues No. 4 and 5 with regard to these knots, although it must be remembered that this book deals with **bushman's** handcrafts, as is clearly indicated by its title.

The reader will no doubt recognise "The Two Thong Turks Head" on page 209 as the 'rolled over' version of the 2-string single-ply Casa-coded 3/2 open-ender, discussed in *The Braider*, issue No. 5, Fig. 87. Hence we can also braid the knot on page 209 by braiding the 3/2 open-ender as indicated in *The Braider*, issue No. 5, and roll the crown over. There are a few other tying methods for this knot on pg. 209, but the 'rolling over' method should be preferred.

Especially the section on "horsehair handcraft" is of great value since there is very little in the literature available concerning this field. Furthermore, this section contains one of the best set of instructions available about horsehair weaving.

This book is not only eminently suited for the novice braider, but it also contains much other valuable information.

THE BRAIDER

No.8 **NOVEMBER 1996.**

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A quarterly publication for the braiding artisan

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Solution to the Question in Issue No. 7

We can divide the various ways in which the two Enlargement Methods may be visualized into two categories: one in which the relevant data of the knot which is to undergo an enlargement (hence p and b) is used as reference, and the other in which the relevant data of the enlarged knot (hence $p + \Delta p$ and $b + \Delta b$) is used as reference.

In the geometrical derivations of the Diophantine equations in *The Braider* No. 6, pg. 109, the relevant data of the knot which underwent an enlargement (hence p and b) was used as reference, while in the analytical derivations on pg. 109 the relevant data of the enlarged knot (hence $p + \Delta p$ and $b + \Delta b$) was used as reference.

In Figs. 139 & 140 the reference used in the geometrical derivations is the relevant data of the enlarged knot, while in the analytical derivations the relevant data of the knot which undergoes an enlargement is used as reference.

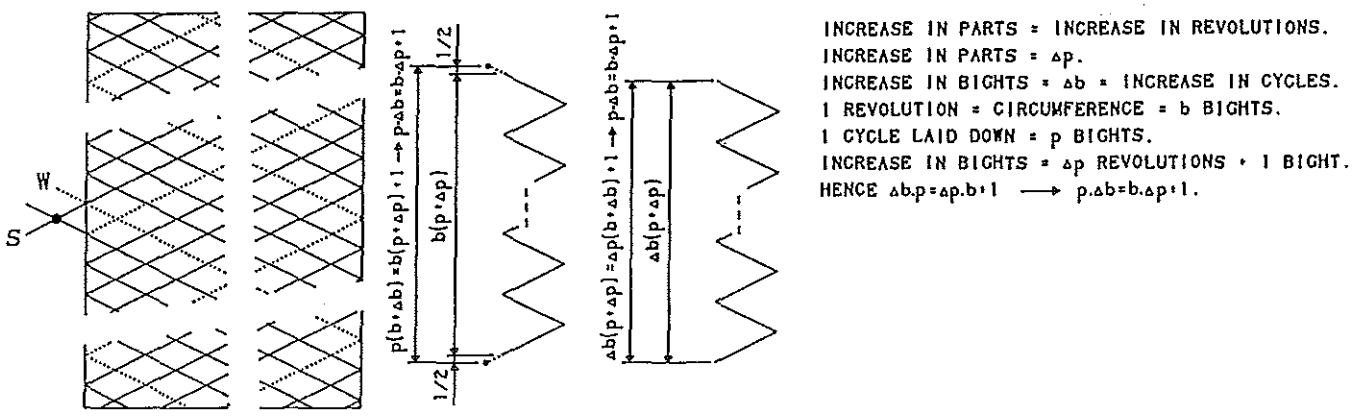

W
S
1/2
p(b•Δb)=b(p•Δp)+1 → p•Δb=b•Δp+1
b(p•Δp)
1/2
Δb(p•Δp)=Δp(b•Δb)+1 → p•Δb=b•Δp+1
Δb(p•Δp)
INCREASE IN PARTS = INCREASE IN REVOLUTIONS.
INCREASE IN PARTS = Δp.
INCREASE IN BIGHTS = Δb = INCREASE IN CYCLES.
1 REVOLUTION = CIRCUMFERENCE = b BIGHTS.
1 CYCLE LAID DOWN = p BIGHTS.
INCREASE IN BIGHTS = Δp REVOLUTIONS + 1 BIGHT.
HENCE Δb.p=Δp.b+1 ⟶ p.Δb=b.Δp+1.

Fig. 139 — The Diophantine equation $p \cdot \Delta b = b \cdot \Delta p + 1$; Enlargement Method I.

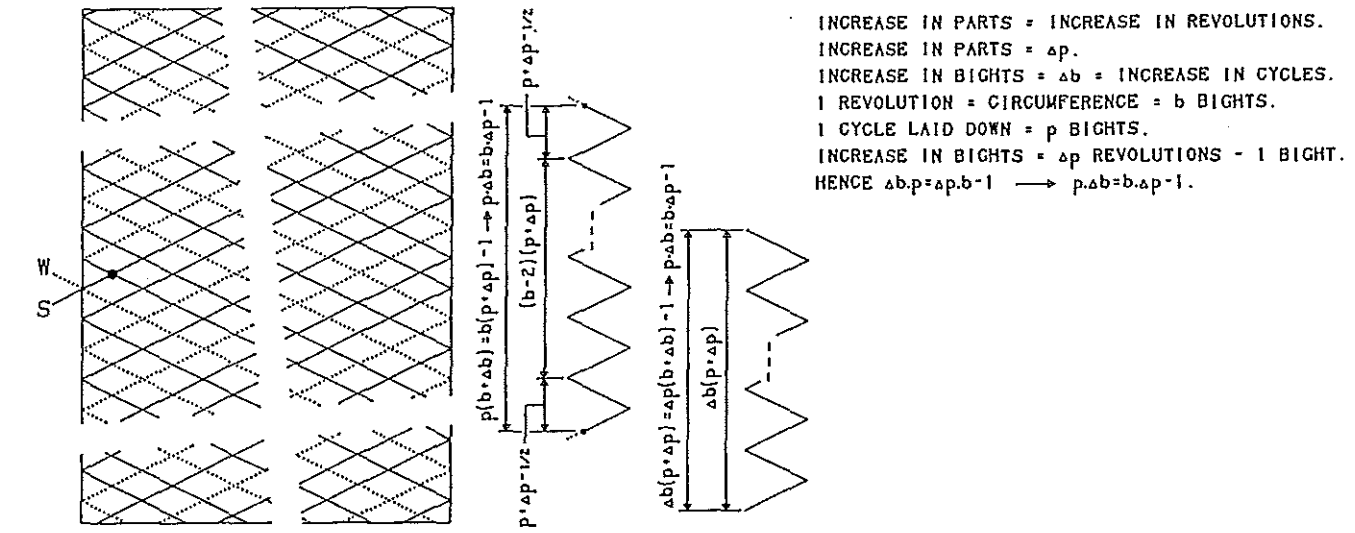

W
S
p•Δp-1/2
p(b•Δb)=b(p•Δp)-1 → p•Δb=b•Δp-1
(b-2)(p•Δp)
p•Δp-1/2
Δb(p•Δp)=Δp(b•Δb)-1 → p•Δb=b•Δp-1
Δb(p•Δp)
INCREASE IN PARTS = INCREASE IN REVOLUTIONS.
INCREASE IN PARTS = Δp.
INCREASE IN BIGHTS = Δb = INCREASE IN CYCLES.
1 REVOLUTION = CIRCUMFERENCE = b BIGHTS.
1 CYCLE LAID DOWN = p BIGHTS.
INCREASE IN BIGHTS = Δp REVOLUTIONS - 1 BIGHT.
HENCE Δb.p=Δp.b-1 ⟶ p.Δb=b.Δp-1.

Fig. 140 — The Diophantine equation $p \cdot \Delta b = b \cdot \Delta p - 1$; Enlargement Method II.

In each of the Figs. 139 & 140 two geometrical derivations are presented. In these derivations part-units and bight-units are used. Recall that in the geometrical derivations on pg. 109 part-units and row-units were used (1 bight-unit = 2 row-units).

For the analytical derivation in Fig. 139 visualize the enlargement as follows: after b-bights have been laid down (hence after returning to the bight-point we started in (the Standing End bight-point)) we continue with the laying down of further cycles **on top** of the already laid down ones till we reach the bight-point immediately **above** the Standing End bight-point. We have now laid down an additional Δb cycles (hence Δb bights). In doing so we moved Δp times around the circumference, **plus** 1 bight. Hence, since the circumference is equal to b bights, we obtain: $\Delta b \cdot p = \Delta p \cdot b + 1$. Finally we join the end of the last cycle with the Standing End and redistribute the bights evenly; this forms then the enlarged knot.

For the analytical derivation in Fig. 140 visualize the enlargement as follows: after b-bights have been laid down (hence after returning to the bight-point we started in (the Standing End bight-point)) we continue with the laying down of further cycles **on top** of the already laid down ones till we reach the bight-point immediately **below** the Standing End bight-point. We have now laid down an additional Δb cycles (hence Δb bights). In doing so we moved Δp times around the circumference, **minus** 1 bight. Hence, since the circumference is equal to b bights, we obtain: $\Delta b \cdot p = \Delta p \cdot b - 1$. Finally we join the end of the last cycle with the Standing End and redistribute the bights evenly; this forms then the enlarged knot.

The reader is furthermore referred to Pamphlet No. 4, pp. 15–18.

Scenes from the Knot Universe

We have seen in the previous issue of *The Braider* how any Casa-coded Regular Knot can be raised from one of four Casa-coded basic Regular Knot types, viz. 1-part knots (embrionic Casa knots), 2-parts knots†, 1-bight knots, 2-bights knots. **In fact it is the only possible way of raising bigger Casa-coded Regular Knots.** Hence any other seemingly different method of braiding or raising Casa-coded Regular Knots disguises in some way the actual process of what is going on. In this article we shall discuss some of these seemingly different processes.

Raising the number of bights without raising the number of parts.

The successive stages in this process are illustrated in Fig. 141 for a 7-parts Regular Knot of which the rightmost intersection-column has a constant coding.

† The 2-parts Column-coded Regular Knots (except the 2-parts/1-bight Regular Knot) may be seen as true Casa knots since their string-run makes an *over–under* movement throughout; but if we view the coding of Casa-coded Regular Knots from the aspect of *a single building block* (hence with $\alpha = 2$ and $\beta = 1$ (see *Braiding — Regular Knots*)), then the 2-parts Column-coded Regular Knots can only be regarded as more advanced embryonic Casa knots. Note that any 1-part Regular Knot (embryonic Casa knot) requires only **one** Enlargement Method I order 1 process to its successive 1-part Regular Knot (embryonic Casa knot); any 1-part Regular Knot (embryonic Casa knot) requires only **one** Enlargement Method II order 1 process to its successive 2-parts Column-coded Regular Knot (Casa knot); any 1-bight Casa knot requires only **one** Enlargement Method II order 1 process to its successive 1-bight Casa knot; a 2-parts/1-bight Regular Knot (Casa knot) requires only **one** Enlargement Method I order 1 process to its successive 3-parts Casa knot (the 3-parts/2-bights Casa knot).

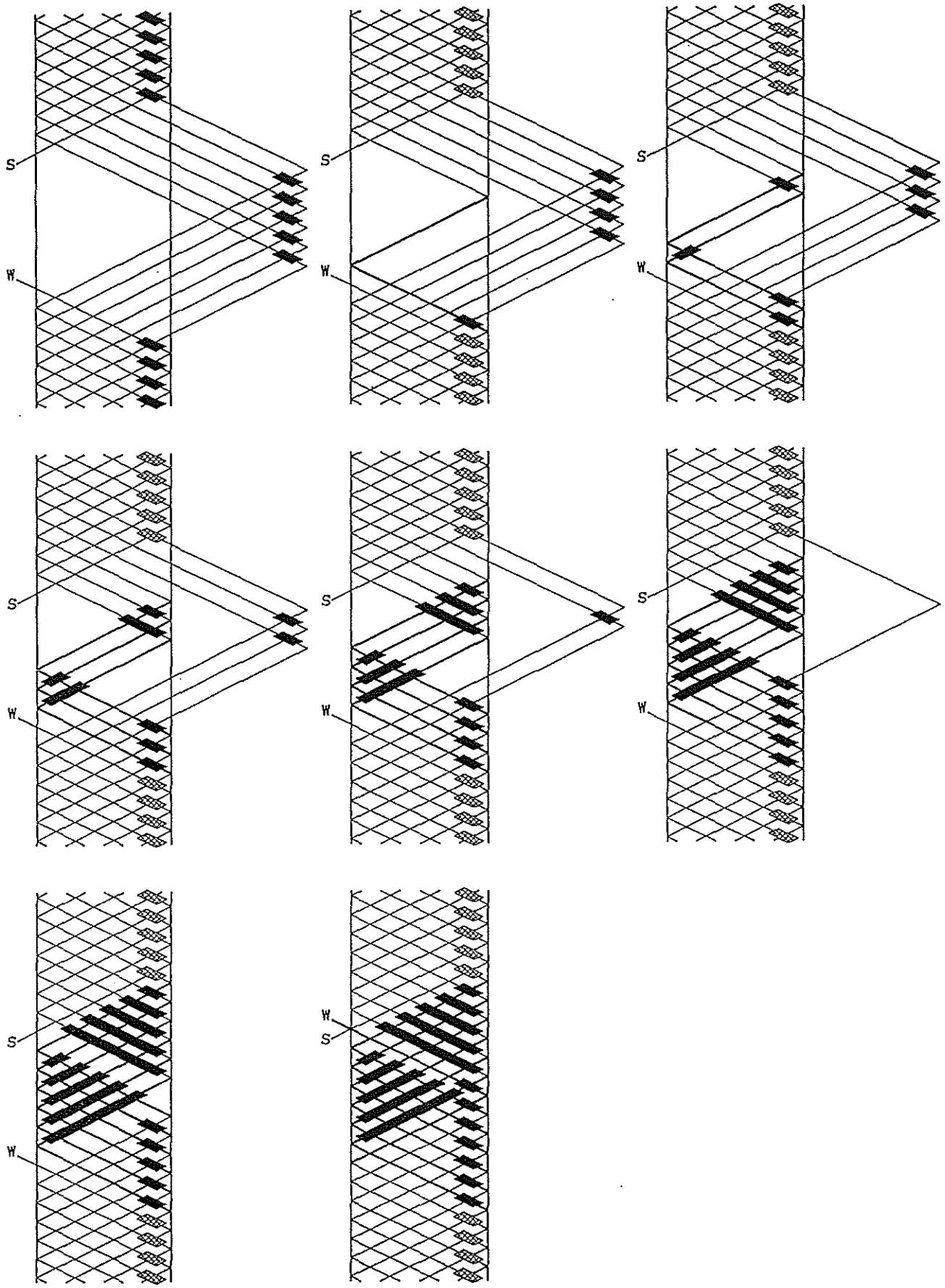


Fig. 141 — Raising the number of bights without raising the number of parts.

We note that for a p-parts Regular Knot, $(p-1)$ adjacent cycles have to be rearranged into $2(p-1)$ cycles. This raises the number of bights with $(p-1)$ bights. Furthermore, the Working-end supplies an additional bight. Hence the number of bights increase by p bights.

The lowermost cycle is readily rearranged: its intersections with the remaining $(p-2)$ cycles are transferred by sliding them along. Then the next cycle is rearranged by sliding its intersections with the remaining $(p-3)$ cycles along and ensuring that its intersection on the rightmost intersection-column complies with the constant coding of this column. This determines the coding of its intersection on the leftmost intersection-column. Then the next cycle is rearranged by sliding its intersections with the remaining $(p-4)$ cycles along and ensuring that its intersection on the rightmost intersection-column complies with the constant coding of this column. This determines the coding of its intersection on the second intersection-column from the right. Its intersections on the two rightmost intersection-columns determine its intersections on the two leftmost intersection-columns. Then the next cycle is rearranged by sliding its intersections with the remaining $(p-5)$ cycles along and ensuring that its intersection on the rightmost intersection-column complies with the constant coding of this column. This determines the coding of its intersection on the second intersection-column from the right, which in turn determines the coding of its intersection on the third intersection-column from the right. Its intersections on the three rightmost intersection-columns determine its intersections on the three leftmost intersection-columns. And so on.

In each of the diagrams in Fig. 141, only the codings on the rightmost constant coded intersection-column and the codings which result from them are indicated.

We note that the innermost consecutive $(p-3)$ intersection-columns do not each have a constant coding. Hence for this number of intersection-columns to be zero, the maximum value for p is 3. Thus:

Raising the number of bights without raising the number of parts in Column-coded Regular Knots while remaining Column-coded is only possible when their number of parts is equal to or less than 3. Their coding is a Casa-coding or embryonic Casa-coding depending on the number of parts.

It should be realized that during this process of increasing the number of bights of the Regular Knot having p-parts and b-bights to p-parts and $(b+p)$-bights, we are in fact catching up to the number of bights which would have been laid down during the successive enlargement stages from the 1/1 Regular Knot to the $p/(b+p)$ Regular Knot. Let $[a_1\,;\,a_2\,,\,a_3\,,\,a_4\,,\,\cdots\,,\,a_n\,,\,1]$ be the path-formula of the p/b Regular Knot, then the path-formula of the $p/(b+p)$ Regular Knot will be $[0\,;\,1\,,\,a_1\,,\,a_2\,,\,a_3\,,\,a_4\,,\,\cdots\,,\,a_n\,,\,1]$. When $a_1 = 0$, then $[0\,;\,1\,,\,a_1\,,\,a_2\,,\,a_3\,,\,a_4\,,\,\cdots\,,\,a_n\,,\,1] = [0\,;\,1\,,\,0\,,\,a_2\,,\,a_3\,,\,a_4\,,\,\cdots\,,\,a_n\,,\,1] = [0\,;\,1+a_2\,,\,a_3\,,\,a_4\,,\,\cdots\,,\,a_n\,,\,1]$[†]. Hence the sequential enlargement steps from the Regular Knot 1/1 to the Regular Knot p/b are identical to the sequential enlargement steps from the Regular Knot 1/2 to the Regular Knot $p/(b+p)$.

In Fig. 142 are depicted the bights-raising processes for 1-part, 2-parts, and 3-parts Column-coded Regular Knots, while the number of parts remain constant. Only the bights-raising process for the 3-parts Casa-coded Regular Knots is of practical value since it is much easier to raise these knots this way rather then from the 1-part Regular Knots. The 3-parts Casa-coded Regular Knots may be braided from either the 3/1

[†] $0\,;\,1\,,\,0\,,\,a_2$ represents in the RKT 1 step to left, 0 steps to the right, a_2 steps to the left. Hence $0\,;\,1\,,\,0\,,\,a_2$ represents in the RKT $(1+a_2)$ steps to the left.

or 3/2 Casa-coded Regular Knot by repeating this bights-raising process the necessary number of times.

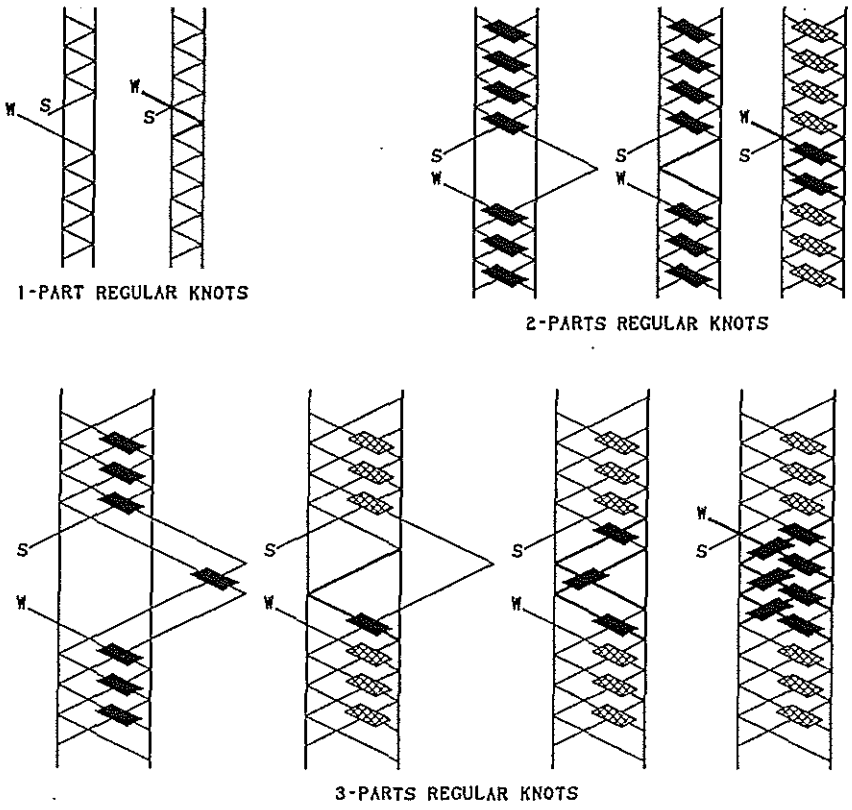


Fig. 142 — Raising the number of bights without raising the number of parts.

Raising the number of parts without raising the number of bights.

The successive stages in this process are illustrated in Fig. 143 for a 7-bights Regular Knot of which the leftmost intersection-column has a constant coding.

For a b-bights Regular Knot, each cycle is extended b-parts.

The first cycle is readily extended. Then the next cycle is extended, ensuring that its intersection on the leftmost intersection-column complies with the constant coding of this column. This determines the coding of its intersection on column-number $(b-1)$ from the left. Then the next cycle is extended, ensuring that its intersection on the leftmost intersection-column complies with the constant coding of this column. This determines the coding of its intersection on the second intersection-column from the left. Its intersections on the two leftmost intersection-columns determine its intersections on the column-numbers $(b-1)$ and $(b-2)$ from the left. Then the next cycle is extended, ensuring that its intersection on the leftmost intersection-column complies with the constant coding of this column. This determines the coding of its intersection on the second intersection-column from the left, which in turn determines the coding of its intersection on the third intersection-column from the left. Its intersections on the three leftmost intersection-columns determine its intersections on the column-numbers $(b-1)$, $(b-2)$ and $(b-3)$ from the left. And so on.

It should be noted that in each extended cycle which has a closed bight at its apex, the string has received a full turn. When extending a cycle, the bight-point in that cycle to be extended must be converted into a crossing (intersection). When this intersection is such that lower-left to upper-right passes *over* lower-right to upper-left, the full turn

will be that of a left-hand screw thread. When lower-left to upper-right passes *under* lower-right to upper-left, the full turn will be that of a right-hand screw thread. Hence only by repeating this parts-raising process a sufficient number of times in such a way that the string in each cycle receives the same number of full right-hand turns as full left-hand turns can we assure that they cancel each other out. This is not only essential for flat braiding material but is also important for round braiding material.

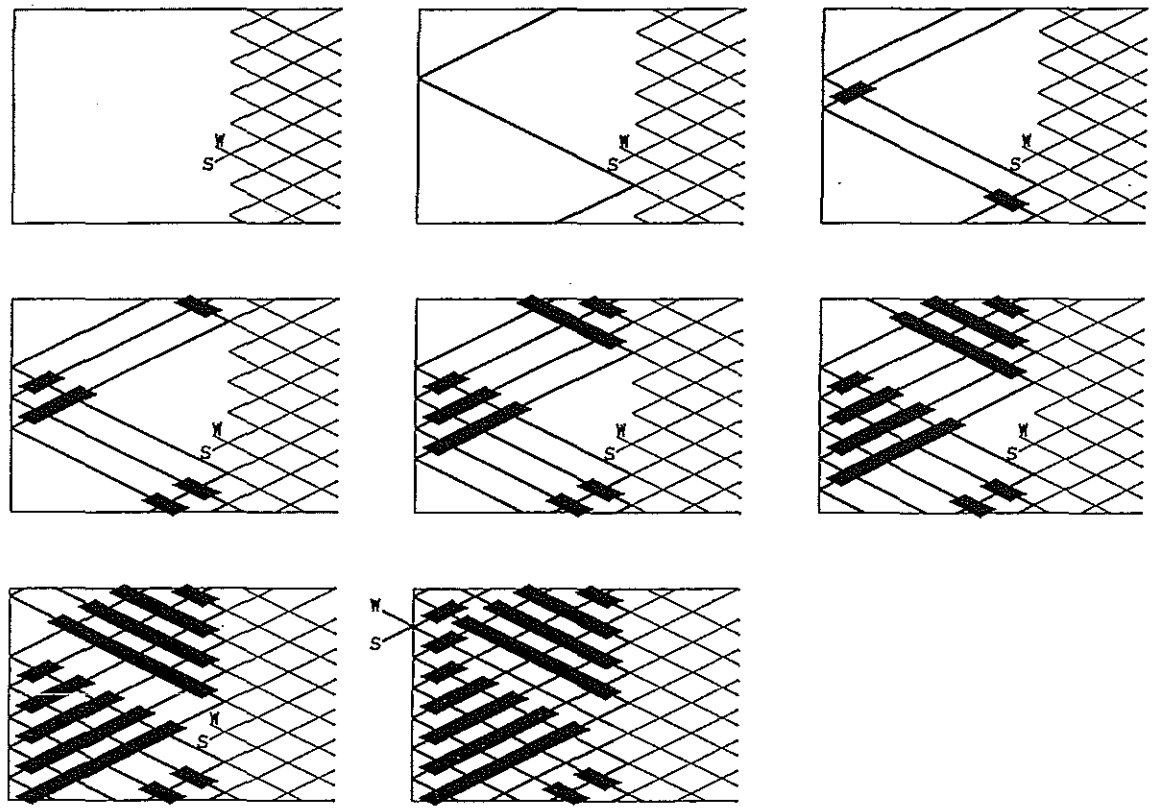


Fig. 143 — Raising the number of parts without raising the number of bights.

In each of the diagrams in Fig. 143, only the codings on the leftmost constant coded intersection-column and the codings which result from them are indicated.

We note that the $(b - 3)$ consecutive intersection-columns next to the leftmost intersection-column do not each have a constant coding. Hence for this number of intersection-columns to be zero, the maximum value for b is 3. Thus:

Raising the number of parts without raising the number of bights in Column-coded Regular Knots while remaining Column-coded is only possible when their number of bights is equal to or less than 3. The process generates b additional consecutive intersection columns, hence: for 1-bight Regular Knots 1 additional intersection-column, the coding of which may be chosen freely; for 2-bights Regular Knots 2 additional intersection-columns,the coding of each may be chosen freely; for 3-bights Regular Knots 3 additional intersection-columns, the codings of the two outermost ones are opposite while the coding of one of them as well as the coding of the third additional intersection-column may be chosen freely.

It should be noted that we are not restricted to a Casa-coding for the Column-coded Regular Knots. However, the 2-bights and 3-bights Regular Knots have respectively one and two extended cycles with a closed bight at their apex. The string in each of

these extended cycles did receive a full turn. Hence we must observe the earlier outlined requirements in this respect.

It should be realized that during this process of increasing the number of parts of the Regular Knot having p-parts and b-bights to $(p+b)$-parts and b-bights, we are in fact catching up to the number of parts which would have been laid down during the successive enlargement stages from the 1/1 Regular Knot to the $(p+b)/b$ Regular Knot. Let $[a_1\,;\,a_2\,,\,a_3\,,\,a_4\,,\,\cdots\,,\,a_n\,,\,1]$ be the path-formula of the p/b Regular Knot, then the path-formula of the $(p+b)/b$ Regular Knot will be $[a_1+1\,;\,a_2\,,\,a_3\,,\,a_4\,,\,\cdots\,,\,a_n\,,\,1]$. Hence the sequential enlargement steps from the Regular Knot 1/1 to the Regular Knot p/b are identical to the sequential enlargement steps from the Regular Knot 2/1 to the Regular Knot $(p+b)/b$.

In Fig. 144 are depicted the parts-raising processes for 1-bight, 2-bights, and 3-bights Column-coded Regular Knots, while the number of bights remain constant. None of these is of any practical value since it is much easier to raise these knots in accordance with the Enlargement Methods I and II. Not only is this parts-raising method much more cumbersome, but to enable us to braid all the 3-bights Casa-coded Regular Knots, we have to know besides the 1/3 and 2/3 Column-coded Regular Knots also the 4/3 and 5/3 Casa-coded Regular Knots in order to prevent full turns in the string.

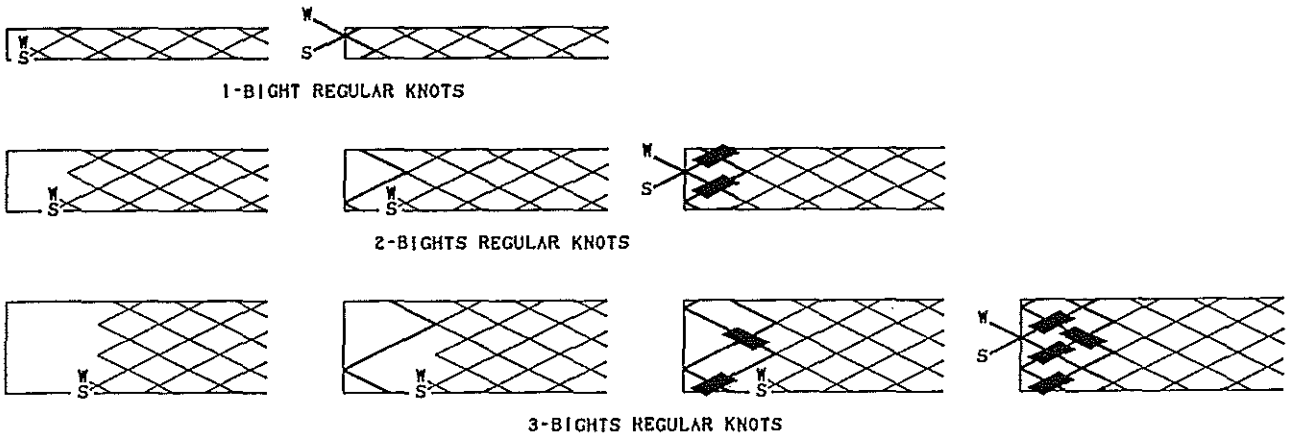


Fig. 144 — Raising the number of parts without raising the number of bights.

Most knotting and braiding procedures have no doubt been discovered by experimentation. Unfortunately in many cases round string has been and still is being used in such experiments: one of the greatest sins an experimenter can commit. Even in Ashley's monumental book *The Ashley Book of Knots* do we find numerous examples of such an erroneous practice which leads to the introduction of twists in the string. As far as physical knots and braids are concerned, a great amount of erroneous theory, conclusions, procedures and results can be found in the existing literature, all emanating from disregarding twists in the string. A neglect which no doubt finds its origin in the use of round string. Since twists in round string do not readily show up, they tend to be overlooked, and hence we cannot stress enough that only flat string should be used in experimental work. **One should realize that a full turn in the string is equivalent to a loop**, and although no sensible person would neglect one or more loops in knots or braids, many round string "practitioners" appear to accept readily any twists in the string, an obvious consequence of "what the eye doesn't see, the heart doesn't grieve over".

Irrespective whether any experimentation is carried out in order to verify and support a theory which is meant to be of an applicable nature, experimentation in its own right can be a satisfying and mentally stimulating occupation which often leads to exciting discoveries. Since from time immemorial man will have been experimenting with a piece of 'string' it is only fair to assume that most, if not all, such exciting discoveries are not first time discoveries. Nevertheless, the excitement experienced and the beauty of a genuine discovery as far as the discoverer is concerned (and no doubt very many 'around' him or her) is not the slightest bit diminished by its earlier, but to him or her (and them) unknown, discovery. Since it is an undeniable fact that it is utterly impossible to determine the time and person who made the initial discovery of the item concerned, any historical data as to person and time associated with it is of no real significance at all. Unfortunately too many people are more concerned about the irrelevant historical data (as to person and time) than about the actual underlying structural principles of the knot or braid and its relationships with other knot or braid forms[†]. Also here we find the so common trait associated with jealousy, namely that the longer an earlier discoverer is dead the more important this discoverer with **his** or **her** discovery becomes, notwithstanding that the discovery is exactly the same as far as the initial and the latest genuine discoverer is concerned[‡].

A discovery is thus intimately associated with the knowledge the discoverer and the people around him or her possess at the time, hence what may appear a discovery to some, may be well-known to others. This of course should not diminish the value of the discovery as far as these people are concerned. Furthermore, an experimental discovery on its own will often turn out to be incomplete, and consequently deductions made from it will also be incomplete or even misleading. **In the field of knotting and braiding, any experimental result should always be followed up and examined with the aid of grid-diagrams.** Grid-diagrams are the only means by which a proper examination can take place, there is just no substitute. A typical example of an experimental discovery which was not followed up with the examination of its related grid-diagrams is **the Masurel algorithm**. It should be pointed out here that this name was not bestowed upon the discovery by the discoverer, but instead by people around him. Furthermore it must be stressed that this discovery should not be looked down upon, nor should the people associated with its naming be scoffed at or belittled in any way. To the contrary, the discoverer should be praised for his experimental efforts and the people around him should be praised for their interest and their effort in letting other knotters and braiders participate in this seemingly exiting algorithm. In order to do so, they necessarily had to give the discovery a name, and they appreciatively named it after the discoverer. This Masurel algorithm, as we shall show below, is nothing but the first order Method I and Method II enlagement process, although of a somewhat restricted nature. In order to illustrate clearly what the discovery really entails, we shall in our discussion below not follow the actual path of the discovery, but instead derive this algorithm from the two enlargement Methods I and II.

The enlargement of a Casa-coded Regular Knot is for the 'body' of the knot (the

[†] Even Ashley's monumental book *The Ashley Book of Knots* suffers conspicuously from this malady.

[‡] In the literature even Ashley has often falsely been accused (supposedly in order to belittle him in favour of earlier discoverers) of stating that he discovered a particular knot, whereas in fact Ashly only stated it to be **probably** original.

section away from the bight-boundaries) a simple matter: add (somehow) two adjacent sets of consecutive cycles, let one set of consecutive cycles to be added follow the weaving-pattern of one of its adjacent consecutive cycles which already exist, and let the other set of consecutive cycles to be added split the two adjacent identical weaving tracks which now have been created. The problem which has to be solved is: what has to be done at the existing bight-boundaries in order to achieve an overall Casa-coding? In order to solve this problem, we draw the grid-diagram of an arbitrary Casa-coded Regular Knot. It is obvious that there are only two ways in which we can continue with adding cycles such that the knot remains a Regular Knot: either we follow the Standing half-cyle **below**, or we follow it **above**. This are respectively the Enlargement Methods I and II. Method I is depicted in Fig. 145, and Method II in Fig. 146.

Let's first examine the uppermost grid-diagrams, which depict the first half-cycle and parts of the second and third half-cycles of the first set of the to be added consecutive cycles for a first order enlargement. The construction-phase, involving the first set of the to be added consecutive cycles, is also known as **the first order track-laying phase**. We observe that in Enlargement Method I each added half-cycle follows in weaving-pattern the half-cycle immediately **above** it, and that in Enlargement Method II each added half-cycle follows in weaving-pattern the half-cycle immediately **below** it. Observe furthermore that in Method I we do not encounter any difficulty in this respect at the start of the first added half-cycle. In Method II however, we have to pass over or under the Standing half-cycle **before** we can start with laying down the first to be added half-cycle. Observe that this crossing-movement of the 'closing' half-cycle with the Standing half-cycle is determined by the sequential continuation of the crossing-movements of the 'closing' half-cycle. We observe that in Method I we encounter no difficulty with the final crossing-movement of an added half-cycle: it is determined by the crossing-movenent of the half-cycle we follow, and is identical to *its* last one. In Method II we encounter no difficulty with the first crossing-movement of an added half-cycle: it is determined by the crossing-movenent of the half-cycle we follow, and is identical to *its* first one. Observe that in Method I the first crossing-movement of a to be added half-cycle (other than the first) will automatically be correct if the string goes **Down** in the first 'hole' when it came **Up** in the previous 'hole' (the last 'hole') of the immediately preceding half-cycle, and vice versa. Observe that in Method II the last crossing-movement of an added half-cycle will automatically be correct if the string goes **Down** in the last 'hole' when it has to come **Up** in the first 'hole' of the following to be added half-cycle, and vice versa. These rules are necessary and sufficient for the adding of half-cycles during the first order track-laying phase associated with the enlargement of Casa-coded Regular Knots. Of course we should be well aware that this are not the only existing necessary and sufficient rules; they in fact turn out to be restrictive. In the first place, the original Casa-coded Regular Knot must have **'holes'** in its weave, hence we cannot apply these rules to embryonic (Casa-coded) Regular Knots (Regular Knots having 1-part). In the second place, these rules don't hold for higher order track-laying operations (refer to the upper diagrams in Figs. 118 – 123, *The Braider*, issue No. 7).

Let's next examine the lowermost grid-diagrams in Figs. 145 & 146, which depict the first half-cycle and parts of the second and third half-cycles of the second set of the to be added consecutive cycles for a first order enlargement. The construction-phase, involving the second set of the to be added consecutive cycles, is also known as **the track-splitting phase** (recall that the track-splitting phase always must me a first order enlargement (see *The Braider*, issue No. 7, pg. 136)).

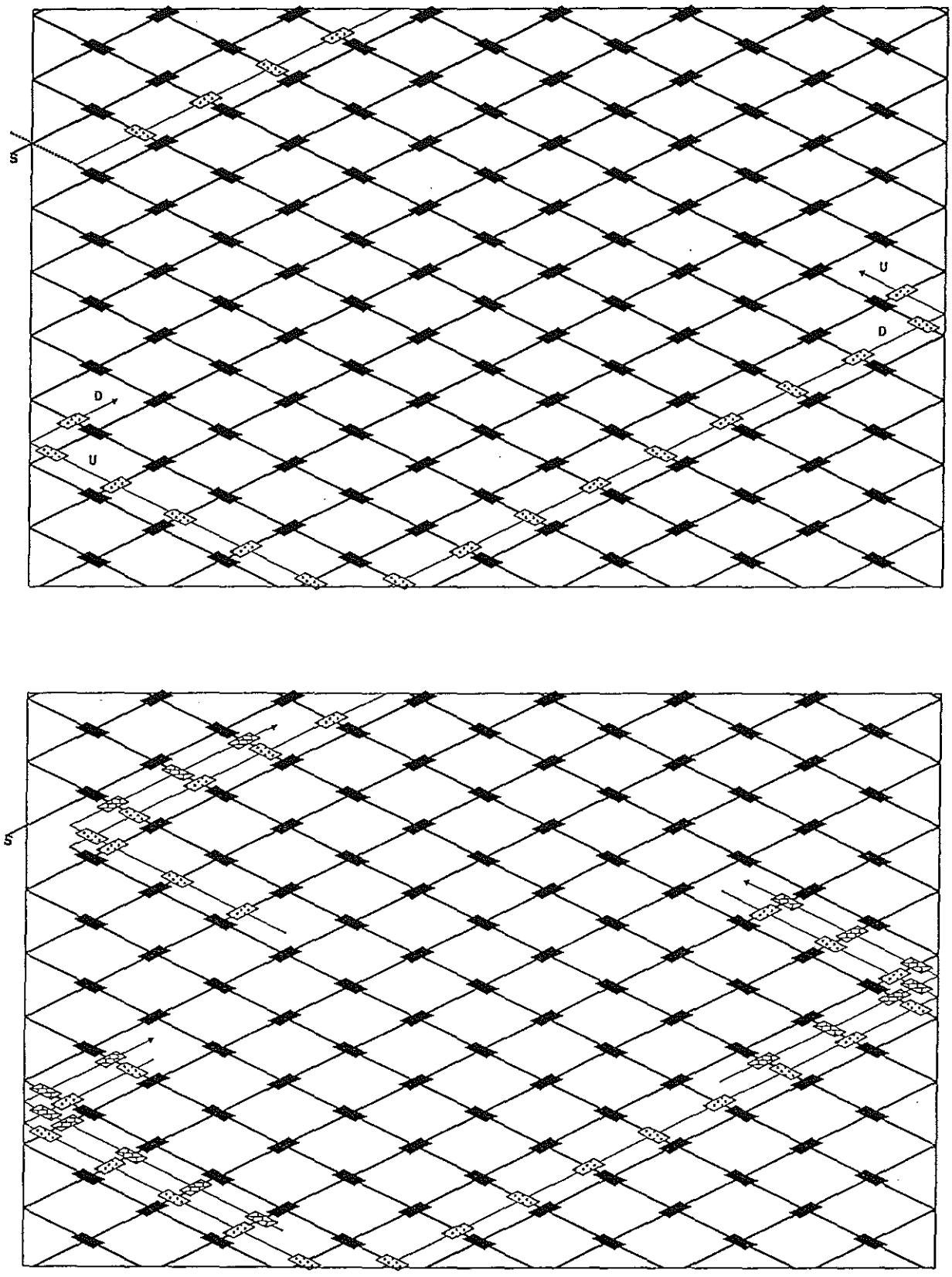


Fig. 145 — First order Method I enlargements for Casa-coded Regular Knots.

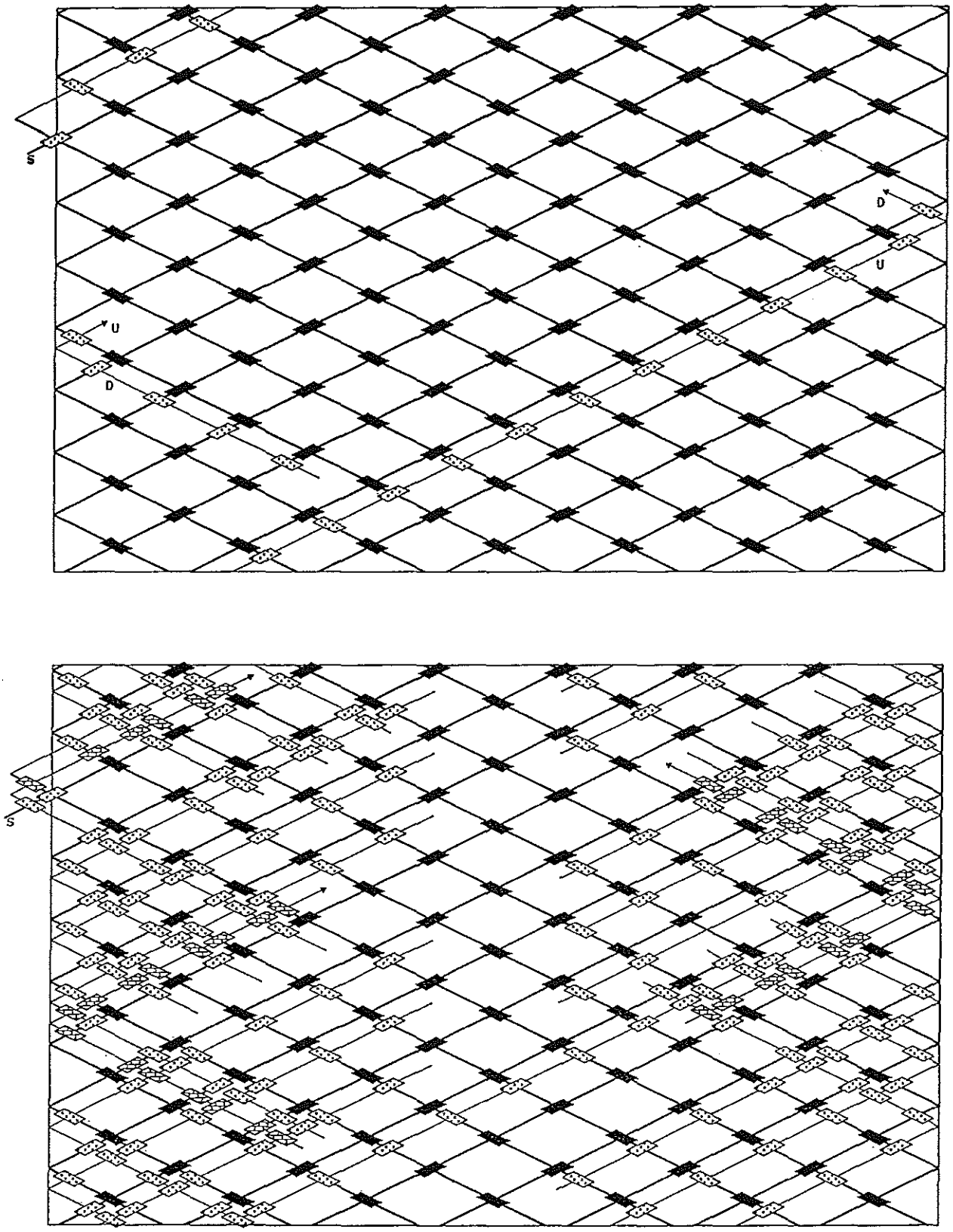


Fig. 146 — First order Method II enlargements for Casa-coded Regular Knots.

In Method I we do not encounter any difficulty at the start of its to be added first track-splitting half-cycle. For all other to be added track-splitting half-cycles, the first crossing is of the opposite nature to that of the last crossing of the immediately preceding track-splitting half-cycle, which in turn is of the opposite nature to *its* immediately preceding crossing which is determined by the track-splitting operation. In Method II we have first to finish off correctly the final to be added track-laying half-cycle before we can start with the track-splitting phase. The last three crossings of the final to be added track-laying half-cycle are alternating; the first of these three is determined by the track-laying process, and the last of these three is with the Standing half-cycle. The first crossing of the first to be added track-splitting half-cycle is of the opposite nature to the immediately preceding crossing (the last crossing of the final track-laying half-cycle, which is a crossing with the Standing half-cycle). For all other to be added track-splitting half-cycles, the first crossing is of the opposite nature to that of the last crossing of the immediately preceding track-splitting half-cycle, which in turn is of the opposite nature to *its* immediately preceding crossing which is determined by the track-splitting operation.

We note that the restrictive nature of these enlargement processes in association with their rules is caused by the track-laying phase. We have seen that this phase, under these rules, can only be of the first order. Furthermore we have seen that these enlargement processes cannot be applied to the embryonic Casa Knots (the 1-part Regular Knots). The interesting special case which constitutes the discovery of the Masurel algorithm is that of the 2-parts Column-coded Regular Knots. We know from the RKT (see *The Braider*, issues No. 6 & 7) that these knots can be enlarged to bigger Casa-coded Regular Knots by two successive first order enlargements (one for track-laying followed by one for track-splitting). These enlarged Casa-coded Regular Knots have 4-parts. It will immediately be observed that in this special case all 'holes' in the weave of the column-coded 2-parts Regular Knot are stacked in one 'column', in fact the weave is a closed overhand or underhand knot, and consequently the movement of the string through those 'holes' during the track-laying phase becomes a nice algorithm.

It should be of no surprise to anyone that this special case was experimentally discovered before, in fact it has most likely been genuinely discovered untold times, and only a few times at most will it have been communicated in the literature, a very normal occurence (for several obvious reasons) for most discoveries, if not all. But, as has been said before, any genuine discovery, irrespective of the number of times it may have been discovered before, does not lose its shine.

Some enchanting aspects of braids

Most people and even most braiders are totally unaware of the fabulous treasure house that is contained within braids. None of us is fully aware of what it all contains, since the eyes of a very few have seen glimpses only of some of its contents. It is not that its contents are hidden, but man's eyesight is very poor and biased which makes him in absolute sense to a large degree blind. Under the heading above we shall from time to time communicate to the reader some of the glimpses we have been privileged to observe. The treasures we will discuss are of course intimately associated with braids, but, very importantly, are not confined to braids. In fact they govern various important aspects

in the natural world around us, maybe they govern the basic fabric of the Universe. It should be clearly understood that we do not pretend to know the underlying reason(s) for the various relationships between braid-properties and phenomena in the world around us, nor do we pretend to know their sphere of influence. Hence we shall present only snippets of intriguing facts; any speculation is left to the philosopher.

There is a sequence of numbers that is associated with a multitude of phenomena in the world around us. Since this sequence of numbers is a very special and sweet one, we shall introduce it via those who make the most perfect sweet, namely the honey bee. The honey bee family is a remarkable family, still little understood. This is not surprising, since it has faculties we don't possess nor understand. For example, we know that bees can see ultraviolet light, but in which way they see it we don't know; we know that in a cylindrical magnetic field they build a cylindrical comb, but how they detect magnetic fields and in which way these fields influence them we don't know; we know that hives under high voltage lines produce more honey, while the bees are more aggressive, but why all this, we don't know; and so we could go on with numerous mysteries surrounding the honey bee family.

We all know that a honey bee family consists of a queen, workers and drones. We also know that the queen and the workers are females and develop from a fertilised egg, while the drones are males and develop from an unfertilised egg.

Human society tends to look down on the drones; maybe a matter of jealousy for are they not the lords of the castle? Let's face it, don't they seem to have the wisdom which enables them to enjoy the pleasures in life without having to bother with work? Does their genealogy contain the 'philosopher's stone' which bestows them with such a favourable position in their castle? All we know is that their genealogy contains that special sweet number sequence. So let's have a look at the genealogy of a drone bee; it is depicted in Fig. 147.

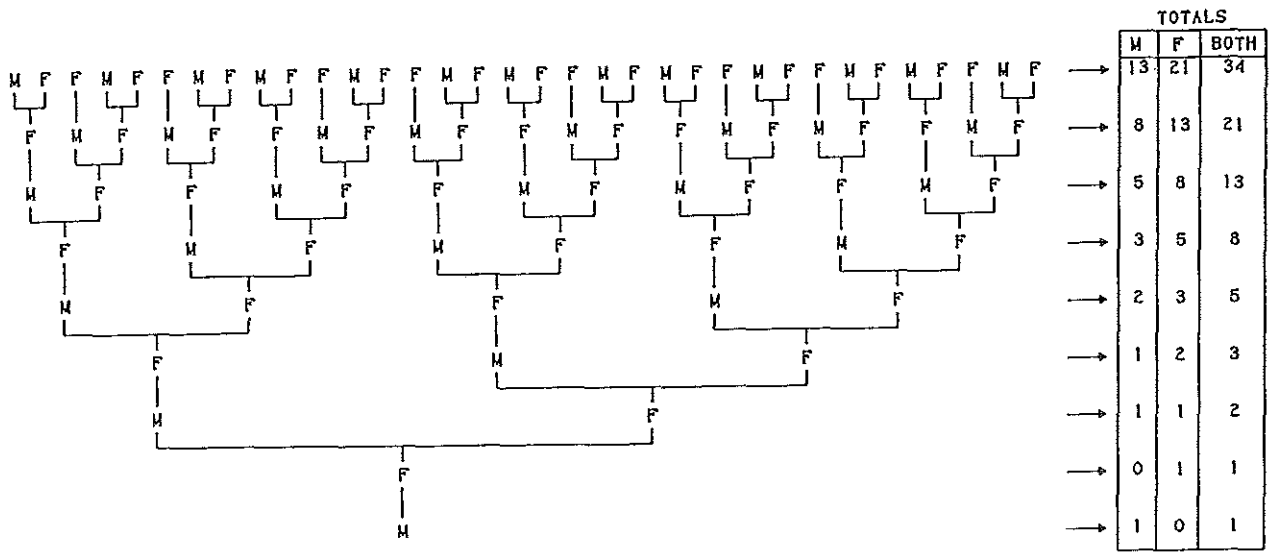


Fig. 147 — The genealogy of a drone bee.

From the table we see that his total number of ancestors in each consecutive earlier generation form the number sequence:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, \cdots .$$

Note that any term after the second term is the sum of its two immediately preceding terms.

This number sequence is known as the **Fibonacci series**, so named by Edward Lucas in 1877. We shall indicate the terms in this series as follows:

$$F_1\,,\ F_2\,,\ F_3\,,\ F_4\,,\ F_5\,,\ F_6\,,\ F_7\,,\ \cdots\,,\ F_n\,,\ \cdots\,.^{\dagger}$$

Since bees did exist well before man appeared on the scene, and hence well before mathematicians did exist, it might have been better if this number series would have been named after the drone rather than after a mathematician; but then again, it is associated with a multitude of phenomena which did exist well before the bee did.‡

Let's expand the table at the right-hand side of Fig. 147 with the ratios male/female, and male/(male + female). Then let's depict these two sets of ratios in the form as shown at the right-hand side of Fig. 148.

TOTALS			RATIOS	
M	F	BOTH	M/F	M/BOTH
13	21	34	13/21	13/34
8	13	21	8/13	8/21
5	8	13	5/8	5/13
3	5	8	3/5	3/8
2	3	5	2/3	2/5
1	2	3	1/2	1/3
1	1	2	1/1	1/2
0	1	1	0/1	0/1
1	0	1	1/0	1/1

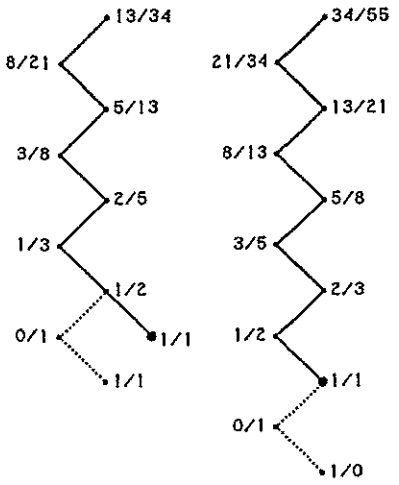


Fig. 148 — The expanded genealogy table of Fig. 147.

We notice that the graphical representations of these two ratio sequences are also to be found in the left half of the RKT (the inverse ratio sequences are in the right half of the RKT); see Fig. 149.

It is of some considerable interest to note that these two ratio sequences (and their inverse) are the only $F_n/F_{(n+m)}$ ratio sequences ($n = 1, 2, 3, \cdots$, while m is constant) in which the two integers of each ratio are coprime.

⋆⋆ *Prove this above statement.*

It is also interesting that, in as far as the Fibonacci series is concerned, only ratios from these two ratio sequences seem to play a role in the world around us. Furthermore,

† Hence $F_4 = 3$; $F_7 = 13$; $F_{13} = 233$.

‡ Not surprisingly, this series was even at the dawn of time intimately associated with a multidue of phenomena in the Universe, hence whether or not man would eventually discover its existence is in real terms irrelevant an hence so are any historical aspects associated with man's discoveries. Nevertheless, what we have said concerning discoveries in the previous article in this issue, holds of course also here when one finds the involvement of the Fibonacci series with phenomena in the world around us. There is even an Association of mathematicians, called **The Fibonacci Association**, which pretends to be primarily interested in applications in which the Fibonacci series plays a role. Although this may well have been its initial aim, today however this Association appears to be of a more degenerative kind with respect to the Fibonacci number series.

the sequence of ratios with $m = 1$ appears to occur much more often than the sequence of ratios with $m = 2$. Does this indicate that in the bee family the drones are more special than the rest, and hence is man's opinion biased due to a self-centred outlook? All those question we obvious leave to the philosopher!

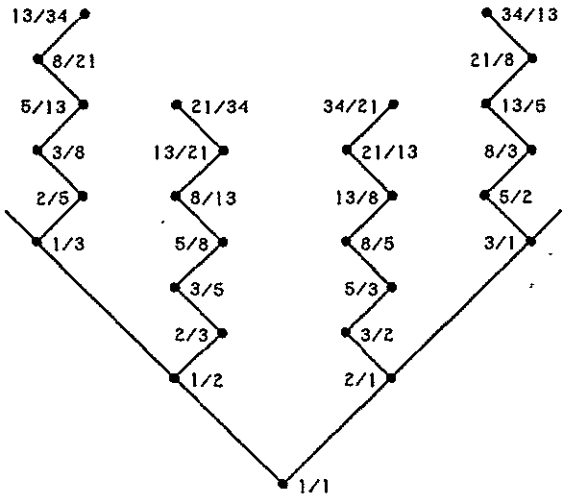


Fig. 149 — The sequences $F_n/F_{(n+1)}$ and $F_n/F_{(n+2)}$, and their inverse, in the RKT.

Of the four upward paths with first order enlargement steps in Fig. 149, the two with $m = 1$ start at the trunk-node, hence at level 0, and the two with $m = 2$ start at 1 level above the trunk-node, hence at level 1 (the nodes 1/2 and 2/1 respectively).

Let's look at some basic properties in the RKT. The left-hand diagram of Fig. 150 depicts the case where p/b was obtained by enlarging $p^*/b*$ by means of a first order Method I enlargement process, and where p/b is enlarged by a first order Method I enlargement process to $(2p - p^*)/(2b - b^*)$, respectively by a first order Method II enlargement process to $(p + p^*)/(b + b^*)$. If the rational number p/b is smaller than the rational number p^*/b^*, then the rational number $(2p - p^*)/(2b - b^*)$ is smaller than the rational number p/b, and the rational number $(p + p^*)/(b + b^*)$ is greater than the rational number p/b.

The right-hand diagram of Fig. 150 depicts the case where p/b was obtained by enlarging p^*/b^* by means of a first order Method II enlargement process, and where p/b is enlarged by a first order Method I enlargement process to $(p + p^*)/(b + b^*)$, respectively by a first order Method II enlargement process to $(2p - p^*)/(2b - b^*)$. If the rational number p/b is greater than the rational number p^*/b^*, then the rational number $(p + p^*)/(b + b^*)$ is smaller than the rational number p/b, and the rational number $(2p - p^*)/(2b - b^*)$ is greater than the rational number p/b.



Fig. 150 — Adjacent nodes and their associated rational numbers.

These relationships between the rational numbers concerned may readily be proved by expressing the rational number $(2p - p^*)/(2b - b^*)$ in the form $b(2p - p^*)/b(2b - b^*)$, and the rational number p/b in the form $p(2b - b^*)/b(2b - b^*)$. The denominators are now identical and hence we have to show for the left-hand diagram in Fig. 150 that $b(2p - p^*)$ is smaller than $p(2b - b^*)$; thus that bp^* is greater than pb^*. This is true since

we assumed that the rational number p/b was smaller than the rational number p^*/b^*, hence that pb^*/bb^* was smaller than bp^*/bb^*, thus bp^* greater than pb^*. For the right-hand diagram in Fig. 150 we have to show that $b(2p-p^*)$ is greater than $p(2b-b^*)$; thus that pb^* is greater than bp^*. This is true since we assumed that the rational number p/b was greater than the rational number p^*/b^*, hence that pb^*/bb^* was greater than bp^*/bb^*, thus pb^* greater than bp^*.

For the left-hand diagram in Fig. 150 we have to show that $b(p+p^*)$ is greater than $p(b+b^*)$; thus that pb^* is smaller than bp^*. This is true since we assumed that the rational number p/b was smaller than the rational number p^*/b^*, hence that pb^*/bb^* was smaller than bp^*/bb^*, thus pb^* smaller than bp^*. For the right-hand diagram in Fig. 150 we have to show that $b(p+p^*)$ is smaller than $p(b+b^*)$; thus that bp^* is smaller than pb^*. This is true since we assumed that the rational number p/b was greater than the rational number p^*/b^*, hence that pb^*/bb^* was greater than bp^*/bb^*, thus bp^* smaller than pb^*.

Since our above assumptions about the relative values of p/b and p^*/b^* are true at the beginning of the RKT, so by induction they are true everywhere in the RKT, and hence:

The rational number associated with a node in the RKT is greater than the rational number associated with its immediately succeeding node to the left, and smaller than the rational number associated with its immediately succeeding node to the right.

In Fig. 151 are depicted two general "hockey-stick" paths in the RKT. It will readily be seen that when n approaches infinity the rational number of its associated node approaches the value of p^*/b^*. Since for the left hockey-stick path the rational number p/b is greater than p^*/b^*, the rational numbers associated with the nodes along the stick decrease with increasing n. For the right hockey-stick path the rational number p/b is smaller than p^*/b^*, and hence the rational numbers associated with the nodes along the stick increase with increasing n.

The rational number associated with a node in the RKT is smaller than the rational numbers associated with its immediately succeeding nodes on a left hockey-stick (the values of the rational numbers associated with these nodes decrease and approach the value p^*/b^* towards infinity), and greater than the rational numbers associated with its immediately succeeding nodes on a right hockey-stick (the value of the rational numbers associated with these nodes increase and approach the value p^*/b^* towards infinity).

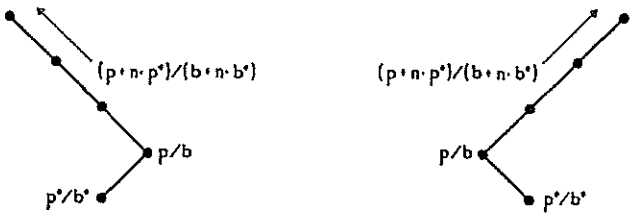


Fig. 151 — Hockey-stick paths in the RKT.

If we now apply these above two rules printed in bold to an arbitrary enlargement path consisting of alternating first order Method I and Method II steps, we obtain the result depicted in Fig. 152. Hence when such a path continues indefinitely, the rational numbers associated with consecutive nodes approach a value between that of the rational

number p/b and the rational number associated with its immediately succeeding node.

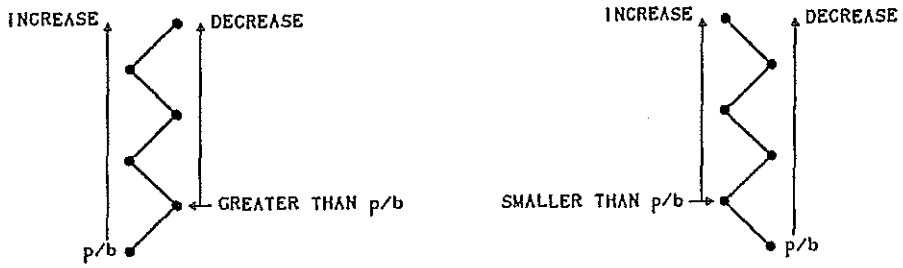


Fig. 152 — Alternating first order Method I and Method II enlargement paths.

Let's now examine the four enlargement paths we depicted in Fig. 149. To what value do the successive nodes $F_{(n+1)}/F_n$ approach when n approaches infinity (second path from the right in Fig. 149)? Then:

$$\frac{F_{(n+1)}}{F_n} = \frac{F_{(n+2)}}{F_{(n+1)}}.$$

$$\frac{F_{(n+1)}}{F_n} = \frac{F_{(n+1)} + F_n}{F_{(n+1)}}, \qquad \text{since } F_{(n+2)} = F_{(n+1)} + F_n.$$

$$\frac{F_{(n+1)}}{F_n} = \frac{\frac{F_{(n+1)}}{F_n} + 1}{\frac{F_{(n+1)}}{F_n}}.$$

Let $\dfrac{F_{(n+1)}}{F_n} = x$, then: $\quad x = \dfrac{x+1}{x}$.

$$x^2 - x - 1 = 0.$$

Hence: $\quad x = \dfrac{1+\sqrt{5}}{2} = 1.618033989\ldots.$

This number is known as the **golden ratio** (often also referred to as the **devine proportion**), and is indicated by the letter ϕ. It plays an important role in a multitude of phenomena, and those who like to read more about it are referred to two exellent publications: (1). *The Curves Of Live* by Theodore Andrea Cook, published by Dover Publications, Inc., New York; (2). *The Devine Proportion* by H.E. Huntley, published by Dover Publications, Inc., New York.

It will be obvious now that the successive nodes $F_n/F_{(n+1)}$ approach the value $1/\phi$ when n approaches infinity (second path from the left in Fig. 149).

The value to which the successive nodes $F_{(n+2)}/F_n$ approach when n approaches infinity (rightmost path in Fig. 149) is equal to $(\phi + 1) = \phi^2$, and consequently the successive nodes $F_n/F_{(n+2)}$ (leftmost path in Fig. 149) approach the value $1/\phi^2$ when n approaches infinity.

We have seen above how the four alternating first order Method I and Method II enlargement paths starting at level 0 and 1 are associated with the golden ratio ϕ. The immediate question is of course to what value the nodes on other alternating first order Method I and Method II enlargement paths approach. It may readily be shown that the nodes on such a path, starting at an arbitrary node p_0/b_0 (see Fig. 153), approach the value:

$$\psi = \frac{p_0 + \phi \cdot p_1}{b_0 + \phi \cdot b_1}.$$

In this formula for ψ we see the very special nature of ϕ clearly. It obviously should not matter which two consecutive nodes on this alternating first order Method I and Method II enlargement path we denote by p_0/b_0 and p_1/b_1 respectively since the path continues forever. We can readily prove that the formula for ψ complies with this obvious requirement concerning the consecutive nodes p_0/b_0 and p_1/b_1 since $\phi + 1 = \phi^2$.†

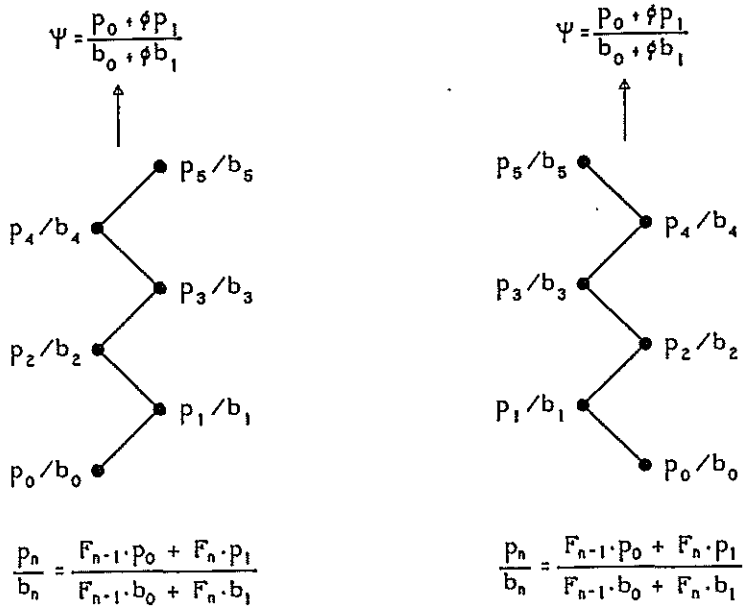


Fig. 153 — General alternating first order Method I and Method II enlargement paths.

The golden ratio ϕ plays a very important role in the RKT, and hence in Regular Knots, in fact it plays an important role in most, if not all, braids since ϕ is intimately associated with the Fibonacci series which in turn plays an important role in braids, examples of which we shall show in due course.

$\star\star$ *We have seen how any Casa-coded Regular Knot may be obtained by enlarging either an 1-part Regular Knot, a 2-parts Column-coded Regular Knot, an 1-bight Casa-coded Regular Knot, or a 2-bights Casa-coded Regular Knot. These enlargements can be categorised according to the successive Method I and Method II processes involved with track-laying and track-splitting. The categorisation may be done in relation to the number of entries in the path-formulae. Recall that each entry in a path-formula can be either odd or even, except the last entry which is always 1, and for knots in which the number of parts is smaller than the number of bights the first entry is 0. What do you discover in the categorisation?*

When a Casa-coded Regular Knot can be raised from both an 1-part Regular Knot and a 3-parts Casa-coded Regular Knot, it is in practice much easier to do it from the 3-parts Casa-coded Regular Knot. What proportion of Casa-coded Regular Knots can only be obtained from 1-part Regular Knots, what proportion from 2-parts Column-coded Regular Knots, what proportion from 3-parts Casa-coded Regular Knots, what proportion from 1-bight Casa-coded Regular Knots, and what proportion from 2-bights Casa-coded Regular Knots?

---

† Since $\phi + 1 = \phi^2$, it follws that $\phi - 1 = \frac{1}{\phi}$. Also here ϕ is unique; it is the only number which, when deminished by unity, becomes its own reciprocal.

Braiding Instructions and their Presentation

In *The Braider* issue No. 5 we discussed *the complementary cyclic bight-number scheme* for Regular Knots and *the algorithm diagram* for Column-coded Regular Knots. In this issue we will look at them in assosiation with Semi-Regular Knots. Recall that Regular Knots are Regular Cylindrical Braids in which the number of parts and the number of bights are **coprime**, and that Semi-Regular Knots are Regular Cylindrical Braids in which the number of parts and the number of bights have a greatest common divisor greater than 1, hence are not coprime. Also recall that for a Regular Cylindrical Braid the greatest common divisor of its parts and bights is equal to the number of strings required in its construction. For easy reference we have in the diagrams placed the starting-points of the strings in adjacent bight-points, however, it should be needless to say that in the construction of the braid we distribute the starting-points of the strings as evenly as possible along a bight-boundary.

The distribution of the actual starting-points has no influence on the complementary cyclic bight-number scheme, nor has it any influence on the algorithm diagram for Column-coded Semi-Regular Knots. Only the string-sequence of braiding affects the complementary cyclic bight-number scheme and hence the algorithm diagram.

The string-run diagrams in Fig. 154 are of Semi-Regular Knots which have 20-parts and 15-bights. Since the greatest common divisor of 20 and 15 is equal to 5, we require five strings in their construction. The strings are numbered in the order in which they will be braided.

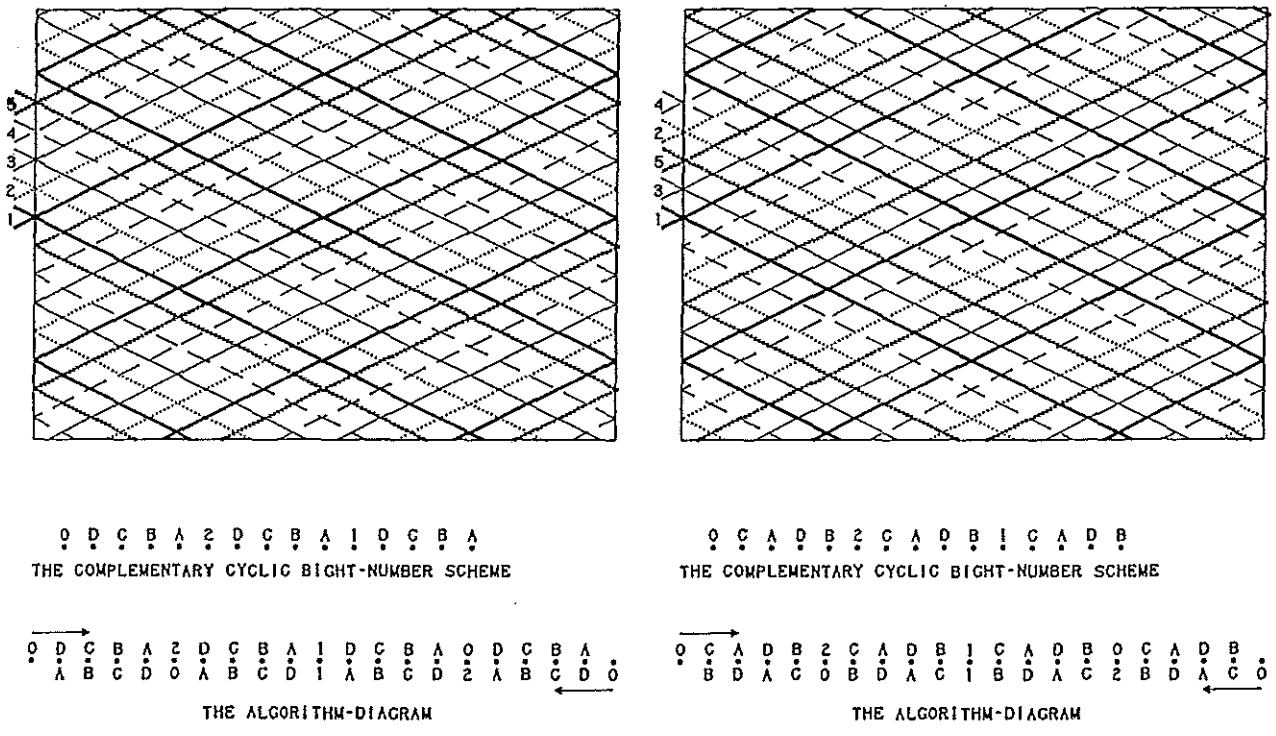

5
4
3
2
1
0 D C B A 2 D C B A 1 D C B A
THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME
0 D C B A 2 D C B A 1 D C B A 0 D C B A
A B C D 0 A B C D 1 A B C D 2 A B C D 0
THE ALGORITHM-DIAGRAM
4
2
5
3
1
0 C A D B 2 C A D B 1 C A D B
THE COMPLEMENTARY CYCLIC BIGHT-NUMBER SCHEME
0 C A D B 2 C A D B 1 C A D B 0 C A D B
B D A C 0 B D A C 1 B D A C 2 B D A C 0
THE ALGORITHM-DIAGRAM

Fig. 154 — Two examples of regular periodic sequences for braiding the strings.

Since any string-sequence of braiding is cyclic, we obtain for n strings a total of $(n-1)!$ distinctly different sequences.† Let there be m natural numbers, each one smaller than n, which are coprime with n. Then the $(n-1)!$ distinctly different string-sequences of braiding the n strings contain m sequences which are **regular** periodic. For example, when $n = 9$ we have $(n-1)! = (9-1)! = 8! = 40320$ distictly different string-sequences, which contain $m = 6$ sequences which are regular periodic (the natural numbers $1, 2, 4, 5, 7, 8$ are coprime with 9). These six regular periodic string-sequences are:

$1, 2, 3, 4, 5, 6, 7, 8, 9.$
$1, 3, 5, 7, 9, 2, 4, 6, 8.$
$1, 5, 9, 4, 8, 3, 7, 2, 6.$
$1, 6, 2, 7, 3, 8, 4, 9, 5.$
$1, 8, 6, 4, 2, 9, 7, 5, 3.$
$1, 9, 8, 7, 6, 5, 4, 3, 2.$

Only for **regular** periodic string-sequences will a single complementary cyclic bight-number scheme suffice, and hence for column-coded Semi-Regular Knots a single algorithm diagram is all that is needed for braiding the knot.

For the diagrams in Fig. 154, the respective string-sequences are $1, 2, 3, 4, 5$ and $1, 3, 5, 2, 4$. These sequences are **regular** periodic. Since $p = 20$ and $b = 15$, we obtain $|-p|_b = |-20|_{15} = 10$. Consequently we obtain the following complementary cyclic bight-number scheme for each single string:

0 2 1
. . . . . .

Of the 15 bight-points only 3 have received a bight-number ($b \div$ (g.c.d. of p and b) $= 15 \div 5 = 3$).

For every string on its own we can use this scheme, and in doing so the '0' point does not stay in the same bight-point of the knot, but shifts to the bight-point of the string under consideration. Strings which have already been laid down occupy their associated bight-points relative to the bight-points of the string under consideration, hence relative to the numbered bight-points in our above complementary cyclic bight-number scheme. Thus for the string-run of the left-hand knot in Fig. 154 we obtain the following: First we lay down string #1; for this string the above complementary cyclic bight-number scheme applies. Then we lay down string #2, but since string #1 already occupies the bight-points immediately **below** those which string #2 is going to occupy, we mark the bight-points immediately to the **left** (in cyclic fashion of course) of those who possess numerical values with A. Hence for string #2 we obtain the complementary cyclic bight-number scheme:

0 A 2 A 1 A
. . . . . .

Next we lay down string #3, but since strings #1 and #2 already occupy the two consecutive bight-points immediately **below** those which string #3 is going to occupy, we mark the bight-points immediately to the **left** (in cyclic fashion of course) of those

† The factorial of a natural number n (the numbers $1, 2, 3, \cdots$), written as $n!$, is the product of that number and all the natural numbers less than it. For example, the factorial of 5 is $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

who possess the letter A with B. Hence for string #3 we obtain the complementary cyclic bight-number scheme:

$$\begin{array}{ccccccccccccccc} 0 & & & B & A & 2 & & & B & A & 1 & & & B & A \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

Although the 'A' points were occupied later than the 'B' points, this is irrelevant for our purpose since both the 'A' points and 'B' points are occupied **before** the laying down of string #3 commences.

Next we lay down string #4, but since strings #1, #2 and #3 already occupy the three consecutive bight-points immediately **below** those which string #4 is going to occupy, we mark the bight-points immediately to the **left** (in cyclic fashion of course) of those who possess the letter B with C. Hence for string #4 we obtain the complementary cyclic bight-number scheme:

$$\begin{array}{ccccccccccccccc} 0 & & C & B & A & 2 & & C & B & A & 1 & & C & B & A \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

Although the 'A' points were occupied later than the 'B' points, which in turn were later occupied than the 'C' points, this is irrelevant for our purpose since the 'A' points, 'B' points and 'C' points are occupied **before** the laying down of string #4 commences.

Finally we lay down string #5, but since strings #1, #2, #3 and #4 already occupy the four consecutive bight-points immediately **below** those which string #5 is going to occupy, we mark the bight-points immediately to the **left** (in cyclic fashion of course) of those who possess the letter C with D. Hence for string #5 we obtain the complementary cyclic bight-number scheme:

$$\begin{array}{ccccccccccccccc} 0 & D & C & B & A & 2 & D & C & B & A & 1 & D & C & B & A \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

Although the 'A' points were occupied later than the 'B' points, which in turn were later occupied than the 'C' points, which in turn were later occupied than the 'D' points, this is irrelevant for our purpose since the 'A' points, 'B' points, 'C' points and 'D' points are occupied **before** the laying down of string #5 commences.

This completes the complementary cyclic bight-number scheme for the knot concerned.

The algorithm diagram is constructed as usual with the aid of the completed complementary cyclic bight-number scheme (the dots have to be replaced by the appropriate coding in accordance with that of their associated columns.

For the the string-run of the right-hand knot in Fig. 154 we obtain the following: First we lay down string #1; its complementary cyclic bight-number scheme is:

$$\begin{array}{ccccccccccccccc} 0 & & & & & 2 & & & & & 1 & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

Then we lay down string #2; its complementary cyclic bight-number scheme is:

$$\begin{array}{ccccccccccccccc} 0 & & A & & & 2 & & A & & & 1 & & A & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{array}$$

Then we lay down string #3; its complementary cyclic bight-number scheme is:

0 A B 2 A B 1 A B

. . . . . .

Then we lay down string #4; its complementary cyclic bight-number scheme is:

0 C A B 2 C A B 1 C A B

. . . . . .

Finally we lay down string #4; its complementary cyclic bight-number scheme is:

0 C A D B 2 C A D B 1 C A D B

. . . . . .

This completes the complementary cyclic bight-number scheme for the rightmost string-run of Fig. 154. The algorithm diagram is constructed again as usual with the aid of the completed complementary cyclic bight-number scheme (the dots have to be replaced by the appropriate coding in accordance with that of their associated columns.

When the periodic string-sequence is **not regular** we require more than one complementary cyclic bight-number scheme, and consequently more than one algorithm diagram as is shown with an example in Fig. 155.

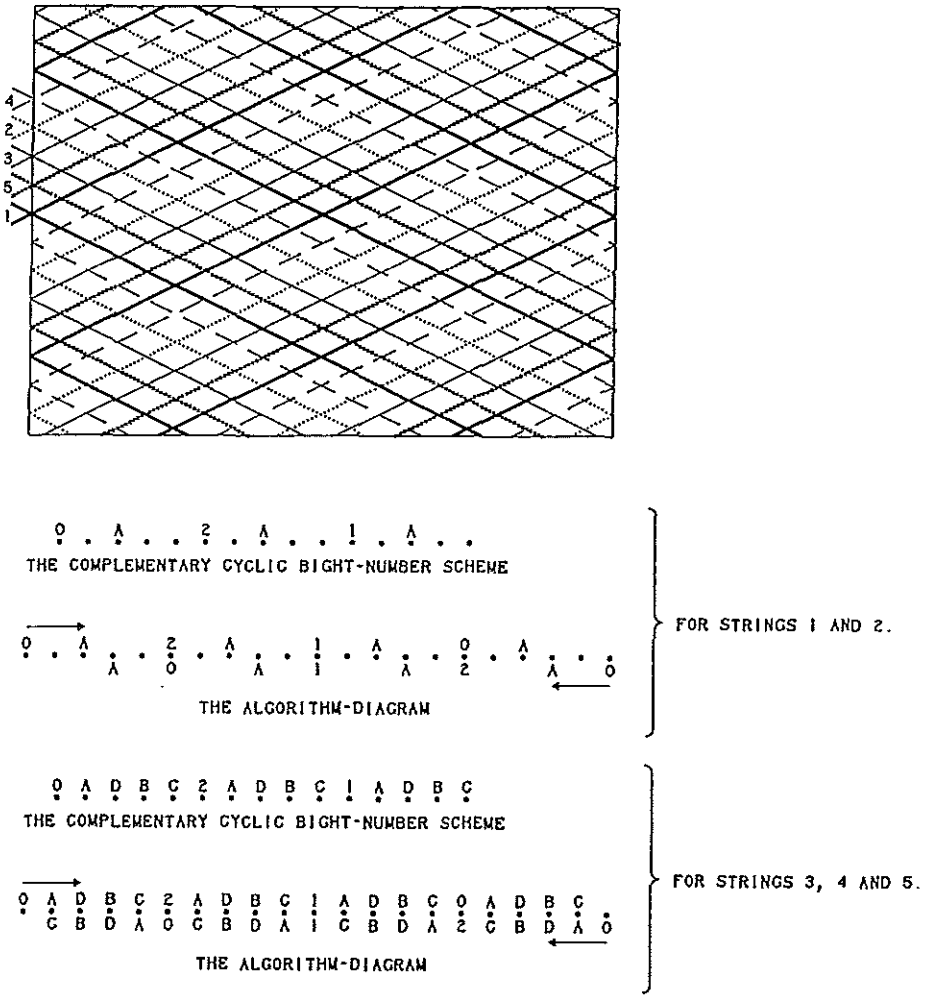


Fig. 155 — An example of a non regular periodic string-sequence.

THE BRAIDER

APPENDIX 1996

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A quarterly publication for the braiding artisan

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Reviews

We hereby like to thank Tom Hall for informing us that his copy of **The Bushman's Handcrafts** differs from the one we reviewed in the August Issue (No. 7), pg. 154. The copy we reviewed was the reprint of 1981; the copy which Tom possesses is the revised edition of 1988. There seems, however, to be some Australian funny business going on since both copies carry exactly the same ISBN number of 0 9599299 0 8. It is well known that, when the publication has been provided with an ISBN number, any change in such a publication as for example a reissue, reprint, revision, different form of presentation such as covers, etc. requires by law a new and different ISBN number. Such a procedure is of course an obvious requirement for the ISBN number to have any significance.

Not only carry the reprint from 1981 and the revised edition of 1988 the same ISBN number, but it appears likely that earlier reprints (or revisions?) carry the identical ISBN number since the first line in the '*Foreword*' of our reprint has the controversial statement: *The Bushman's Handcrafts is the revision of a previous edition issued over 20 years ago.* This tends to suggest that our copy is not as stated a reprint but a revised edition, however, the publication list specifies: *First published 1943, reprinted 1948, reprinted 1950, reprinted 1953, reprinted 1954, reprinted 1966, reprinted 1972, reprinted 1973, reprinted 1975, reprinted 1977, reprinted 1979, and reprinted 1981.* Tom's copy has the following additions to this list: *reprinted 1984, revised edition 1988.* It is thus obvious that the ISBN number in *The Bushman's Handcrafts* is totally meaningless.

The illustrations and pages are somewhat rearranged in the 1988 revised edition, although the comment we made with regards the error in the instructions for the "Whip Handle Turks Head" applies also here.

While we are on the subject, there is an interesting illustration on pg. 217 of the 1981 reprint (or on pg. 171 of the 1988 revised edition). This illustration is reproduced in Fig. 1 below.

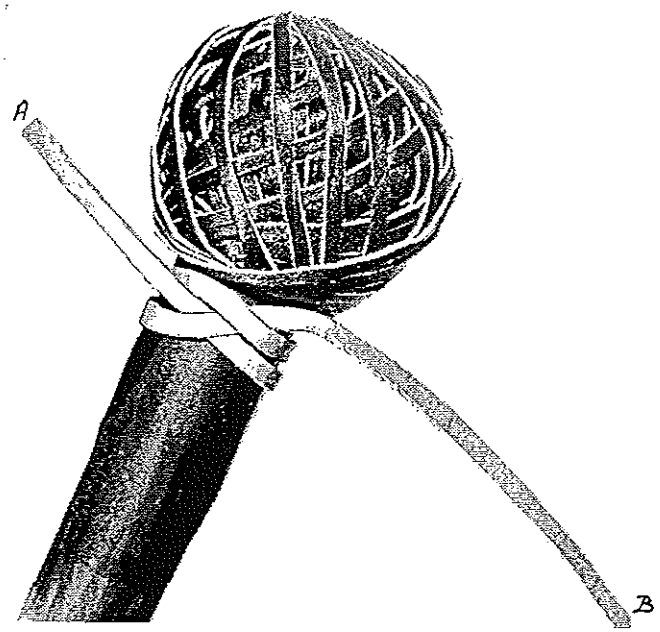


Fig. 1 — The Knot in question.

The subject of interest is the knot below the knob. Fortunately the text indicates that this knot is a **clove hitch**, and hence we know what it is supposed to be. Nevertheless, as is so often the case with three dimensional pictures, we encounter here a good example of an ambiguous illustration.

• Take string-end A as the Standing End: we will obtain the knot whose grid-diagram is depicted on the extreme left in Fig. 2. This knot will readily transform into a clove hitch (second grid-diagram from the left). This is obviously the knot intended.
• Take string-end B as the Standing End: we will obtain the knot whose grid-diagram is depicted on the extreme right in Fig. 2. This knot cannot be transformed into a clove hitch, and hence is obviously not the knot intended.

The problem here is that there is a crossing at the back of the illustration, but since it is invisible we are not aware of its existence and consequently we don't know the type of crossing. Such problems will never arise when grid-diagrams are used!!!

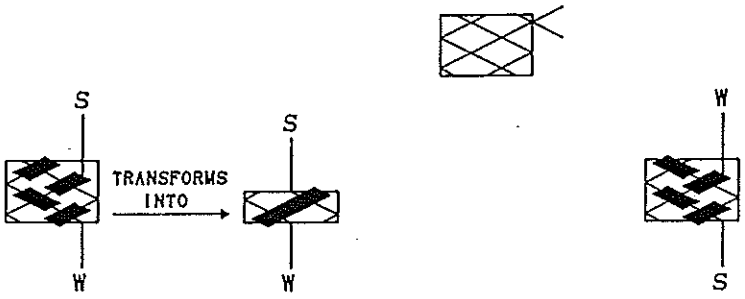


Fig. 2 — The two possible knots represented by Fig. 1.

A Reminder

Please don't forget that we like to receive your solutions to the questions posed in *The Braider*. Your participation in this aspect will of course be acknowledged. Furthermore, if you have any queries, comments, etc., please don't hesitate to drop us a line; you will promptly receive a personal reply. If we feel that a query or comment may lead to explanations which could be of benifit to other readers, we will also deal with them in a often more general way in either a future Issue of *The Braider* or in the yearly Appendices.

Queries Received

We only received one query. Does this really indicate that the articles are so clear that to most readers there are no question marks??? We don't believe it, in fact read under the next heading!!!

Since it is most unlikely that the person who poses a question is the only one who would like to see some further clarification, we will briefly discuss these questions for the benefit of those who also wonder why, but don't ask for further clarification.

Again we like to thank Tom Hall for his question, which refers to our statement under *Reviews* in Issue No. 7, pg. 154, why the 'rolling over' method, for the construc-

tion of *The Two Thong Turk's Head* (on pg. 209 in *The Bushman's Handcrafts* of our reprinted edition of 1981, should be preferred. The reason for preferring the 'rolling over' construction procedure is that the "crowning" and "walling" operations are very basic and hence often used in the construction of many different knots. Consequently the "crowning" and "walling" operations are much easier to remember than specific construction methods associated with a specific knot. Only when such specific knots are tied often enough will the average person be able to remember the specific procedures involved. But, of course, there is nothing against an individual wanting to use one of the few specific construction methods for the knot in question. A further reason for using the 'rolling over' procedure is to be at all times well aware of the properties of the 'rolled over' knot and hence not to use it in an incorrect position.

Where are the Queries we should have received, but didn't???????

Sure, all our subscribers (there are only a small dozen though) reside in the Northern Hemisphere, and there it was and still is hibernation-time when they received the November 1996 Issue of *The Braider*. But, ... braiders are not supposed to hibernate, well ... that's as far as we know, and since we know blow all, please forgive us our ignorance.

As you all know, each issue is ready well before its publication date, hence to be honest, we didn't expect questions since most people don't seem to ask any (some too shy and some can't be bothered). Hence we thought: let's see if we can trick our readers with something which is correct but incomplete. It appears we succeeded!!! Of course discounting the smart aleck who saw the discrepency (most likely after reading this Appendix), but didn't want to embarress us; forget that next time and let us know, because there will be a next time some time, just to see if we are all still awake. Let those who don't see it not worry, we will make the story complete in the appropriate Appendix. But then on the other hand, we could have really fallen asleep!!!

So, let's see where the trick was this time.

On pg. 158, the fifth line from the top states: *"The lowermost cycle is readily rearranged: its intersection with the remaining* $(p-2)$ *cycles are transferred by sliding them along."*. From this statement it follows that we have at least **two** cycles to start with (the lowermost cycle + the minimum number of $(p-2)$ cycles, which is 1). Hence the knot has at least **two** bights. But the line above the footnote on the same page states: *"The 3-parts Casa-coded Regular Knots may be braided from either the* **3/1** *or 3/2 Casa-coded Regular Knot by repeating this bight-raising process the necessary number of times."*. The **3/1** Casa-coded Regular Knot, however, has only **one** bight.

The proof we presented is correct for Regular Knots with two or more bights, but does not cover Regular Knots with one bight. The limitation was due to the grid-diagram we employed in our proof. For a proper and complete proof we should have used the grid-diagram in Fig. 3. The sequence of steps depicted should make the procedure clear.

For the 1-part, 2-parts and 3-parts Casa-coded Regular Knots the first set of sequential steps for raising the number of bights are depicted in Fig. 4.

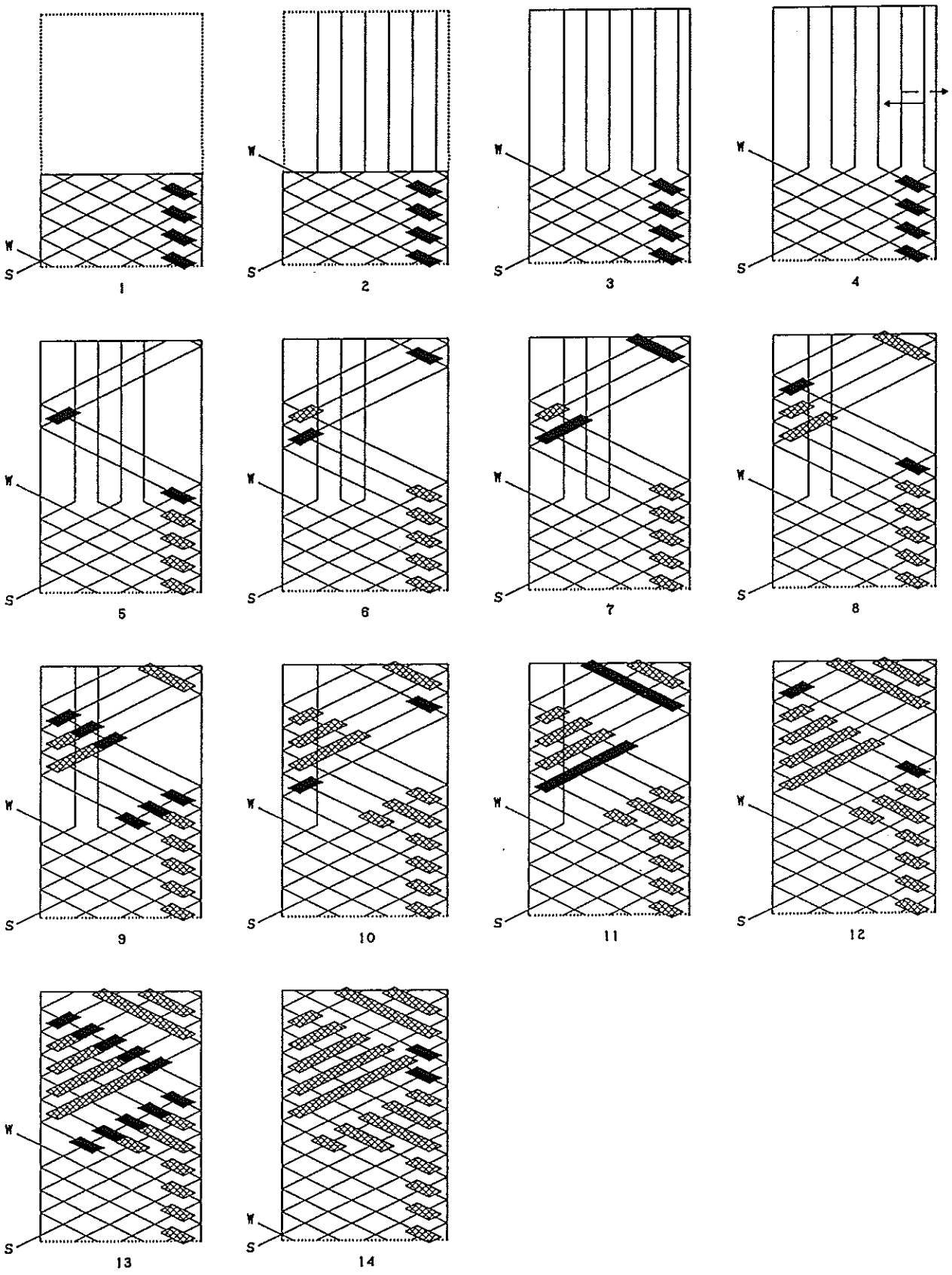


Fig. 3 — Raising the number of bights without raising the number of parts.

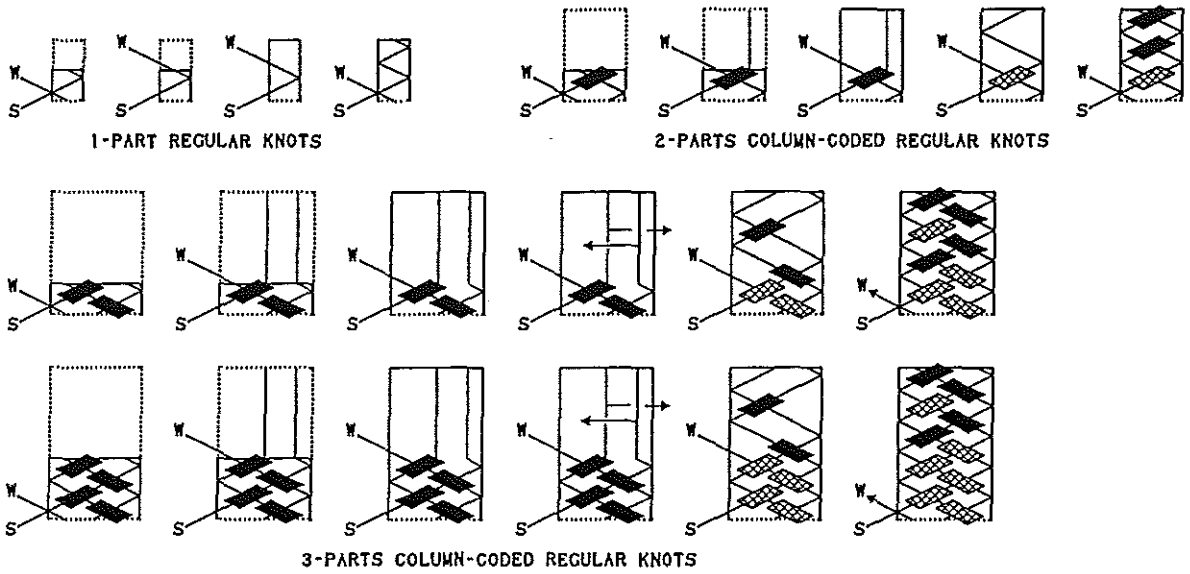


Fig. 4 — Raising the number of bights without raising the number of parts.

We furthermore like to draw the reader's attention to what has been stated on pp. 160 & 161 with regards the parts-raising process concerning the 2-bights and 3-bights Regular Knots (**not necessarily Casa-coded!!!**).

With respect to a **Casa-coding**, the requirement for avoiding full turns in the string does not imply that it is possible to construct any or all of the 2-bights and 3-bights **Casa-coded** Regular Knots without such twists in their string. In fact only the further 1-bight and 3-bights Casa-coded Regular Knots without twists in the string can be produced with the *"the parts-raising procedure without raising the bights"* by repeating the parts-raising process a sufficient number of times in such a way that the string receives the same number of full right-hand turns as full left-hand turns.

It was Tom Hall who pointed out that at least some readers might read more in our statements than it actually contained, and consequently might erroneously think that further 2-bights Casa-coded Regular Knots without twists in the string can be produced by repeating this parts-raising process a sufficient number of times in such a way that the string receives the same number of full right-hand turns as full left-hand turns. This is impossible since all full turns in the string are of the same kind, and hence will have the same helix, due to the required Casa-coding for 2-bights Casa-coded Regular Knots.

Note: In some copies of *The Braider*, No. 8, there is a typing error on pg. 161, 12th line from the top: 'bights-raising' should be 'parts-raising'.

Errata — *The Braider*

Some typing errors were noted in at least some copies of the following Issues of *The Braider*. Our grateful thanks to Doug Van Tassel who detected most of them. The corrections required are indicated below in bold:

No. 1

pg. 3, line 9 — **it** is
pg. 3, line 10 — **unnecessarily**
pg. 3, line 17 — in order **to** present
pg. 12, line 5 under "An Introduction to Flat Braids" — Four pages are devoted **to**
pg. 16, line 9 under "Reviews" — **instructor**

No. 2

pg. 21, line 2 — delete **that**
pg. 28, line 6 — However, **a** one colour
pg. 38, line 18 under "Reviews" — **they** are first class

No. 3

pg. 42, line 2 — Naturally, **queries** not directly

No. 4

pg. 76, line 5 — **phenomenon**
pg. 81, line 16 — **phenomenon**

No. 5

pg. 87, line 3 from bottom of page — **exclude**
pg. 106, line 11 — **locality**

No. 6

pg. 129, line 15 — 2-**colour**

No. 7

No. 8

pg. 158, line 3 — Hence **the** number of bights
pg. 161, line 12 — In Fig. 144 are depicted the **parts-raising** processes

---

Subscribers to *The Braider*

Only a small number of braiders receive this quarterly publication; at the moment a total of eleven. This small number clearly indicates that the progress in braiding is slow. By reflecting to the past, this is of course not surprising at all, in fact it was expected to be the case. We do expect progress in general to remain at a snail's pace for many years to come. Nevertheless, those braiders whose desire it is to know more about braiding than "the old stick-in-the-mud" might be interested in knowing who their "kinsmen" are. For them the up to date full address list in alphabetical order of all those who receive *The Braider* is as follows:

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A quarterly publication for the braiding artisan

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Solutions to the Questions in Issue No. 8

Question on pg. 168.

Since for $m = 1$ and $m = 2$ all ratios $F_n/F_{(n+m)}$, where $n = 1, 2, 3, \cdots$, occur in the RKT, it follows that F_n and $F_{(n+m)}$ are coprime for $m = 1$ and for $m = 2$. Hence we only have to investigate the status between F_n and $F_{(n+m)}$ for m greater than 2, and show that there are values of n for which F_n and $F_{(n+m)}$ are **not coprime.**

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	$\cdots$
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	$\cdots$

$$
\begin{aligned}
F_{1+m} &= F_{m-1} + F_m = F_1 \cdot F_{m-1} + F_2 \cdot F_m \,. \\
F_{2+m} &= F_m + F_{m+1} = F_{m-1} + 2F_m = F_2 \cdot F_{m-1} + F_3 \cdot F_m \,. \\
F_{3+m} &= F_{m+1} + F_{m+2} = 2F_{m-1} + 3F_m = F_3 \cdot F_{m-1} + F_4 \cdot F_m \,. \\
F_{4+m} &= F_{m+2} + F_{m+3} = 3F_{m-1} + 5F_m = F_4 \cdot F_{m-1} + F_5 \cdot F_m \,. \\
F_{5+m} &= F_{m+3} + F_{m+4} = 5F_{m-1} + 8F_m = F_5 \cdot F_{m-1} + F_6 \cdot F_m \,. \\
F_{6+m} &= F_{m+4} + F_{m+5} = 8F_{m-1} + 13F_m = F_6 \cdot F_{m-1} + F_7 \cdot F_m \,. \\
\vdots &= \qquad\qquad \vdots \\
F_{n+m} &= F_n \cdot F_{m-1} + F_{n+1} \cdot F_m \,.
\end{aligned}
$$

A more elegant proof for the relationship $F_{n+m} = F_n \cdot F_{m-1} + F_{n+1} \cdot F_m$ may be accomplished by means of induction:

Assume the relationship to be true for n and for $(n+1)$, hence:

$$
\begin{aligned}
F_{n+m} &= F_n \cdot F_{m-1} + F_{n+1} \cdot F_m \,. \\
F_{n+1+m} &= F_{n+1} \cdot F_{m-1} + F_{n+2} \cdot F_m \,.
\end{aligned}
$$

Then, since $F_{n+2+m} = F_{n+m} + F_{n+1+m} = (F_n + F_{n+1}) \cdot F_{m-1} + (F_{n+1} + F_{n+2}) \cdot F_m = F_{n+2} \cdot F_{m-1} + F_{n+3} \cdot F_m$, the relationship is also true for $(n+2)$. Since the relationship is true for $n = 1$ and for $n = 2$, the relationship is true for all natural numbers n.

$$
\begin{aligned}
F_{2m} &= F_{m+m} = F_m F_{m-1} + F_{m+1} F_m = F_m(F_{m-1} F_1 + F_{m+1}) \,. \\
F_{3m} &= F_{2m+m} = F_{2m} F_{m-1} + F_{2m+1} F_m = F_m(F_{m-1}^2 F_1 + F_{m-1} F_{m+1} + F_{2m+1}) \,. \\
F_{4m} &= F_{3m+m} = F_{3m} F_{m-1} + F_{3m+1} F_m \\
&= F_m(F_{m-1}^3 F_1 + F_{m-1}^2 F_{m+1} + F_{m-1} F_{2m+1} + F_{3m+1}) \,. \\
F_{5m} &= F_{4m+m} = F_{4m} F_{m-1} + F_{4m+1} F_m \\
&= F_m(F_{m-1}^4 F_1 + F_{m-1}^3 F_{m+1} + F_{m-1}^2 F_{2m+1} + F_{m-1} F_{3m+1} + F_{4m+1}) \,. \\
\vdots &= \qquad\qquad \vdots \\
F_{(s+1)m} &= F_m(F_{m-1}^s F_1 + F_{m-1}^{s-1} F_{m+1} + F_{m-1}^{s-2} F_{2m+1} + F_{m-1}^{s-3} F_{3m+1} + \cdots + \\
&\quad + F_{m-1}^2 F_{(s-2)m+1} + F_{m-1} F_{(s-1)m+1} + F_{sm+1}) \\
&= F_m\left(F_{sm+1} + \sum_{k=1}^{k=s} F_{m-1}^k F_{(s-k)m+1}\right) \,.
\end{aligned}
$$

The sequence of the ratios $F_n/F_{(n+m)}$, where $n = 1, 2, 3, \cdots$ and m is constant, contains ratios in which n is equal to sm, where $s = 1, 2, 3, \cdots$. For such ratios $F_n/F_{(n+m)} = F_{sm}/F_{(s+1)m}$.

Since
$$\frac{F_{sm}}{F_{(s+1)m}} = \frac{F_m\left(F_{(s-1)m+1} + \sum_{k=1}^{k=s-1} F_{m-1}^k F_{(s-1-k)m+1}\right)}{F_m\left(F_{sm+1} + \sum_{k=1}^{k=s} F_{m-1}^k F_{(s-k)m+1}\right)},$$

F_n and F_{n+m} have for $n = sm$ a common divisor F_m, and hence are **not coprime** when m is greater than 2.†

Question on pg. 172.

The best general overview of Casa-coded Regular Knot enlargements is obtained by categorising these knots by a set of categorisation-paths for each number of straight path-elements in their enlargement-paths. For the **left half-tree** this has been done in Figs. 156 – 160 for enlargement-paths which have one to eight straight path-elements.

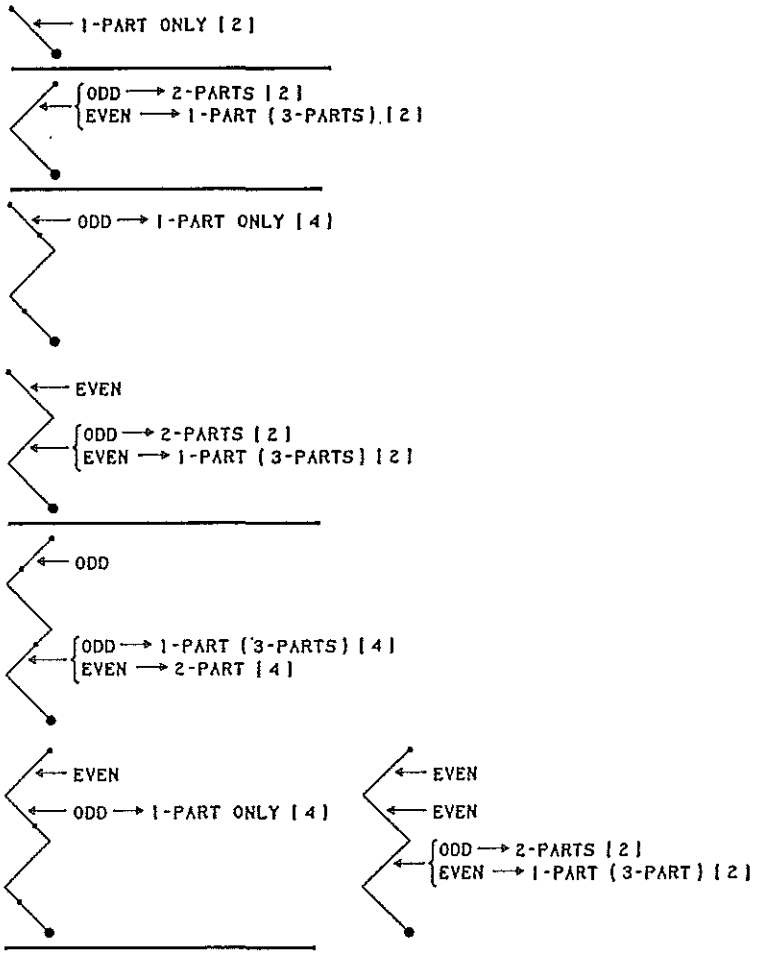


Fig. 156 — Categorisation for the 1 to 4 consecutive straight path-elements.

† $\sum_{k=1}^{k=s} F_{m-1}^k F_{(s-k)m+1}$ stands for the summation ($\sum$) of the terms $F_{m-1}^k F_{(s-k)m+1}$ in which k takes the successive values $1, 2, 3, \cdots, (s-2), (s-1), s$.

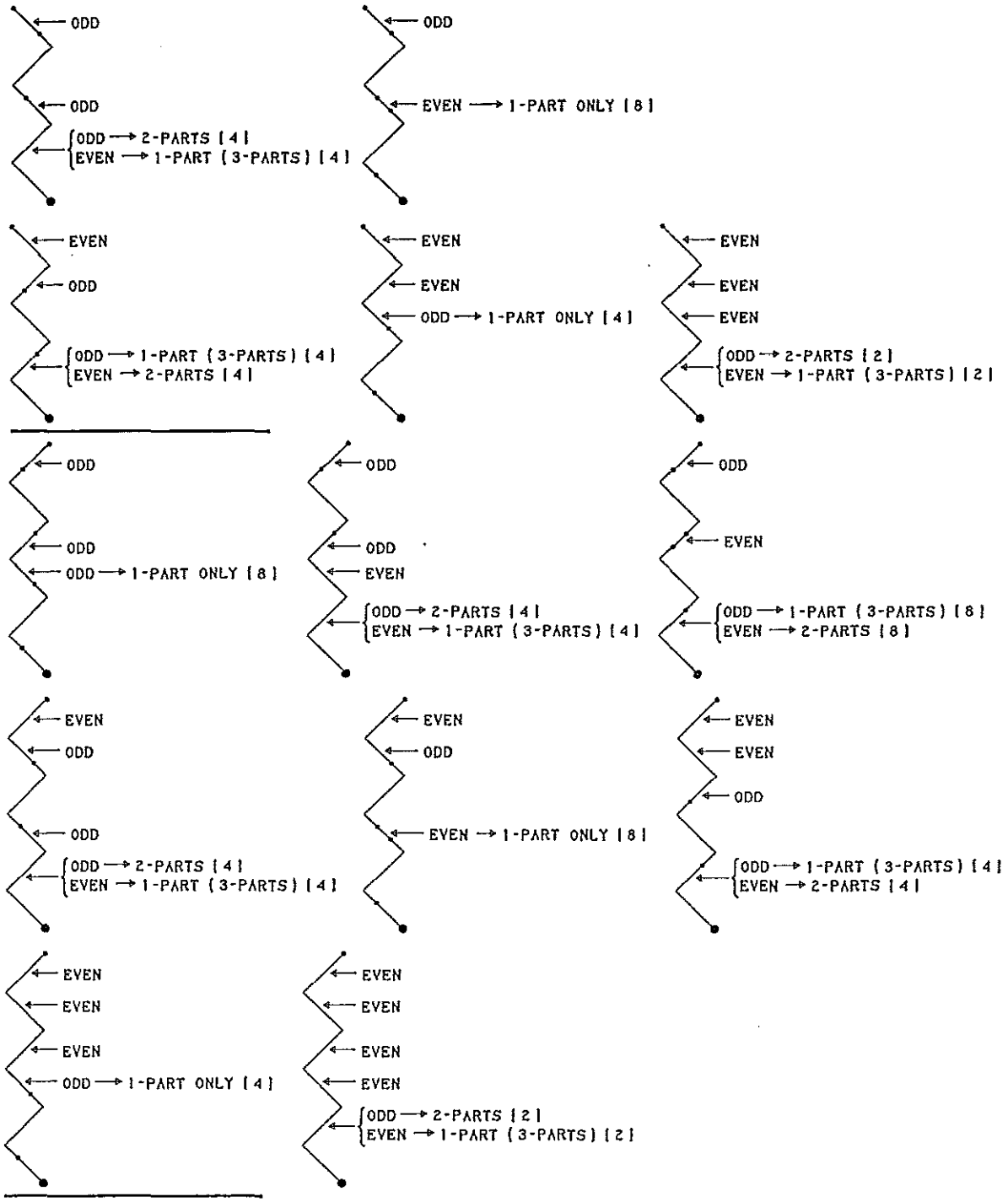


Fig. 157 — Categorisation for the 5 and 6 consecutive straight path-elements.

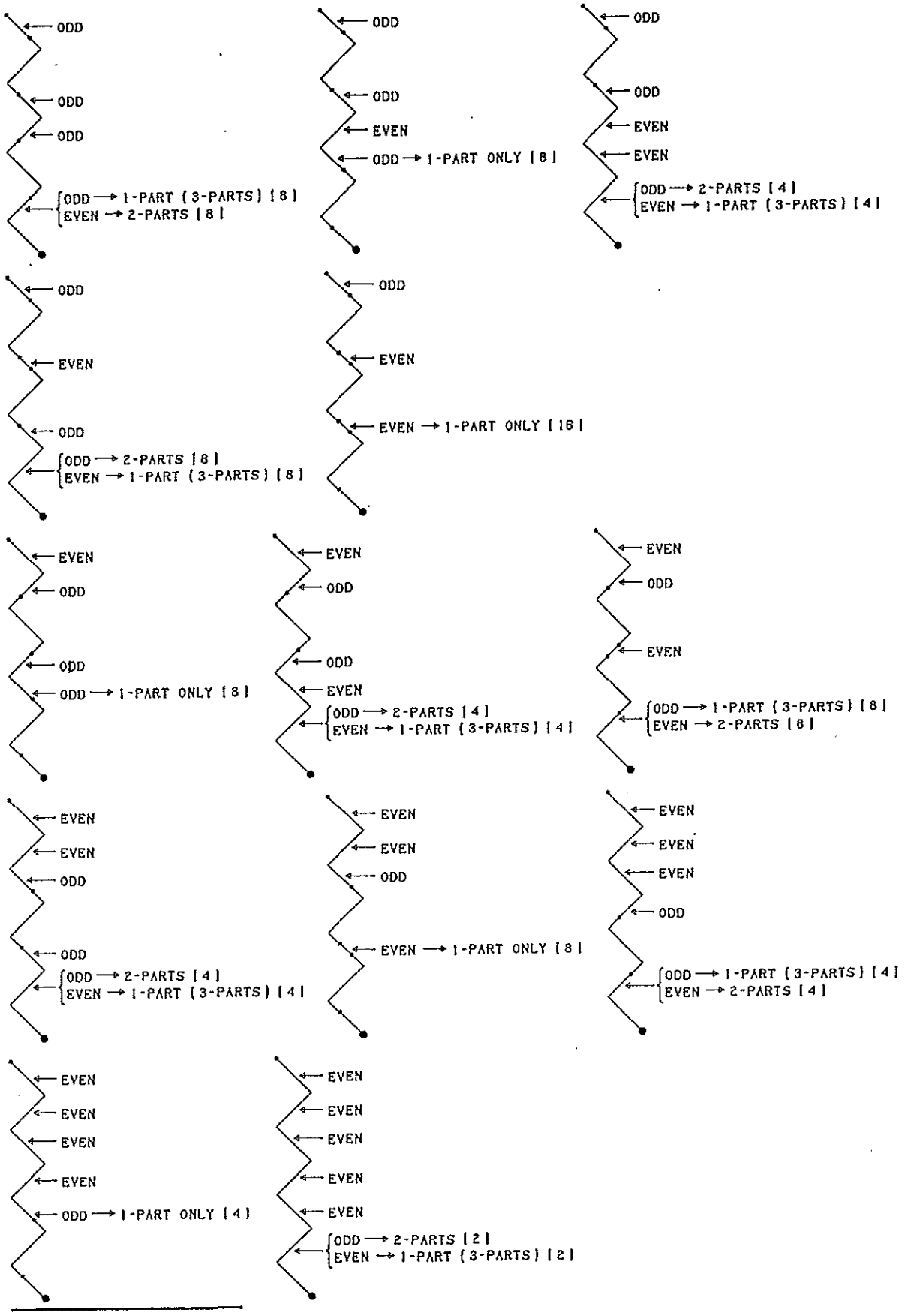


Fig. 158 — Categorisation for the 7 consecutive straight path-elements.

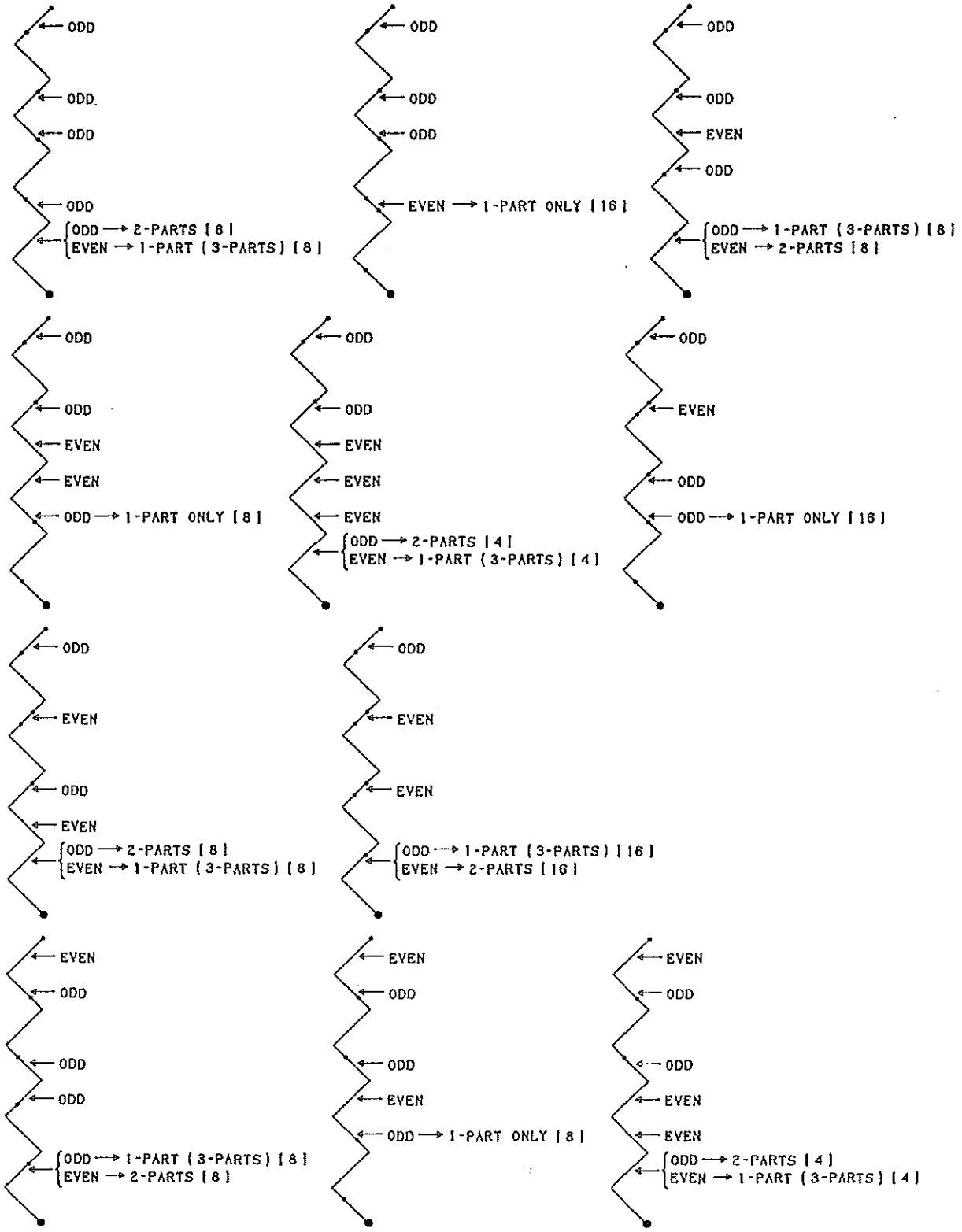


Fig. 159 — Categorisation for the 8 consecutive straight path-elements (cont. in Fig. 160).

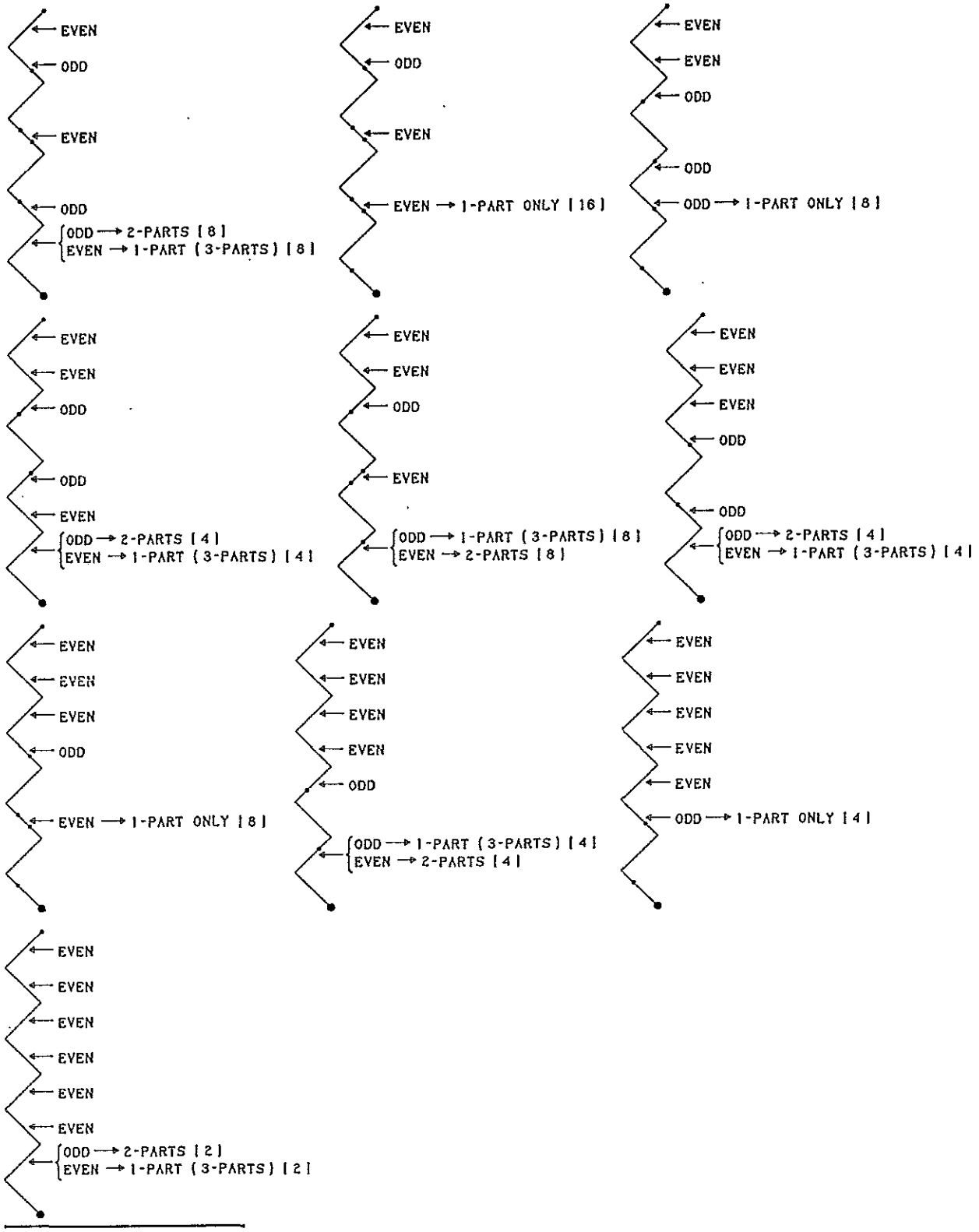


Fig. 160 — Categorisation for the 8 consecutive straight path-elements (cont. from Fig. 159).

In these categorisations of the Casa-coded Regular knots associated with the left-half of the RKT, *'ODD'* indicates an **odd** number of unit-steps along its associated straight path-element, and *'EVEN'* indicates an **even** number of unit-steps along its associated straight path-element. Such straight path-elements have thus only one possible parity, namely the parity indicated. Note that the **actual number** of unit-steps in each straight path-element is **not limited**, only the parity (*odd* or *even*) of the number plays a role. Straight path-elements without an *'ODD'* or *'EVEN'* indication, may be *'ODD'* or *'EVEN'*, and hence have two possible parities In each categorisation-path diagram one *'ODD'* and/or one *'EVEN'* parity indication is accompanied by a **parts** statement followed by a number between brackets. The parts statement indicates the number of parts of the smallest Casa-coded Regular Knot (including embryonic ones) with which the enlargement process starts. The 1-part starts which go through the 3-parts Casa phase, and hence may be started from 3-parts Casa-coded Regular knots, are indicated as 3-parts between parentheses. The number between brackets indicates the number of different sequential path parities, associated with the starting knot, within the categorisation-path concerned.

We observe that we can categorise the enlargements of Casa-coded Regular Knots in a set of categorisation-paths for each number of consecutive straight enlargement path-elements.

For one straight path-element we require one categorisation-path for categorisation.

For two consecutive straight path-elements we require also one categorisation-path for categorisation.

For three consecutive straight path-elements we require two categorisation-paths for categorisation.

For four consecutive straight path-elements we require three categorisation-paths for categorisation.

For five consecutive straight path-elements we require five categorisation-paths for categorisation.

For six consecutive straight path-elements we require eight categorisation-paths for categorisation.

For seven consecutive straight path-elements we require thirteen categorisation-paths for categorisation.

For eight consecutive straight path-elements we require twenty one categorisation-paths for categorisation.

And so on.

In the categorisation, the number of categorisation-paths per number of straight path-elements follows the Fibonacci series. In fact starting from 3 consecutive straight path-elements, we observe that for each number of consecutive straight path-elements the set of categorisation-paths is divided into two subsets: the "odd" subset in which the final straight path-element has an **odd** number of unit-steps, and the "even" subset in which the final straight path-element has an **even** number of unit-steps. If we take for each number of consecutive straight path-elements (starting with 3) the ratio of the number of categorisation-paths in the "odd" subset over the number of categorisation-paths in the "even" subset, we obtain the sequence:

$$\frac{1}{1}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \frac{13}{21}, \cdots.$$

If we take for each number of consecutive straight path-elements (starting with 3) the

ratio of the number of categorisation-paths in the "odd" subset over the total number of categorisation-paths, we obtain the sequence:

$$\frac{1}{2}, \frac{1}{3}, \frac{2}{5}, \frac{3}{8}, \frac{5}{13}, \frac{8}{21}, \cdots .$$

This are the same sequences we met before with the bees, and which we meet over and over again in a multitude of other phenomena in the world around us.

In the right-half of the RKT an identical type of categorisation is obtained whereby the categorisation-paths are the mirror images of those in the left-half of the RKT with 1-part replaced by 1-bight while 3-parts are deleted, and 2-parts replaced by 2-bights.

The categorisation results for Casa-coded Regular Knot enlargements are tabulated in Figs. 161 and 162 for enlargement-paths with one to eight straight path-elements in respectively the left half-tree and the right half-tree.

We like to stress again that in the categorisation the actual number of unit-steps in each straight path-element is unlimited, only the parity (odd or even) of the number of unit-steps plays a role.

NUMBER OF CONSECUTIVE STRAIGHT PATH-ELEMENTS	NUMBER OF PATHS	PARITY OF FINAL STRAIGHT PATH-ELEMENT	NUMBER OF PATHS WITH DIFFERENT CONSECUTIVE PARITIES				
			3-PARTS	1-PART ONLY	1-PART TOTAL	2-PARTS	TOTAL
1	1	O/E		2	2		2
2	1	O				2	2
		E	2		2		2
3	1	O		4	4		4
	1	E	2		2	2	4
4	1	O	4		4	4	8
	2	E	2	4	6	2	8
5	2	O	4	8	12	4	16
	3	E	6	4	10	6	16
6	3	O	12	8	20	12	32
	5	E	10	12	22	10	32
7	5	O	20	24	44	20	64
	8	E	22	20	42	22	64
8	8	O	44	40	84	44	128
	13	E	42	44	86	42	128

Fig. 161 — Tabulation of the enlargement-paths in the left-half of the RKT.

Note that for i consecutive straight path-elements the total number of paths with different consecutive parities is equal to 2^i, and that this number is equally divided between the "odd" subset and the "even" subset of categorisation-paths. Furthermore note that in the RKT we find exactly the same relationship for the number of Regular Knots in level i above the trunk-node.†

† Be careful not to confuse level i above the trunk-node in the RKT with i consecutive straight path-elements. Each of the straight path-elements contains an infinite number of levels since the number of unit-steps in a straight path-element is unlimited.

NUMBER OF CONSECUTIVE STRAIGHT PATH-ELEMENTS	NUMBER OF PATHS	PARITY OF FINAL STRAIGHT PATH-ELEMENT	NUMBER OF PATHS WITH DIFFERENT CONSECUTIVE PARITIES		
			1-BIGHT	2-BIGHTS	TOTAL
1	1	O/E	2		2
2	1	O		2	2
		E	2		2
3	1	O	4		4
	1	E	2	2	4
4	1	O	4	4	8
	2	E	6	2	8
5	2	O	12	4	16
	3	E	10	6	16
6	3	O	20	12	32
	5	E	22	10	32
7	5	O	44	20	64
	8	E	42	22	64
8	8	O	84	44	128
	13	E	86	42	128

Fig. 162 — Tabulation of the enlargement-paths in the right-half of the RKT.

In Fig. 163 we have tabulated for consecutive numbers of straight path-elements the summations of their respective results. Although we have limited the table to ten straight path-elements, the number of straight path-elements obviously continues indefinitely in order to include all Casa-coded Regular Knots. The tabulated results clearly show that in the left half-tree 1/3 of all Casa-coded Regular Knots are raised from 1-part Regular Knots only, 1/3 of all Casa-coded Regular Knots may be raised from 3-parts Casa-coded Regular Knots, and 1/3 of all Casa-coded Regular Knots are raised from 2-parts Column-coded Regular Knots. These ratios have to be halved when all Casa-coded Regular Knots in the whole RKT are considered.

NUMBER OF CONSECUTIVE STRAIGHT PATH-ELEMENTS EQUAL AND SMALLER THAN			1	2	3	4	5	6	7	8	9	10
TOTAL NUMBER OF ENLARGEMENT-PATHS WITH DIFFERENT CONSECUTIVE PARITIES	LEFT HALF-TREE	1-PART ONLY	2	2	6	10	22	42	86	170	342	682
		3-PARTS	0	2	4	10	20	42	84	170	340	682
		2-PARTS	0	2	4	10	20	42	84	170	340	682
		TOTAL	2	6	14	30	62	126	254	510	1022	2046
	RIGHT HALF-TREE	1-BIGHT	2	4	10	20	42	84	170	340	682	1364
		2-BIGHTS	0	2	4	10	20	42	84	170	340	682
		TOTAL	2	6	14	30	62	126	254	510	1022	2046

Fig. 163 — Tabulated summations for consecutive straight path-elements.

In the above discussion we have considered the **parities** of the number of unit-steps in each straight path-element in order to account for **all** Casa-coded Regular Knots. We found that 1-part Regular Knots, 2-parts Column-coded Regular Knots, and 3-parts Casa-coded Regular Knots are of the same importance in the raising of bigger Casa-coded Regular Knots.

In order to show the role of 1-part Regular Knots, 2-parts Column-coded Regular Knots, and 3-parts Casa-coded Regular Knots in the raising of Casa-coded Regular Knots associated with consecutive **levels** in the RKT, we have tabulated their involvement in Fig. 164. Since final 1-part Regular Knots have no practical application on their own they have been discarded in the numbers between parentheses. Except for very low levels in the RKT, we observe similar relationships as before, hence also here we find that 1-part Regular Knots, 2-parts Column-coded Regular Knots, and 3-parts Casa-coded Regular Knots are near enough of the same importance in the raising of bigger Casa-coded Regular Knots.

KNOTS PER LEVEL					
LEVEL	3-PARTS	1-PART ONLY	1-PART TOTAL	2-PARTS	TOTAL
1	0	1 (0)	1 (0)	0	1 (0)
2	0	1 (0)	1 (0)	1	2 (1)
3	2	1 (0)	3 (2)	1	4 (3)
4	2	3 (2)	5 (4)	3	8 (7)
5	6	5 (4)	11 (10)	5	16 (15)
6	10	11 (10)	21 (20)	11	32 (31)
7	22	21 (20)	43 (42)	21	64 (63)
8	42	43 (42)	85 (84)	43	128 (127)
9	86	85 (84)	171 (170)	85	256 (255)
10	170	171 (170)	341 (340)	171	512 (511)
11	342	341 (340)	683 (682)	341	1024 (1023)
12	682	683 (682)	1365 (1364)	683	2048 (2047)
13	1366	1365 (1364)	2731 (2730)	1365	4096 (4095)
14	2730	2731 (2730)	5461 (5460)	2731	8192 (8191)
15	5462	5461 (5460)	10923 (10922)	5461	16384 (16383)
16	10922	10923 (10922)	21845 (21844)	10923	32768 (32767)
17	21846	21845 (21844)	43691 (43690)	21845	65536 (65535)
18	43690	43691 (43690)	87381 (87380)	43691	131072 (131071)
19	87382	87381 (87380)	174763 (174762)	87381	262144 (262143)
20	174762	174763 (174762)	349525 (349524)	174763	524288 (524287)

KNOTS UP TO AND INCLUSIVE LEVEL					
LEVEL	3-PARTS	1-PART ONLY	1-PART TOTAL	2-PARTS	TOTAL
1	0	1 (0)	1 (0)	0	1 (0)
2	0	2 (0)	2 (0)	1	3 (1)
3	2	3 (0)	5 (2)	2	7 (4)
4	4	6 (2)	10 (6)	5	15 (11)
5	10	11 (6)	21 (16)	10	31 (26)
6	20	22 (16)	42 (36)	21	63 (57)
7	42	43 (36)	85 (78)	42	127 (120)
8	84	86 (78)	170 (162)	85	255 (247)
9	170	171 (162)	341 (332)	170	511 (502)
10	340	342 (332)	682 (672)	341	1023 (1013)
11	682	683 (672)	1365 (1354)	682	2047 (2036)
12	1364	1366 (1354)	2730 (2718)	1365	4095 (4083)
13	2730	2731 (2718)	5461 (5448)	2730	8191 (8178)
14	5460	5462 (5448)	10922 (10908)	5461	16383 (16369)
15	10922	10923 (10908)	21845 (21830)	10922	32767 (32752)
16	21844	21846 (21830)	43690 (43674)	21845	65535 (65519)
17	43690	43691 (43674)	87381 (87364)	43690	131071 (131054)
18	87380	87382 (87364)	174762 (174744)	87381	262143 (262125)
19	174762	174763 (174744)	349525 (349506)	174762	524287 (524268)
20	349524	349526 (349506)	699050 (699030)	349525	1048576 (1048555)

Fig. 164 — Raising Casa-coded Regular Knots in relation to levels in the left half-tree.

For the right half-tree, Fig. 165 shows the involvement of 1-bight and 2-bights Casa-coded Regular Knots in the raising of bigger Casa-coded Regular Knots.

KNOTS PER LEVEL			
LEVEL	1-BIGHT	2-BIGHTS	TOTAL
1	1	0	1
2	1	1	2
3	3	1	4
4	5	3	8
5	11	5	16
6	21	11	32
7	43	21	64
8	85	43	128
9	171	85	256
10	341	171	512
11	683	341	1024
12	1365	683	2048
13	2731	1365	4096
14	5461	2731	8192
15	10923	5461	16384
16	21845	10923	32768
17	43691	21845	65536
18	87381	43691	131072
19	174763	87381	262144
20	349525	174763	524288

KNOTS UP TO AND INCLUSIVE LEVEL			
LEVEL	1-BIGHT	2-BIGHTS	TOTAL
1	1	0	1
2	2	1	3
3	5	2	7
4	10	5	15
5	21	10	31
6	42	21	63
7	85	42	127
8	170	85	255
9	341	170	511
10	682	341	1023
11	1365	682	2047
12	2730	1365	4095
13	5461	2730	8191
14	10922	5461	16383
15	21845	10922	32767
16	43690	21845	65535
17	87381	43690	131071
18	174762	87381	262143
19	349525	174762	524287
20	699050	349525	1048575

Fig. 165 — Raising Casa-coded Regular Knots in relation to levels in the right half-tree.

On pp. 83–84 we have seen that the **odd** and **even parities** of the number of unit steps in the final straight path-elements of Casa-coded Regular Knot enlagement-paths and the *seemingly* unrelated **males** and **females** of the honeybee have something in common with respect to their genealogy. There is here one striking difference though: for the honeybee the genealogical aspects are descending (going to the past), while for the Casa-coded Regular Knot they are ascending (going to the future). This commonly encountered difference in the sphere of "genealogy" between the living world around us and knots, is in fact a manifestation of the universal **Complemental Law**. At some future time we shall show some most fascinating braiding examples of this law.

Braiding Instructions
and their
Presentation

In *The Braider* issue No. 8 we discussed for Column-coded Semi-Regular Knots the construction procedures of their algorithm diagrams in general terms. Hence let's now have a look at some specific Examples.

Example 1:

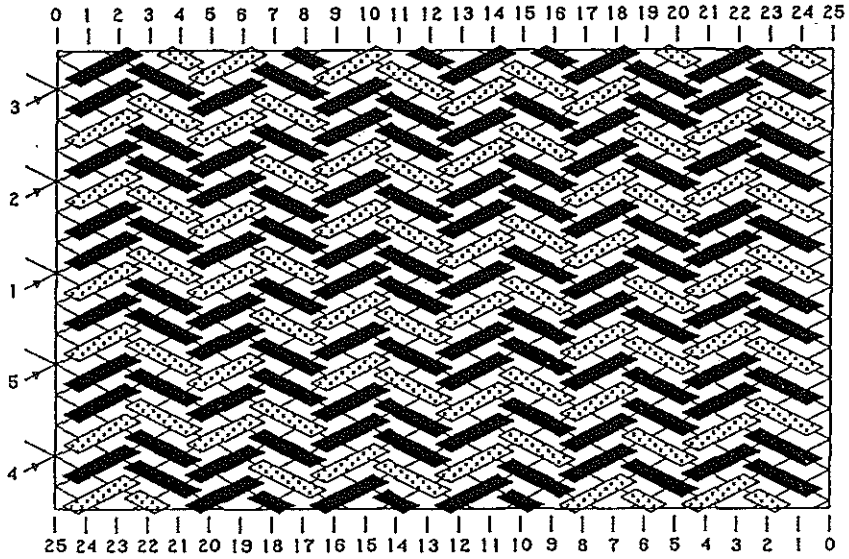


Fig. 166 — 25-parts/15-bights 2-pass Semi-Regular Gaucho Knot; string sequence 1.

0 *C A D B* 1 *C A D B* 2 *C A D B* 0 *C A D B* 1 *C A D B*
· / / \ \ / / \ \ / / \ \ / / \ \ / / \ \ / / \ \ ·
B D A C 1 *B D A C* 0 *B D A C* 2 *B D A C* 1 *B D A C* 0

Example 2:

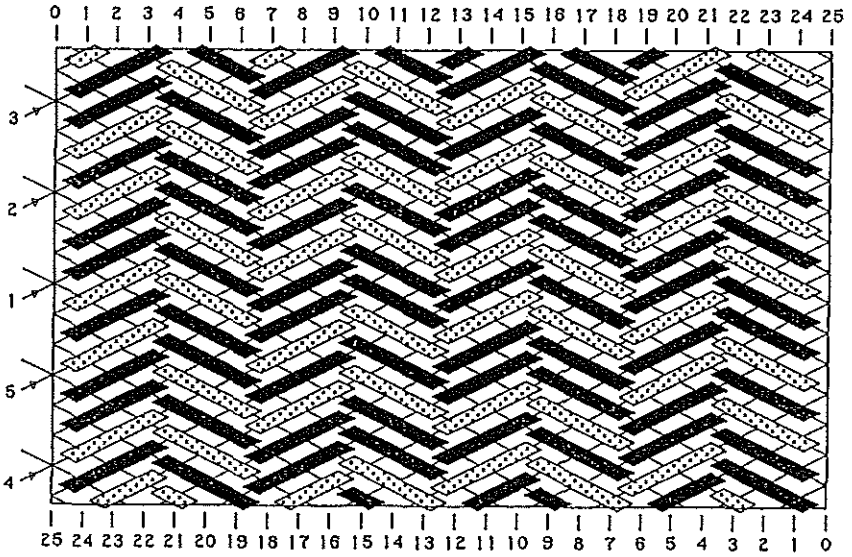


Fig. 167 — 25-parts/15-bights 3-pass Semi-Regular Gaucho Knot; string sequence 1.

0 *C A D B* 1 *C A D B* 2 *C A D B* 0 *C A D B* 1 *C A D B*
· / / / \ \ \ / / / \ \ \ / / / \ \ \ / / / \ \ \ ·
B D A C 1 *B D A C* 0 *B D A C* 2 *B D A C* 1 *B D A C* 0

Example 3:

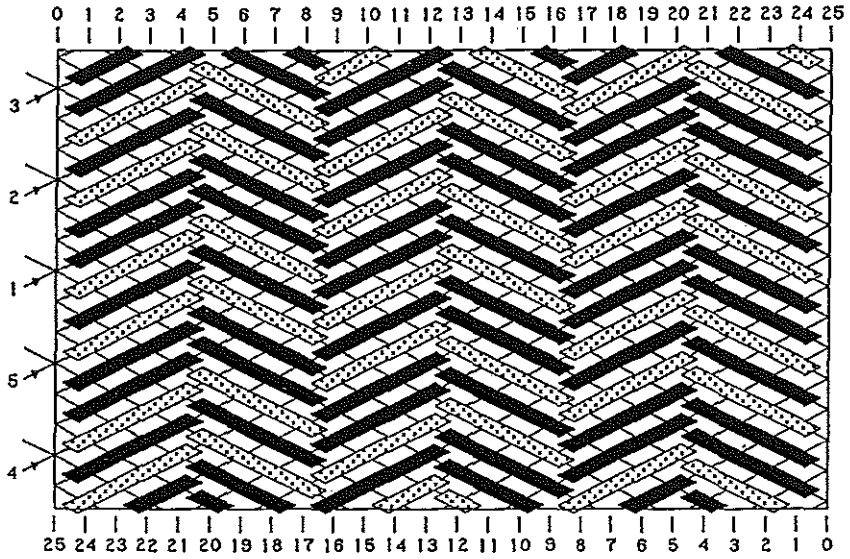


Fig. 168 — 25-parts/15-bights 4-pass Semi-Regular Gaucho Knot; string sequence 1.

```
0 C A D B 1 C A D B 2 C A D B 0 C A D B 1 C A D B
· / / / / \ \ \ \ / / / / \ \ \ \ / / / / \ \ \ \ ·
  B D A C 1 B D A C 0 B D A C 2 B D A C 1 B D A C 0
```

Example 4:

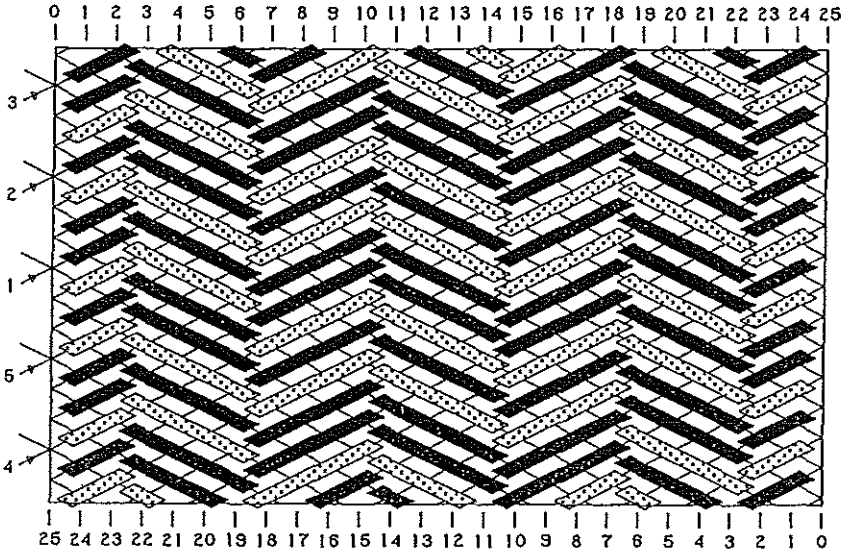


Fig. 169 — 25-parts/15-bights Semi-Regular Knot; string sequence 1.

```
0 C A D B 1 C A D B 2 C A D B 0 C A D B 1 C A D B
· / / \ \ \ \ / / / / \ \ \ \ / / / / \ \ \ \ / / ·
  B D A C 1 B D A C 0 B D A C 2 B D A C 1 B D A C 0
```

The Semi-Regular Knot with p-parts and b-bights requires n strings in its construction when the g.c.d. (greatest common divisor) of p and b is equal to n. Each of the n strings will go through $b \div n = s$ regularly spaced bight-points on each of the two bight-boundaries. Hence those bight-points of a string are spaced $b \div s = n$ bights apart along a bight-boundary.

Hence only when n and s are coprime are we able to distribute the Standing Ends of the n strings regularly over a bight-boundary.

With a regular distribution of the Standing Ends over a bight-boundary, the Standing Ends are spaced $b \div n = s$ bights apart. Note that regular string-sequence periodicity is **independent** of the distribution of the Standing Ends along a bight-boundary, and hence only with a regular periodic string-sequence will a single algorithm diagram suffice.

Example 5:

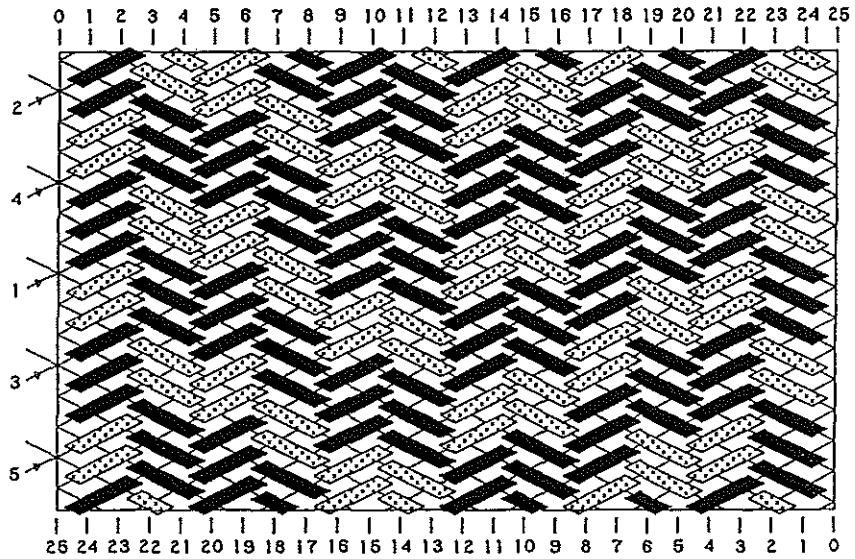


Fig. 170 — 25-parts/15-bights 2-pass Semi-Regular Gaucho Knot; string sequence 2.

```
0 D C B A 1 D C B A 2 D C B A 0 D C B A 1 D C B A
· / / \ \ / / \ \ / / \ \ / / \ \ / / \ \ / / \ \ ·
  A B C D 1 A B C D 0 A B C D 2 A B C D 1 A B C D 0
```

Example 6:

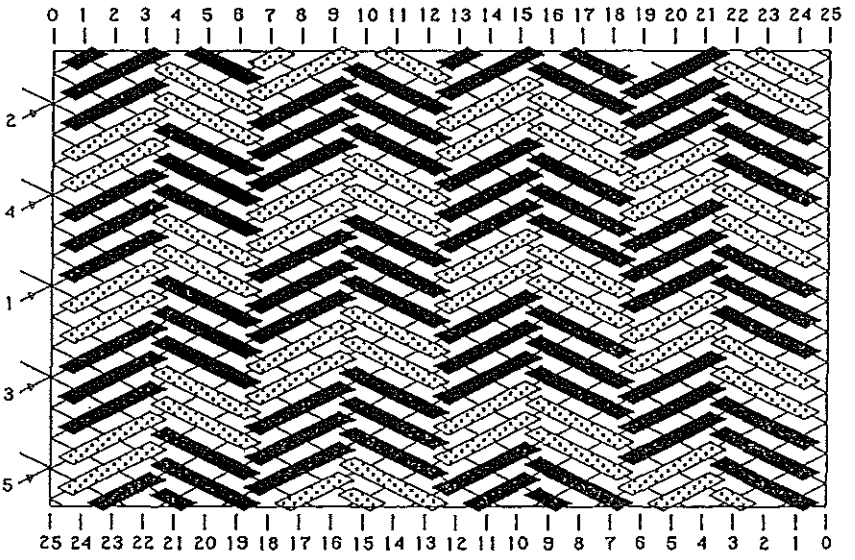


Fig. 171 — 25-parts/15-bights 3-pass Semi-Regular Gaucho Knot; string sequence 2.

```
0 D C B A 1 D C B A 2 D C B A 0 D C B A 1 D C B A
· / / / \ \ \ / / / \ \ \ / / / \ \ \ / / / \ \ \ ·
  A B C D 1 A B C D 0 A B C D 2 A B C D 1 A B C D 0
```

Example 7:

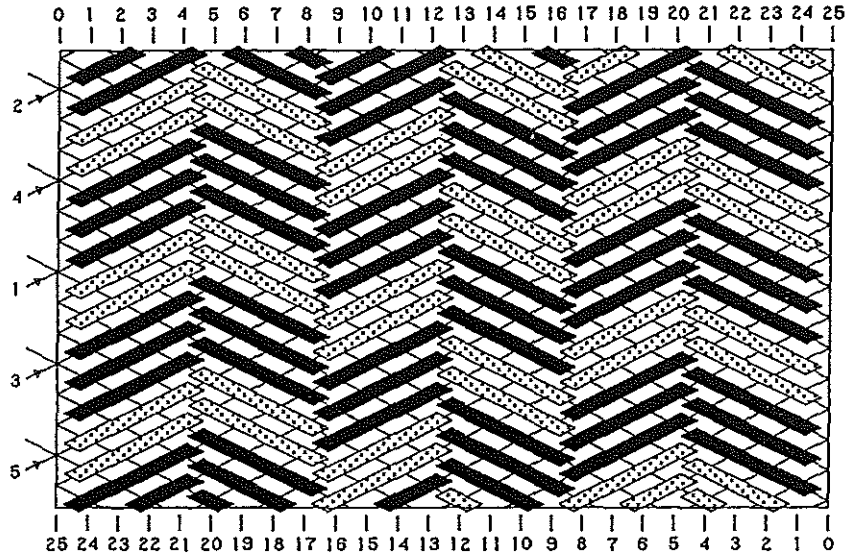


Fig. 172 — 25-parts/15-bights 4-pass Semi-Regular Gaucho Knot; string sequence 2.

```
0 D C B A 1 D C B A 2 D C B A 0 D C B A 1 D C B A
· / / / / \ \ \ \ / / / / \ \ \ \ / / / / \ \ \ \ ·
  A B C D 1 A B C D 0 A B C D 2 A B C D 1 A B C D 0
```

Example 8:

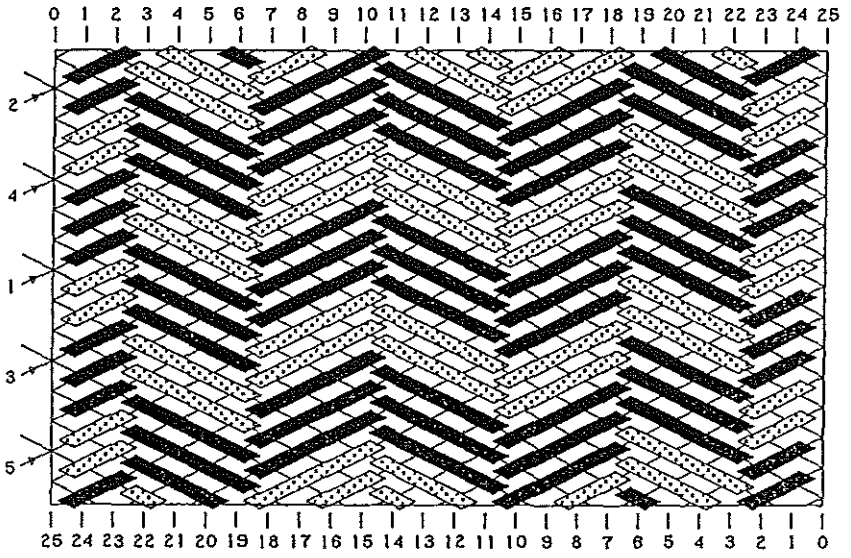


Fig. 173 — 25-parts/15-bights Semi-Regular Knot; string sequence 2.

```
0 D C B A 1 D C B A 2 D C B A 0 D C B A 1 D C B A
· / / \ \ \ \ / / / / \ \ \ \ / / / / \ \ \ \ / / ·
  A B C D 1 A B C D 0 A B C D 2 A B C D 1 A B C D 0
```

Generally it is preferable to braid *all* the strings of the same colour successively before starting with a string of another colour, provided that the coding of the knot is such that this does not make the braiding process more difficult. Consequently the regular periodic string-sequences for the knots in Examples 5, 6, 7, 8 and for the knots in Examples 1, 2, 3, 4 are different.

Although the colour-patterns of the knots in Examples 9, 10, 11, 12 differ from the respective colour-patterns of the knots in Examples 5, 6, 7, 8, we can however use the same regular periodic string-sequence, and hence since the respective codings are the same, so are the respective algorithm diagrams the same. The difference between the Examples 9, 10, 11, 12 and 5, 6, 7, 8 is obviously the colour of string #4 only.

Example 9 :

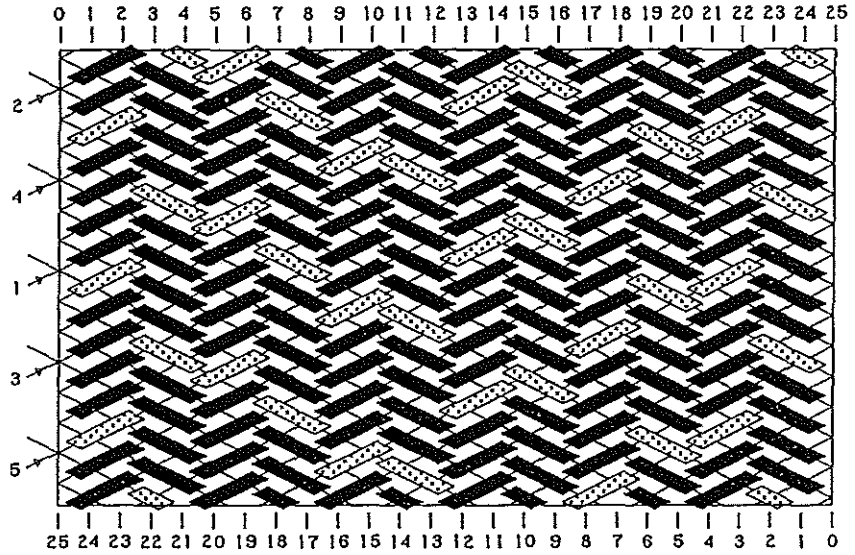


Fig. 174 — 25-parts/15-bights 2-pass Semi-Regular Gaucho Knot; string sequence 2.

0 *D C B A* 1 *D C B A* 2 *D C B A* 0 *D C B A* 1 *D C B A*
· / / \ \ / / \ \ / / \ \ / / \ \ / / \ \ / / \ \ ·
A B C D 1 *A B C D* 0 *A B C D* 2 *A B C D* 1 *A B C D* 0

Example 10 :

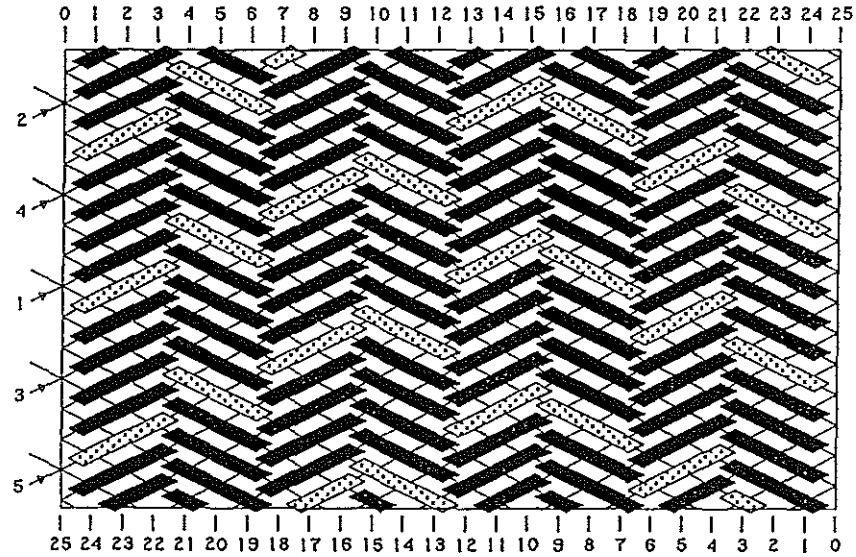


Fig. 175 — 25-parts/15-bights 3-pass Semi-Regular Gaucho Knot; string sequence 2.

0 *D C B A* 1 *D C B A* 2 *D C B A* 0 *D C B A* 1 *D C B A*
· / / / \ \ \ / / / \ \ \ / / / \ \ \ / / / \ \ \ ·
A B C D 1 *A B C D* 0 *A B C D* 2 *A B C D* 1 *A B C D* 0

Example 11:

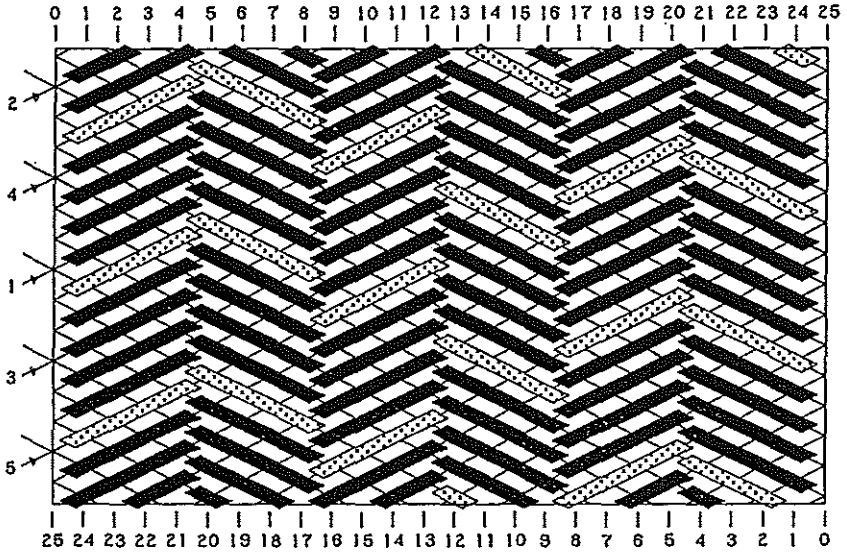


Fig. 176 — 25-parts/15-bights 4-pass Semi-Regular Gaucho Knot; string sequence 2.

```
0 D C B A 1 D C B A 2 D C B A 0 D C B A 1 D C B A
· / / / / \ \ \ \ / / / / \ \ \ \ / / / / \ \ \ \ ·
  A B C D 1 A B C D 0 A B C D 2 A B C D 1 A B C D 0
```

Example 12:

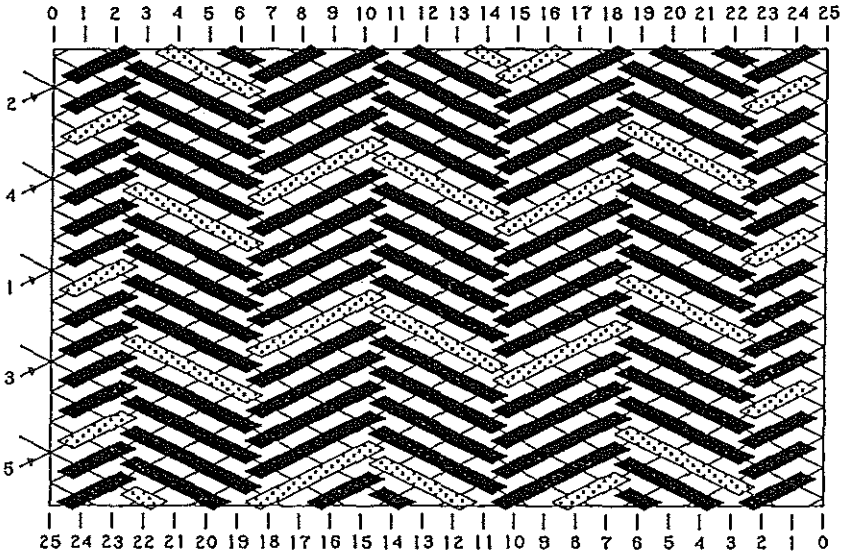


Fig. 177 — 25-parts/15-bights Semi-Regular Knot; string sequence 2.

```
0 D C B A 1 D C B A 2 D C B A 0 D C B A 1 D C B A
· / / \ \ \ \ / / / / \ \ \ \ / / / / \ \ \ \ / / ·
  A B C D 1 A B C D 0 A B C D 2 A B C D 1 A B C D 0
```

The Semi-Regular Knot in the following Example has 20-parts and 16-bights, hence $n = 4$ and $s = 4$. Consequently n and s are not coprime, and hence the Standing Ends of the four strings cannot be regularly distributed over a bight-boundary.

Let the distance in bights between the Standing Ends of the strings #1 and #2 be equal to t, then:

The necessary condition for a possible regular periodic string-sequence is that t and n are coprime.

Say we want to braid the two "black" strings first, then the periodicity of any string-sequence cannot be regular since $t = 2 + 4N$ (where $N = 0, 1, 2, 3.$) and $n = 4$; hence t and n are not coprime.

Example 13:

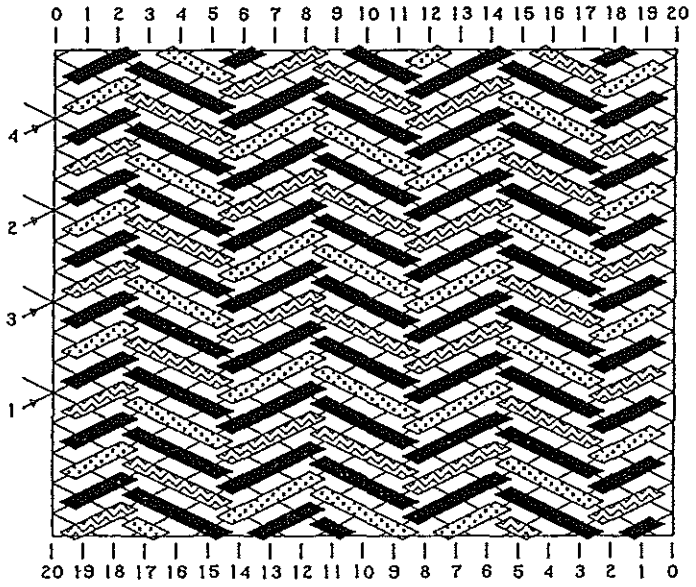


Fig. 178 — 20-parts/16-bights Semi-Regular Knot of Example 13.

Algorithm diagram for strings #1 and #2:

```
0     A     3     A     2     A     1     A     0     A
·  /  /  \  \  \  /  /  /  \  \  \  /  /  /  \  \  \  /  /  ·
      A     0     A     1     A     2     A     3     A     0
```

Algorithm diagram for strings #3 and #4:

```
0  B  C  A  3  B  C  A  2  B  C  A  1  B  C  A  0  B  C  A
·  /  /  \  \  \  /  /  /  \  \  \  /  /  /  \  \  \  /  /  ·
   A  C  B  0  A  C  B  1  A  C  B  2  A  C  B  3  A  C  B  0
```

This type of knot is very suitable as a long knot ($p = 12a + 8$ (where a is a natural number), and b such that the g.c.d. of p and b is equal to 4 (or 2 for two colours only)). It gives, with a sufficiently large value for a, a nice spiralling pattern.

⋆⋆ *For high class braidwork, what should be carefully watched? For which values of a makes each '3-overs spiral', as near as possisible, full revolutions? Give a general relationship for p and b.*

The following final Example shows a Semi-Regular Knot, with 21-parts and 15-bights, which is neither Column-coded nor Row-coded. The coding used here makes this knot very suitable for round string. Since $n = 3$ is coprime with $s = 5$, the Standing Ends of the three strings required can be distributed regularly over a bight-boundary. Since the coding-sequences along all similar half-cycles, associated with the same string, are identical, only $n = 3$ algorithm diagrams are required for this knot. It should, however, be noted that each algorithm diagram is specific for its associated half-cycle string-run directions.

Example 14:

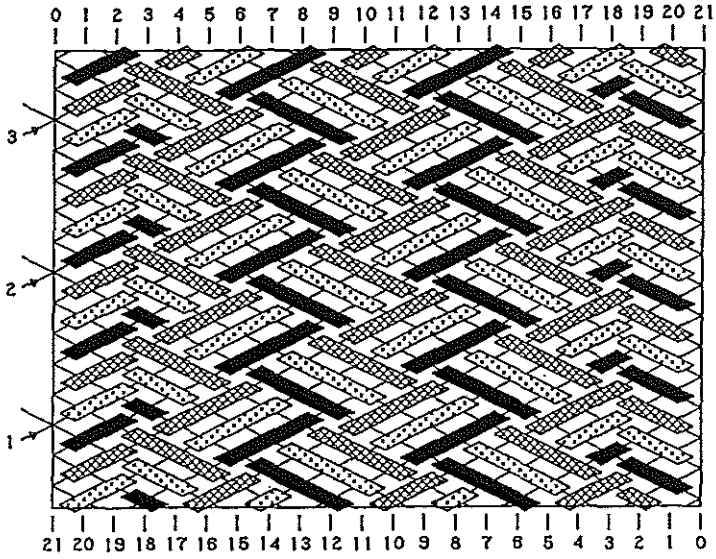


Fig. 179 — 21-parts/15-bights Semi-Regular Knot of Example 14.

Algorithm diagram for string #1:

```
0       2       4       1       3       0       2
·  /  /  \  \  /  /  /  \  \  \  /  /  /  \  \  \  /  /  \  \  ·
         2       0       3       1       4       2       0
```

Algorithm diagram for string #2:

```
0  A     2  A     4  A     1  A     3  A     0  A     2  A
·  /  /  \  \  \  /  /  /  \  \  \  /  /  /  \  \  \  /  \  \  ·
---------------------------------------------------------------
·  /  /  \  /  /  /  \  \  \  /  /  /  \  \  \  /  /  /  \  \  ·
      A  2     A  0     A  3     A  1     A  4     A  2     A  0
```

Algorithm diagram for string #3:

```
0  A  B  2  A  B  4  A  B  1  A  B  3  A  B  0  A  B  2  A  B
·  /  /  \  /  /  /  \  \  \  /  /  /  \  \  \  /  /  /  \  \  ·
---------------------------------------------------------------
·  /  /  \  \  \  /  /  /  \  \  \  /  /  /  \  \  \  /  \  \  ·
   B  A  2  B  A  0  B  A  3  B  A  1  B  A  4  B  A  2  B  A  0
```

In Example 14 the normal conventions for a half-cycle string-run have been used, namely: a half-cycle from left to right runs from lower-left to upper-right, and a half-cycle from right to left runs from lower-right to upper-left (See *The Braider* issue No. 5, pg. 94). For string #1 these two types of half-cycles follow the same coding-sequence, respectively from left to right and from right to left. Consequently, in the same way as in the previous Examples, one coding-sequence is used in the algorithm diagram for both types of half-cycles. For string #2 and string #3, however, the respective coding-sequences differ. Consequently in their algorithm diagrams the two types of half-cycles with their coding-sequences are separated by a horizontal line.

Scenes from the Knot Universe

Fig. 180 depicts the string-run of a **single string Rectangular Mat.** We shall call a single string rectangular mat a **Regular Rectangular Mat.** The **Semi-Regular Rectangular Mat** requires more than one string in its construction.

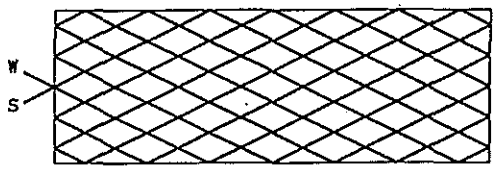


Fig. 180 — The string-run of a Regular Rectangular Mat with $b_h = 7$, $b_v = 5$.

Concerning rectangular mats, it appears that there is in the literature nothing available which is of real informative value. Furthermore, it also appears that an important application of rectangular mats has been overlooked. *The Ashley book of knots* devotes less than two full pages on them (#2261 – #2268), and they haven't even specifically been mentioned in the index. In some other publications one might be able to find short tables which give b_h ; b_v combinations which will yield Regular Rectangular Mats (single string rectangular mats), but nothing in general concerning this appears to be available. As shown in *The Ashley book of knots*, Regular Rectangular Mats can be enlarged to bigger Regular Rectangular Mats in a variety of ways, but again nothing of significance has been stated. In fact, Ashley states in #2265: *"The method of raising these knots appears to have endless ramifications and I have made no effort to exhaust the possibilities"*. Raising Regular Rectangular Mats may, to many braiders, *seem* to have endless ramifications (see Fig. 181), but this is in actual fact not the case at all.

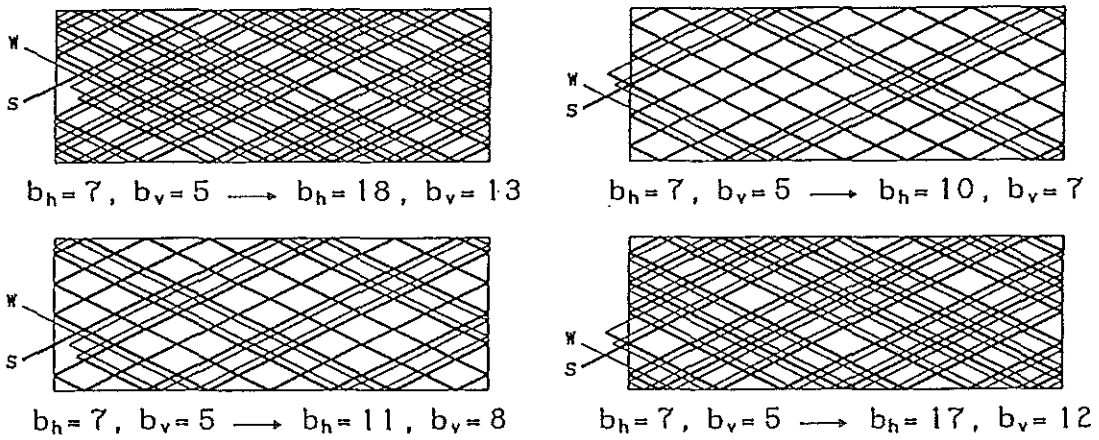


Fig. 181 — Examples of raising Regular Rectangular Mats.

The entries #2261 to #2268 in Ashley's book do suggest that Ashley devoted some time on this raising aspect, but was not successful in finding the consistent general method for raising Regular Rectangular Mats. He obviously did not realise that one of his discussed items presented the solution. It must, however, be kept in mind that Ashley did not have at his disposal the tools which should now hopefully be familiar to the readers of *The Braider*.† In order to provide the reader with a test whether or not the footnote is applicable, we like to pose the following questions:

† If not, then some serious study of the previous issues is urgently called for!!!

⋆⋆ Find the relationship between b_h and b_v for Regular Rectangular Mats (single string rectangular mats). [Hint: Use string-run diagrams, and discover how Regular Rectangular Mats may be seen. Then the real nature of Regular Rectangular Mats should become evident. The way in which Regular Rectangular Mats may be seen plays a vital role in developing an easy systematic braiding procedure for them.]

Find with the aid of string-run diagrams the consistent general raising methods for Regular Rectangular Mats. What can you tell about the other raising methods?

There is little doubt that if Ashley would have been in the possession of the information contained in the previous issues of *The Braider*, he would have solved the apparent raising mystery surrounding the Regular Rectangular Mats.

There are no doubt some braiders who may think that there are virtually no interesting aspects of these mats, and that their field of application is solely as *mats*. Often, those braiders can only imagine these mats as being Casa-coded, normally in multi-ply form. However, Rectangular Mats find an important application in appliqué work, and especially in this field various coding forms may be used to great effect. Before we can discuss this particular field of application, we first have to develop an easy sytematic method for braiding the Rectangular Mats. But first we will in the next issue have a look at the answers to the questions posed above.

Reviews

Braiding Rawhide Horse Tack ISBN 0-87033-333-X 124 pages 227x151mm., soft cover, glued only spine.

Author: Robert L. Woolery.

Publisher: Cornell Maritime Press, Inc., Centreville, Maryland 21617, U.S.A.

From the onset it should be stated that the author sees himself as a **reporter** (Introduction, pg. 3). Apart from the actual braiding processes, the book has some very good sections.

Although it has been stated in the *"Introduction"* that one of the goals of the book is to provide a step by step presentation of braiding techniques for the novice braider (even on the back-cover we find the statement: *"Here is an instruction manual for the novice worker in rawhide."*), a novice braider would, however, be ill-adviced to acquire a copy of this book for that purpose.

The braiding procedures discussed are typically those which are being used by braiders with a very limited ability as far as braiding processes are concerned. This is not surprising of course since their braiding-skills, passed on from one generation to the next, were originally obtained by purely experimental means only. Although their braid-work was of a very restricted nature, it should be clearly understood that this does not mean that it was therefore of an inferior quality. Like a partially sighted man: he can walk the streets, but his movements are very restricted. It is therefore very important for those whose desire it is to become a braiding artisan, to acquire a proper understanding of general braiding techniques in order to establish a proper foundation on which to build their braiding skills.

The first two chapters, which deal with the preparation of rawhide and the fabrication of rawhide-strings are exellent (17 pages). The only other Chapters which contain some information of value are the Chapters 10 to 14 (34 pages).

Three books are referenced as *recommended* reading. It depends for what reason(s) recommendations are made, but if the intention was with the novice in mind then only the *Encyclopedia of Rawhide and Leather Braiding* by Bruce Grant should have been mentioned. The other two books recommended: *Encyclopedia of Knots and Fancy Rope Work* by Raoul Graumont and John Hensel, and *Whips and Whipmaking: With a Practical Introduction to Braiding* by David Morgan, have nothing of value to offer.

Although opinions differ, a long seviceable life expectancy for a book with a glued only spine is very doubtful, hence to call it a *manual* is rather far-fetched.

Rope & Twine Crafts ISBN 0-85091-555-4 63 pages + 8 pages of colour photographs 274x210mm., soft cover, glued only spine.

Author: Grahame (Gypsy) Hussey-Smith.

Publisher: Thomas C. Lothian Pty Ltd., 11 Munro Street, Port Melbourne, Victoria 3207, Australia.

The first 28 pages are devoted to basic techniques and knots. The descriptions are clear and the illustations are very good. Covered are: common whipping, seaman's whipping, back splice, eye splice, cut splice, short splice, Solomon's bar, tiddling, Lark's Head coxcombing, half-hitch spiralling, overhand knot, reef knot, bowline, single sheet bend, figure eight, Eskimo bowstring, monkey chain, half hitch, clove hitch, Lark's Head, constrictor knot, crown knot, wall knot, Matthew Walker knot, diamond knot.

These basic knots and techniques are then used in the following projects: Turk's Head mat, pot stand, round mat, round basket, paunch mat, rectangular mat, monkey's fist, quoit, rope games, rope ladder, rope dog, elephant, crocodile, monkey, bell rope, dog leash, garden swing, rope hammock, cord belt. Each of these projects contains a clear set of instructions which are accompanied by several very good illustrations.

This book is especially well suited for children and the novice knotter. For single-string rectangular mats a table is provided for various b_h ; b_v combinations (the table entry in the seventh row from top, second column from the left should be deleted however). The method by which the rectangular mats are constructed is rather cumbersome, and although quite suitable for children, should not be used by those whose desire it is to become braiding artisans.

The Essential Knot Book ISBN 0-87742-221-4 85 pages 215x135mm., soft cover, glued only spine.

Author: Colin Jarman; Drawings by: Bill Beavis.

Publisher: International Marine Publishing Company, Route 1, P.O. Box 220,Camden, Maine 04843, U.S.A.

This booklet covers the construction and use of 27 knots, 4 whippings and 12 splices. The text is illustrated not only with beautifully detailed drawings but also with photographs which clearly illustrate the use of the various knots and splices. Especially the photographs of actual applications is a special very valuable feature of this little book.

Although it has been specifically written for the yachtsman in order to provide him with the practicalities of the knots which are essential in his endeavours, any braider or knotter will love this publication.

THE BRAIDER

No.10 **MAY 1997.**

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A quarterly publication for the braiding artisan

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New Zealand.

Solutions to the Questions in Issue No. 9

Question on pg. 194.

It is of the utmost importance that high class braidwork complies exactly with the intrinsic geometry of such braidwork. Hence in cylindrical braids, the bight-points at the left-hand and right-hand edges must be positioned in exact accordance with the braid-pattern. Consequently the geometry of a braid-pattern should be analysed before braiding commences.

The geometry of the braid-pattern of the Semi-Regular Knot in Fig. 178 (pg. 194) is as depicted in Fig. 182, where the vertial distances are in *bight*-units and the horizontal distance is in *part*-units.

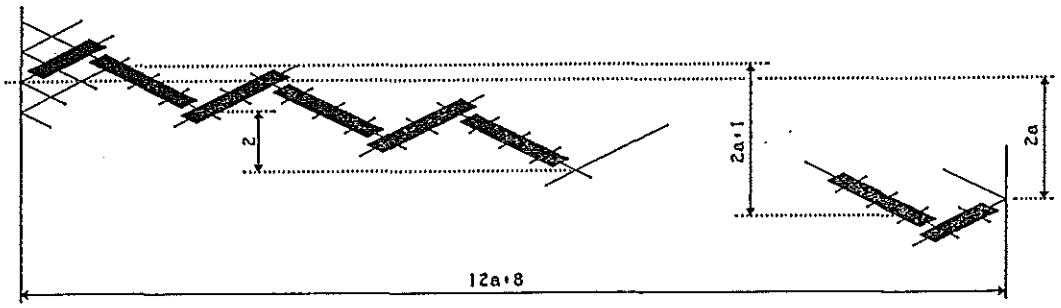


Fig. 182 — Braid-pattern dimensions of the Semi-Regular Knot in Fig. 178.

For high class braidwork the colour of the pattern demands that the circumferential distance between the left-hand bight-point and the right-hand bight-point of a colour spiral is equal to $|2a|_b$, where, with a a natural number, b is the number of bights and $p = (12a + 8)$ is the number of parts of the knot.

It will readily be seen from the dimension $(2a + 1)$ that as near as possible full revolutions of each '3-overs spiral' are obtained when either $2a$ or $(2a + 2)$ is divisible by b; the number of near full revolutions is then respectively $\frac{2a}{b}$ or $\frac{2a+2}{b}$.

Let the number of near full revolutions be r, and let $b = 4z$. Then for $2a = rb$ we obtain $a = 2rz$ and $p = 6rb + 8 = 4(6rz + 2)$. Hence for g.c.d.$(p, b) = 4$, it follows that z and $(6rz + 2)$ must be coprime. Thus z must be **odd.**

For $(2a + 2) = rb$ we obtain $a = (2rz - 1)$ and $p = 6rb - 4 = 4(6rz - 1)$. Hence for g.c.d.$(p, b) = 4$, it follows that z and $(6rz - 1)$ must be coprime. Thus z can be any natural number (positive integer).

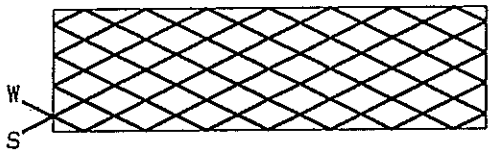
Question on pg. 197.

It has been mentioned before that string-run diagrams form the most important tool in braiding.† A string-run diagram of a braid-form will be able to tell us all the *basic* properties of that braid-form.

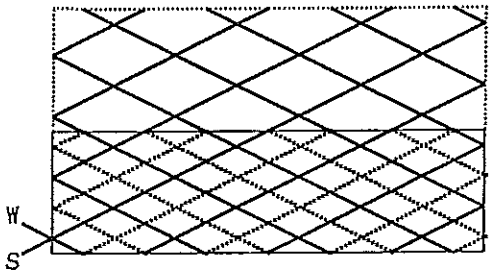
A string-run diagram of a Regular Rectangular Mat may be seen to represent a string-run diagram of a flattened Regular Knot.

† The coding of a braid is only of secondary importance. As far as a braid is concerned, mathematics is of no technical importance at all: mathematical expressions merely serve as a means by which relationships can be communicated in an easy unambiguous manner. Mathematics is therefore of no greater importance than the photocopier on which this page was photocopied.

This obviously leads to the conversion process depicted in Fig. 183 which clearly shows the correspondence between a Regular Rectangular Mat with b_h and b_v and a Regular Knot with $p = b_h$ and $b = b_v$. Hence for Regular Rectangular Mats (1-string Rectangular Mats) b_h and b_v must be coprime. Consequently the properties of the RKT apply to the Regular Rectangular Mats, whereby $p = b_h$ and $b = b_v$. Hence enlargements of Regular Rectangular Mats follow in the RKT paths which are calculated and depicted in a similar way as the enlargement-paths for Regular Knots.



AN 1-STRING RECTANGULAR MAT
(A REGULAR RECTANGULAR MAT)
WITH b_h BIGHTS HORIZONTAL AND
b_v BIGHTS VERTICAL.
CONVERT MAT INTO A CYLINDRICAL
KNOT ——————————→



THIS CYLINDRICAL KNOT IS AN 1-STRING REGULAR KNOT. THE NUMBER OF PARTS OF THIS KNOT IS EQUAL TO b_h AND THE NUMBER OF BIGHTS IS EQUAL TO b_v. HENCE b_v AND b_h MUST BE COPRIME FOR AN 1-STRING RECTANGULAR MAT.

BY REVERSING THE CONVERSION PROCESS WE CAN CONVERT ANY REGULAR KNOT INTO AN 1-STRING RECTANGULAR MAT. IT WILL BE SELF EVIDENT THAT THE REGULAR KNOT TREE AND ITS PROPERTIES ALSO APPLY TO THE REGULAR RECTANGULAR MATS.

Fig. 183 — The Regular Rectangular Mat compared with the Regular Knot.

The Regular Knot is one of the most simple braid-forms, and although the Regular Rectangular Mat is closely related to it, it is however quite a bit more complicated. This becomes immediately apparent from the enlargement processes as depicted in Fig. 181 (issue No. 9, pg. 196). These enlargement processes are compared in Fig. 184 with the corresponding enlargements of the related Regular Knot. The thick solid lines in these diagrams depict the string-run of the Regular Rectangular Mat with $b_h = 7$ and $b_v = 5$. The thick dotted lines and their thick solid line continuations to the lowermost 'boundary' depict the string-run of the Regular Knot with $p = 7$ and $b = 5$. The thin dotted lines in the mat-section of the diagrams $b_h = 7$, $b_v = 5 \rightarrow b_h = 18$, $b_v = 13$ and $b_h = 7$, $b_v = 5 \rightarrow b_h = 17$, $b_v = 12$ depict the 1st-order enlargements while the thin dotted and thin solid lines together in these two mat-sections of the diagrams depict the 2nd-order enlargements. The thin solid lines in the diagrams $b_h = 7$, $b_v = 5 \rightarrow b_h = 10$, $b_v = 7$ and $b_h = 7$, $b_v = 5 \rightarrow b_h = 11$, $b_v = 8$ depict the 1st-order enlargements.

These enlargement examples show clearly that the increase in b_h and b_v depends on the position of the bight-point where the Standing and Working Ends are placed. For the Regular Rectangular Mat with $b_h = 7$, $b_v = 5$ these examples show that:

A 1st-order Method I enlargement is obtained when the Standing and Working Ends are placed in the 2nd bight-point, left-hand side, from the lower horizontal bight-boundary. A 1st-order Method II enlargement is obtained when the Standing and Working Ends are placed in the 3rd bight-point, left-hand side, from the lower horizon-

tal bight-boundary. A 2nd-order Method I enlargement is obtained when the Standing and Working Ends are placed in the 3rd bight-point, left-hand side, from the lower horizontal bight-boundary. A 2nd-order Method II enlargement is obtained when the Standing and Working Ends are placed in the 2nd bight-point, left-hand side, from the lower horizontal bight-boundary.

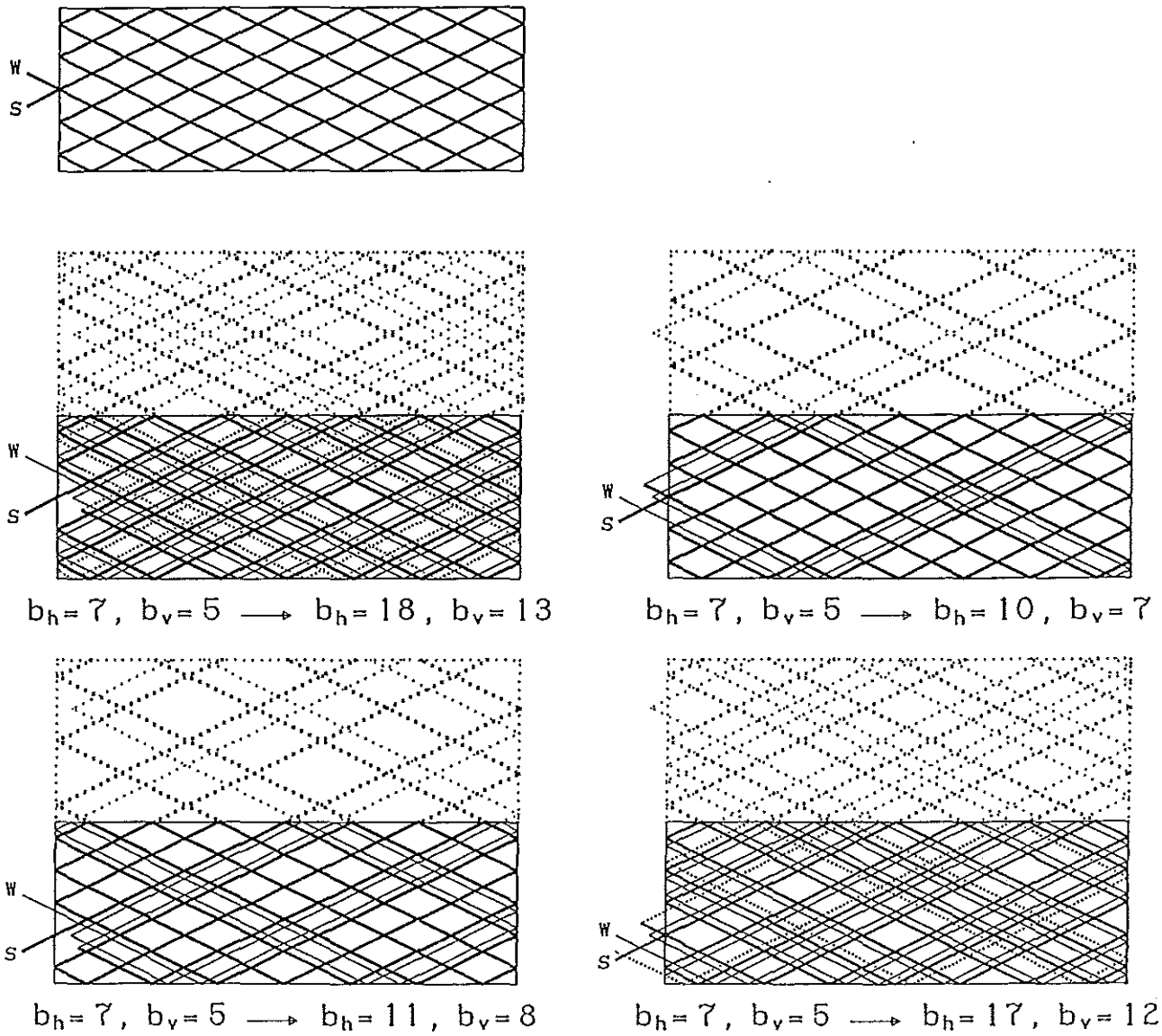

W
S
W
S
$b_h = 7,\ b_v = 5 \longrightarrow b_h = 18,\ b_v = 13$
W
S
$b_h = 7,\ b_v = 5 \longrightarrow b_h = 10,\ b_v = 7$
W
S
$b_h = 7,\ b_v = 5 \longrightarrow b_h = 11,\ b_v = 8$
W
S
$b_h = 7,\ b_v = 5 \longrightarrow b_h = 17,\ b_v = 12$

Fig. 184 — Enlargement examples of the Regular Rectangular Mat with $b_h/b_v = 7/5$.

The string-runs associated with a 1st-order Method I enlargement when the Standing and Working Ends are placed in the 2nd bight-point, left-hand side, from the lower horizontal bight-boundary, and respectively with a 1st-order Method II enlargement when the Standing and Working Ends are placed in the 3rd bight-point, left-hand side, from the lower horizontal bight-boundary are depicted in Fig. 185 in greater detail. Similarly in Fig. 186 are depicted the string-runs associated with a 1st-order Method I enlargement when the Standing and Working Ends are placed in the 3rd bight-point, left-hand side, from the lower horizontal bight-boundary, and respectively with a 1st-order Method II enlargement when the Standing and Working Ends are placed in the 2nd bight-point, left-hand side, from the lower horizontal bight-boundary.

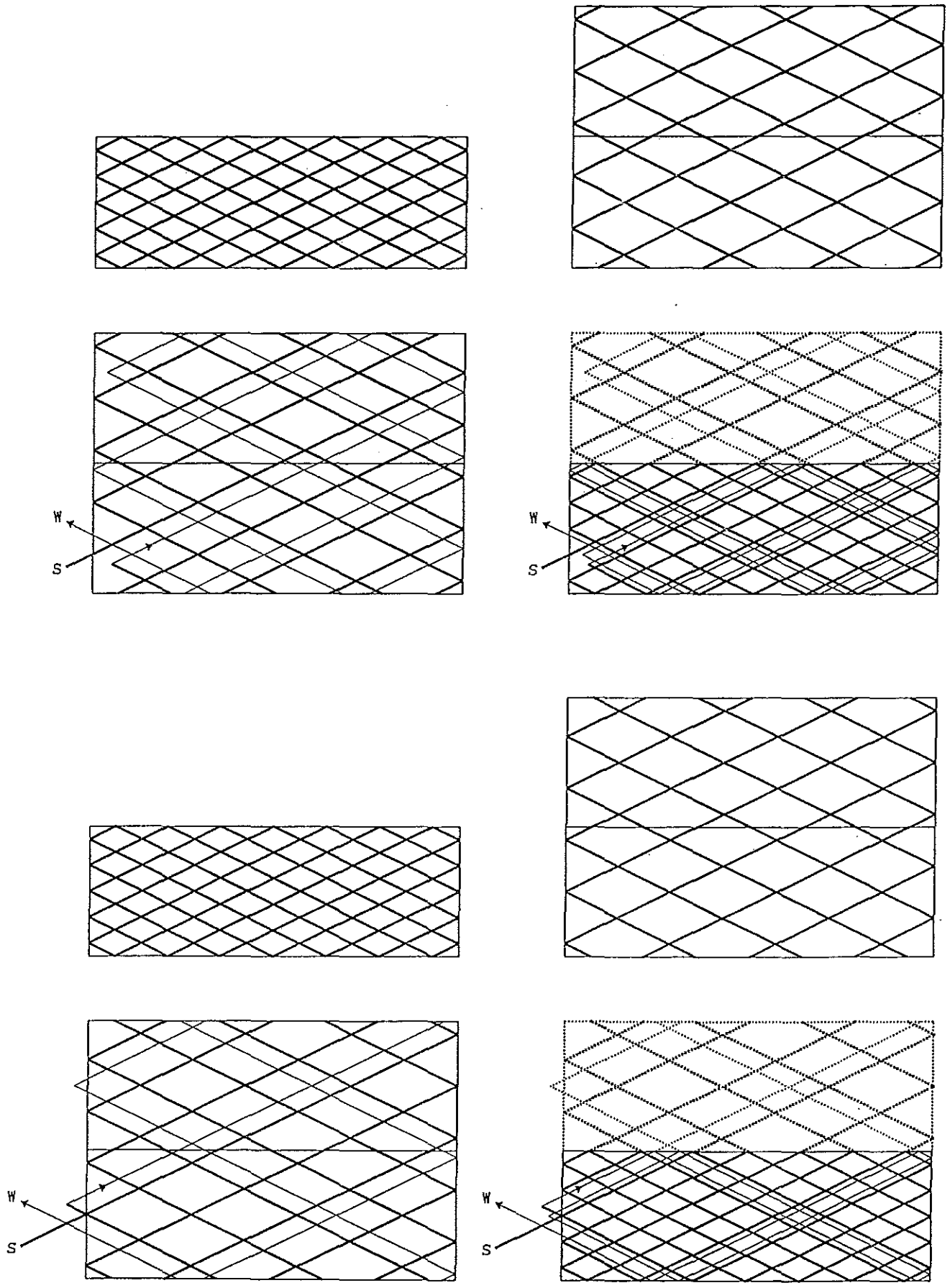


Fig. 185 — The string-run of 1st-order enlargements; $b_h/b_v = 7/5$.

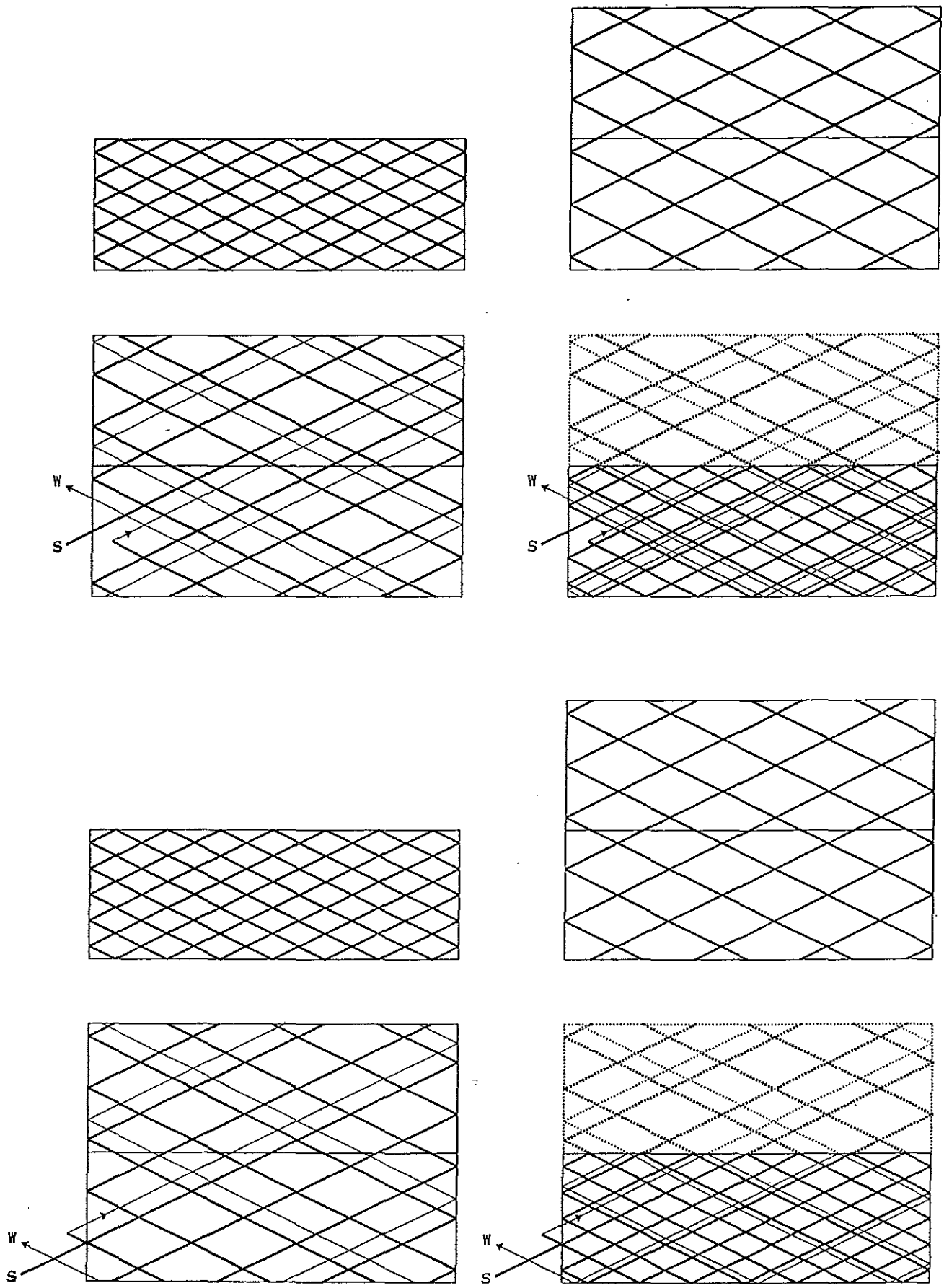


Fig. 186 — The string-run of 1st-order enlargements; $b_h/b_v = 7/5$.

For the Regular Rectangular Mat with $b_h/b_v = 7/5$, the 1st-order Method II enlargement associated with the leftmost upper bight-point and the 1st-order Method I enlargement associated with the leftmost lower bight-point are depicted in Fig. 187. These enlargements have to continue with respectively a subsequent 1st-order Method I enlargement and a 1st-order Method II enlargement in order for the Working End to finish correctly at the Standing End. Hence the overall enlargements are therefore of the 2nd-order.

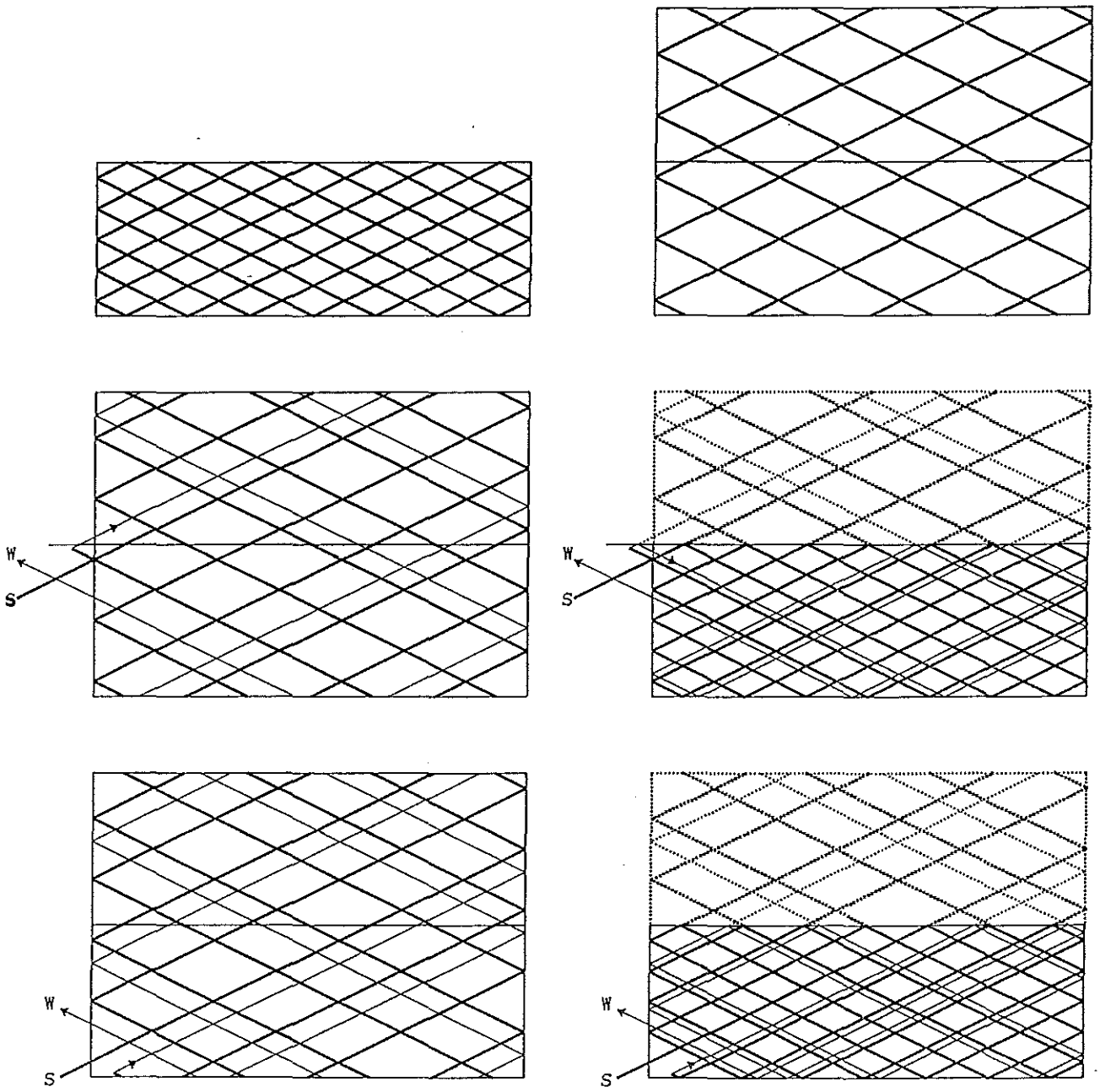


Fig. 187 — The string-run of 1st-order enlargements; $b_h/b_v = 7/5$.

Although there are general patterns in these enlargements, none is consistently repeatable with the new bight-position in which the Standing and Working Ends finish up after each successive enlargement.

For the Regular Rectangular Mat with $b_h/b_v = 7/5$, the 1st-order Method I enlargement associated with the leftmost upper bight-point is depicted in Fig. 188. The

Working End finishes correctly with respect to the Standing End. The general consistent repeatable nature of this enlargement process follows immediately from the geometric layout of its string-run in the associated Regular Knot.

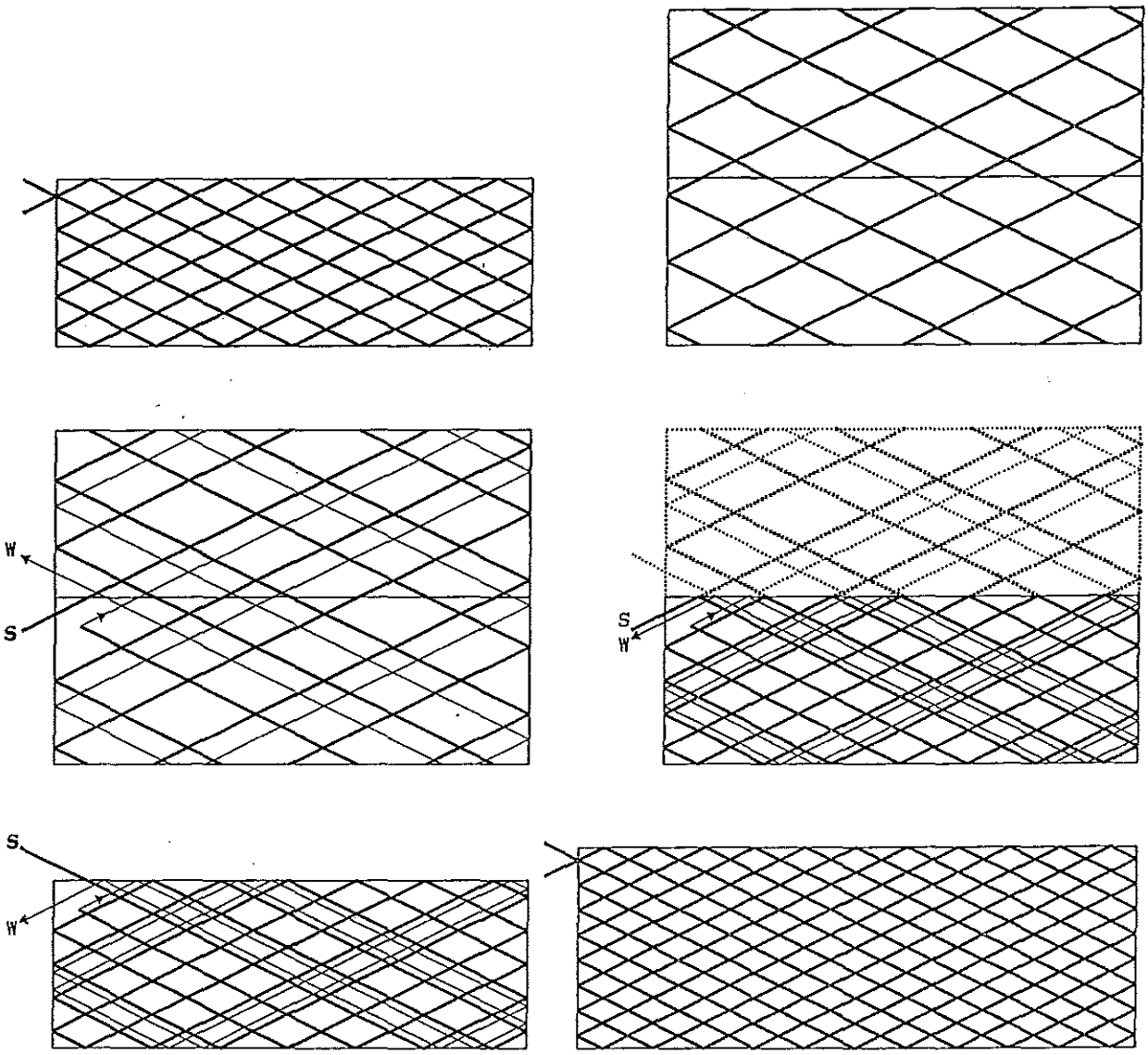


Fig. 188 — The 1st-order Method I enlargement associated with the leftmost upper bight-point.

For convenience sake this general consistent 1st-order Method I enlargement process is transferred to the leftmost lower bight-point as depicted in Fig. 189. The two lowermost string-run diagrams in Fig. 189 depict the 1st-order Method II enlargement process associated with the leftmost lower bight-point. Its general consistent repeatable nature follows again immediately from the geometric layout of its string-run in the associated Regular Knot.

Although we shall only use the general consistent 1st-order Method I and Method II enlargement processes in association with the leftmost lower bight-point as depicted in Fig. 189, it will be obvious that they can be transferred to the other six extreme bight-points: the leftmost upper bight-point, the uppermost left bight-point, the uppermost

right bight-point, the rightmost upper bight-point, the rightmost lower bight-point and the lowermost left bight-point.

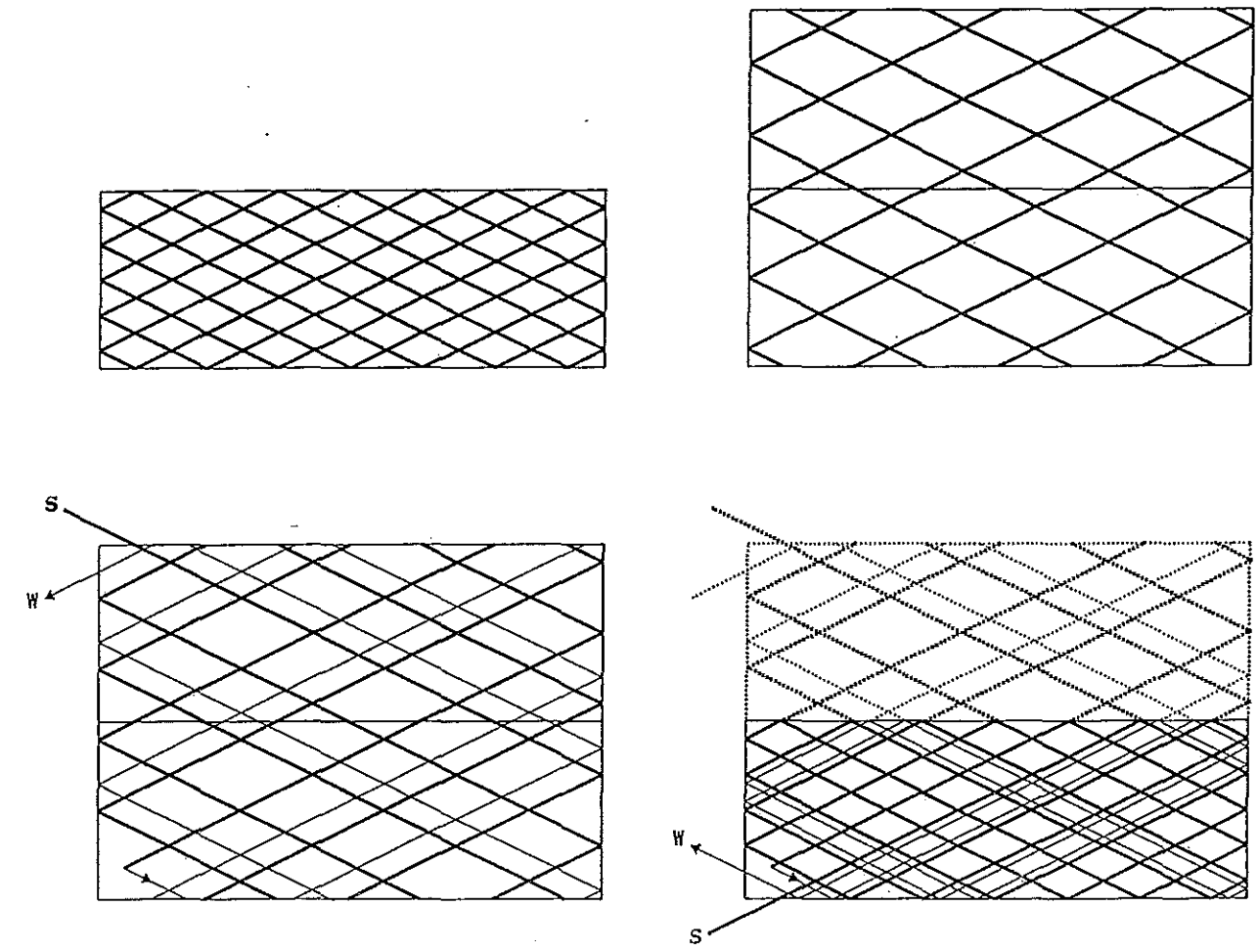


Fig. 189 — The consistent 1st-order enlargement processes.

In Fig. 190 are depicted the successive enlargement stages from $b_h/b_v = 1/1$ to $b_h/b_v = 10/7$ by using the consistent 1st-order enlargement processes.

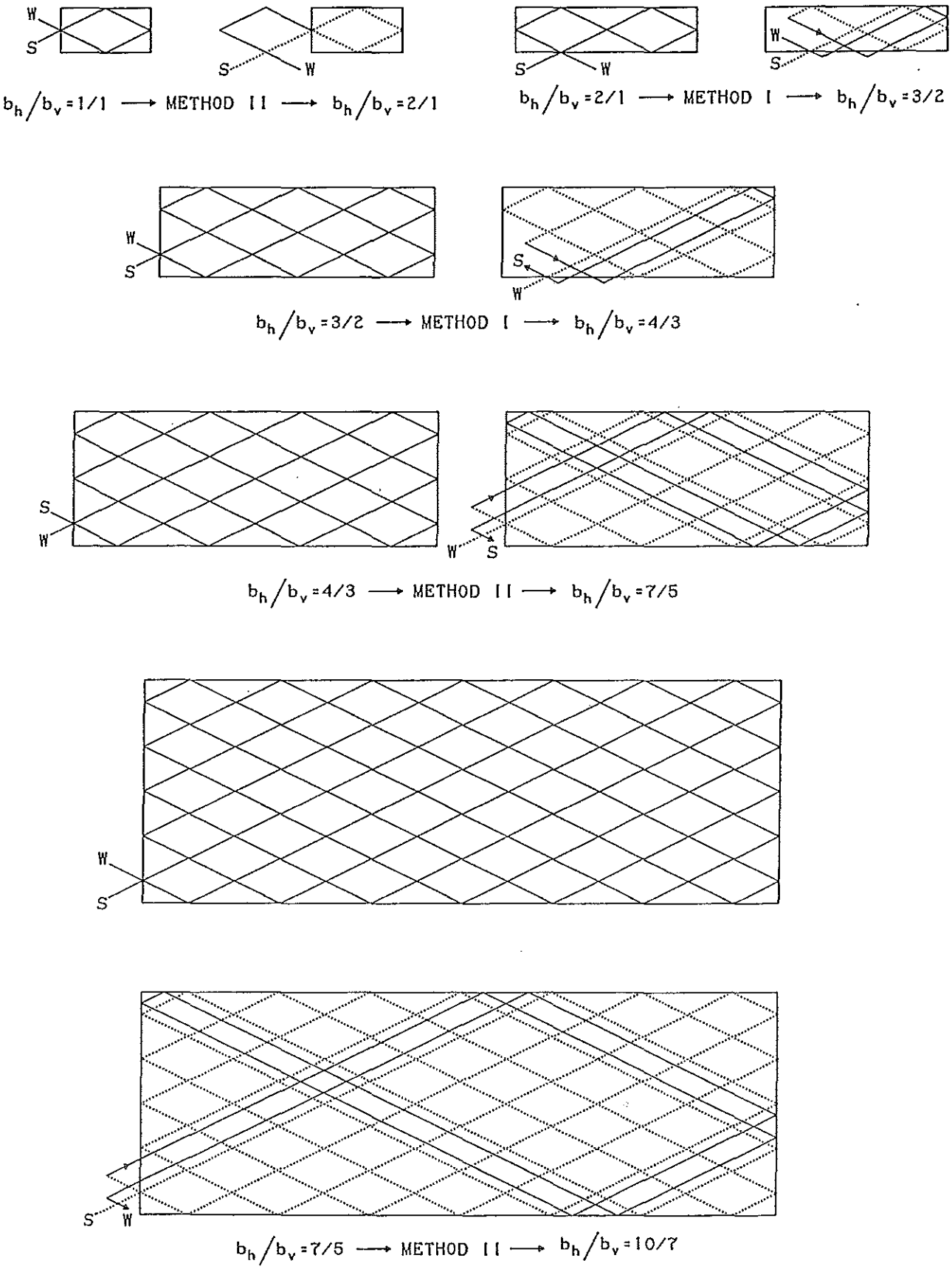


Fig. 190 — The successive 1st-order enlargements from $b_h/b_v = 1/1$ to $b_h/b_v = 10/7$.

For clarity, the essential string-run sections of the **consistent** 1st-order enlargements have once more been depicted in Fig. 191.

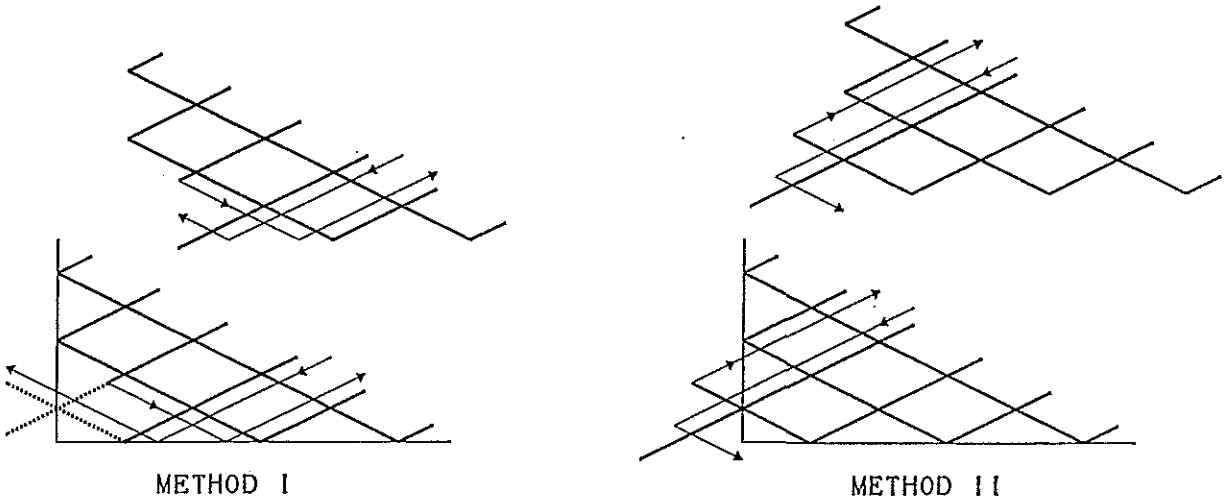


Fig. 191 — The consistent 1st-order enlargement processes.

Readers who have access to *The Ashley book of knots* will see that the raising method mentioned in #2268 uses these consistent 1st-order enlargement methods. Unfortunately however, it appears that Ashley did not realise the significance of the successive raising steps since nothing has been mentioned about them in relation to other dimensioned Regular Rectangular Mats. In fact, had he realised the significance, much of the other entries concerning Regular Rectangular Mats would have been obsolete and could have been replaced with much more interesting facets of these braids.

The essential string-run section of the **nonconsistent** enlargement processes are depicted in Fig. 192; they may be associated with 1st-order and 2nd-order enlargements.

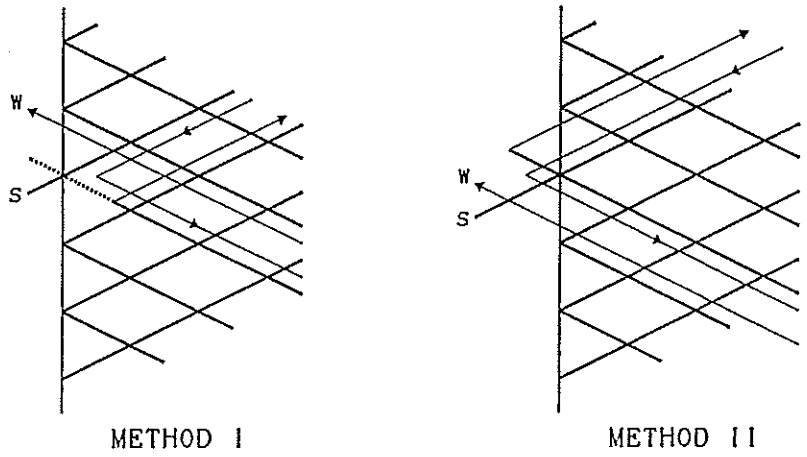
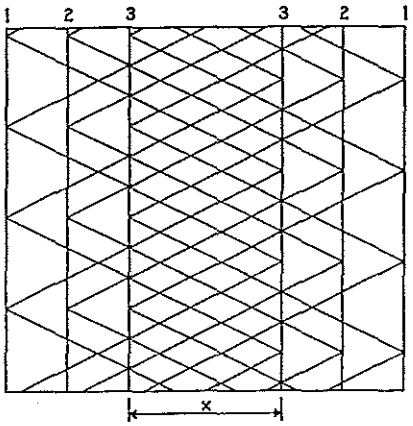


Fig. 192 — The nonconsistent enlargement processes.

⋆⋆ *Find and prove the relationships with which we can determine whether a nonconsistent enlargement process is of the* 1st*-order or of the* 2nd*-order. Which bight-points and associated enlargement processes (other than those associated with the consistent enlargements) are very special, and what is their special nature? As usual, the Complemental Law exerts its powerful influence. Where and how is its influence displayed?*

THE BRAIDER'S NOTEBOOK

Whipmakers are often portrayed as braiders, but are they? In the true sense of the subject *braiding*, most whipmakers are definitely not *braiders* but are, from the braiding point of view, *handymen* which are very skilled in making whips. This does of course not imply that their braid-work is of an inferior quality, although from the braiding point of view the standard of their braid-work leaves in most cases something to be desired. In order to be a braider, one should not only be able to produce first-class braid-work, but should also fully understand the properties of braids and their various associated construction procedures. It are these two requirements which are normally lacking in whipmakers (it is therefore not surprising that most whipmakers still belong to the flock which worships secrecy). A typical example of their deficient understanding of braids and construction procedures may be found in an article by Ron Edwards which deals with the braiding of single-string imitation multi-ply Turk's Head knots (imitation multi-ply Casa-coded) for whiphandle knobs (*The Australian Whipmaker* No. 41 (December 1995), pp. 764–766). Hence, especially for the novice braider, we shall give a brief overview of the construction procedures associated with single-string imitation 3-ply 3-parts/4-bights Casa Knots. It is of course most important to be able to draw the string-run diagram of a multi-ply "Regular" Knot. The string-run is in fact that of a **Regular Nested Knot** (see Fig. 193).



NUMBER OF INTERWOVEN REGULAR KNOTS = A.
NUMBER OF PARTS OF EACH INTERWOVEN REGULAR KNOT = p.
TOTAL NUMBER OF PARTS OF FINAL KNOT = Ap
THE COLUMN-DISTANCE BETWEEN THE INNERMOST BIGHT-BOUNDARIES = x
HENCE x=A(p-2)+2.

FOR p=3 AND A=3 ⟶ x=5.

Fig. 193 — The string-run of a multi-ply "Regular" Knot.

On the bight-edges the consecutive bight-boundaries are 2 columns apart, and the inner bight-boundaries are x columns apart. On the left-hand bight-edge the bight-boundaries are numbered $1, 2, 3, \cdots$ from left to right, and on the right-hand bight-edge the bight-boundaries are numbered $1, 2, 3, \cdots$ from right to left.

An A-ply "Regular" Knot consists of A interwoven Regular Knots, each of which have p-parts and b-bights; hence the total number of parts is Ap, and the total number of bights is Ab. The column-distance x between the innermost left-hand and right-hand bight-boundaries is equal to $A(p-2)+2$. The string-run of the leftmost interwoven Regular Knot runs between the left-hand bight-boundary 1 and the right-hand bight-boundary A. The next interwoven Regular Knot to the right runs between the left-hand bight-boundary 2 and the right-hand bight-boundary $(A-1)$. The next interwoven Regular Knot to the right runs between the left-hand bight-boundary 3 and the right-hand bight-boundary $(A-2)$. And so on.

As far as multi-ply Casa Knots are concerned we can superimpose two coding-

patterns on this string-run as is shown for 3-ply 3-parts/4-bights Casa Knots in Fig. 194.

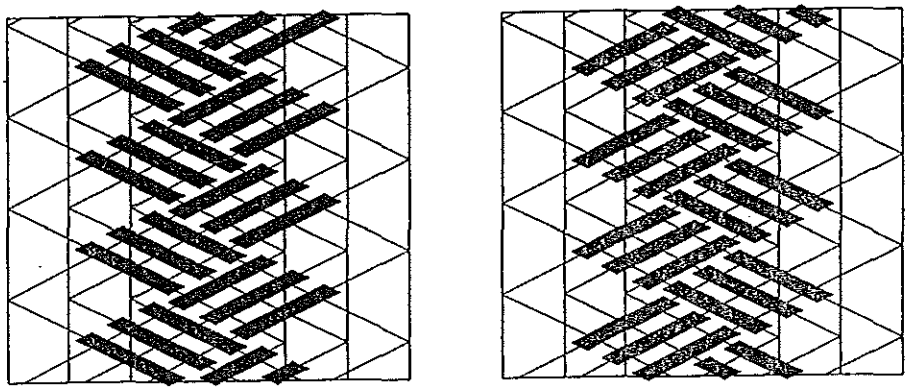


Fig. 194 — 3-ply 3parts/4-bights Casa-coded "Regular" Knots.

We shall assume that in the diagrams the top of the whiphandle knob is towards the right.

After the multi-ply Casa-coding has been superimposed, we can modify the string-run in order to make it comply with a single-string imitation. The modifications must fall within a coding-block set where the movement of the modified half-cycles is an **under**-movement. Since the string-ends are preferably worked away near the bottom side of the knob (hence the left-hand side in the diagrams), we shall only draw the diagrams in which this will be the case. With these conditions for the string-ends, the various construction methods for a single-string imitation of each of the two 3-ply 3-parts/4-bights Casa-coding patterns have been depicted in the Figs. 195 and 196 respectively. Each of the diagrams 1 to 4 depicts a construction process which starts at a string end: the starting point is indicated by a small black dot. Each of the diagrams 5 to 8 depicts a construction process which starts at the 1/3 string-length position: the starting point is indicated by the letter S. Each of the diagrams 9 to 10 depicts a construction process which starts at the 1/2 string-length position: the starting point is indicated by the letter S. Thus we have here 10 different construction processes.

With the above mentioned restrictions for the final positions of the string-ends it is more or less self evident, and hence easy to prove, that for $A > 1$, there are for a single-string imitation of each of the two A-ply p-parts/b-bights Casa-coding patterns $(2A+4)$ different construction processes when $A = odd$, and $(2A + 2)$ different construction processes when $A = even$.†

Preferably we employ a process which starts at either the 1/2 or the $\frac{A-1}{2A}$ string-length position. When we start at the 1/2 string-length position, we have for a single-string imitation of each of the two A-ply p-parts/b-bights Casa-coding patterns 2 different construction processes at our disposal, and when we start at the $\frac{A-1}{2A}$ string-length position, we have 4 different construction processes at our disposal. It should, however, be realised that the different processes are not all of the same quality standard, in fact half are of a better standard then the rest. Of inferior standard are those in which a string-end emerges from the outermost bight-boundary after its passage under the last coding-block set. Hence in Figs. 195 and 196 the processes depicted in the diagrams 2, 4, 6, 8 and 10 are of an inferior standard, and hence should preferably not be used.

† When $A = even$ the two processes which start at the 1/2 string-length position are already included among those whose S-point is positioned at the beginning of a modified half-cycle.

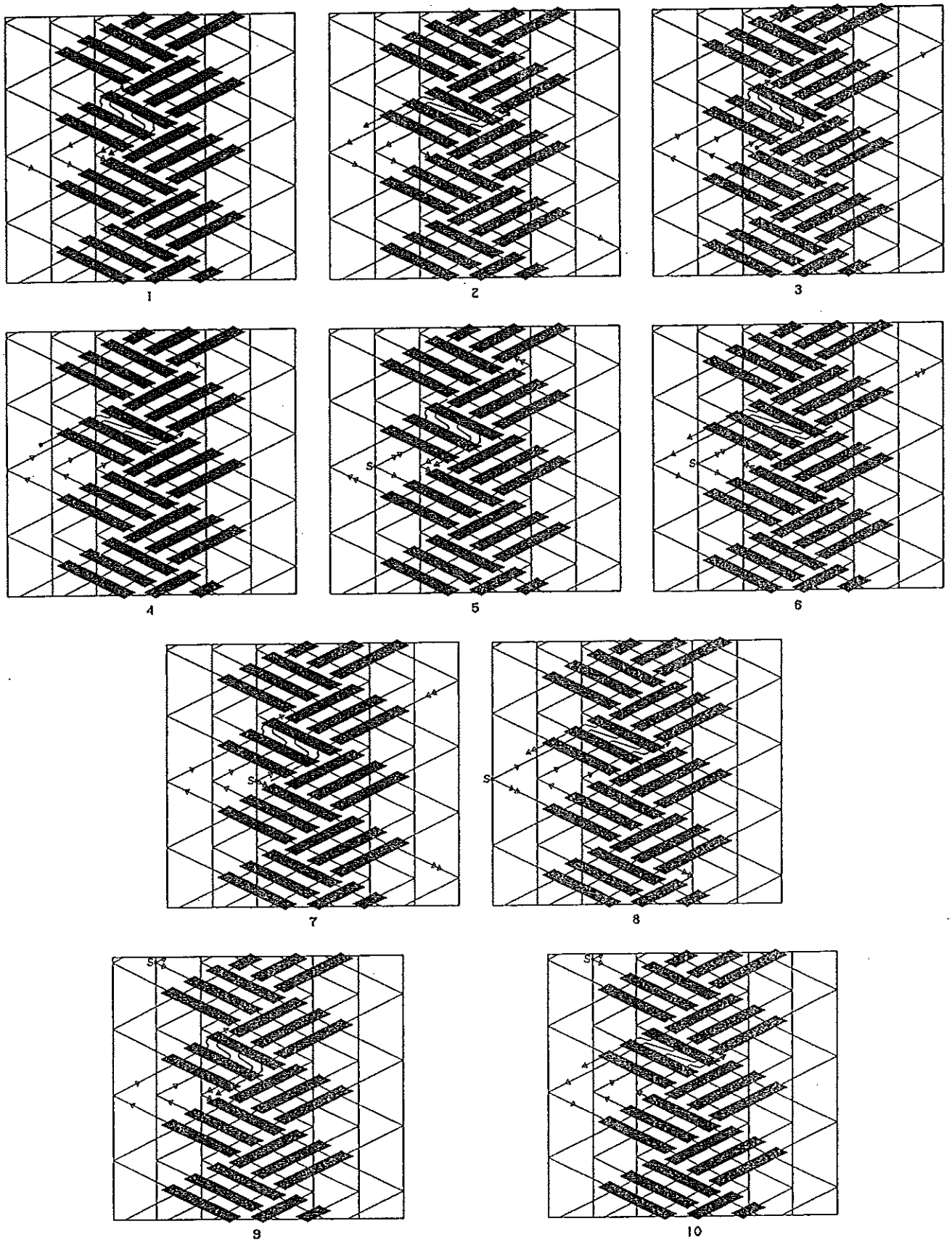


Fig. 195 — 10 construction processes for a single-string imitation 3-ply 3/4 Casa Knot.

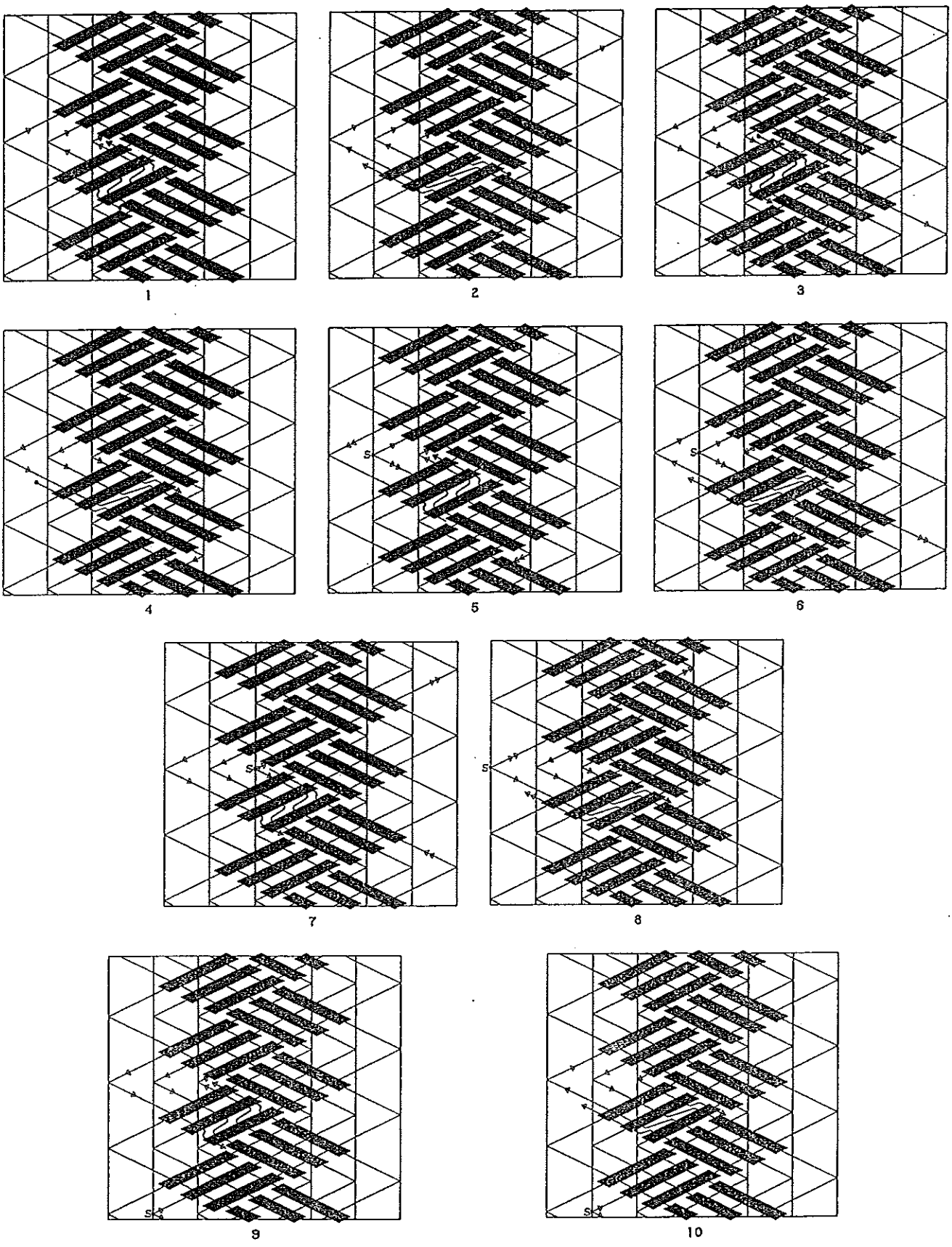


Fig. 196 — 10 construction processes for a single-string imitation 3-ply 3/4 Casa Knot.

From the grid-diagrams in Figs. 195 and 196 it will be noted that only after braiding the foundation 3-parts/4-bights Casa Knot we will have to chose whether we continue with the next half-cycle above or below the Standing half-cycle. **This is in essence a completely free choice and not in any way influenced by the way in which we start the foundation 3-parts/4-bights Casa Knot. Only the level of desired standard influences the choice.**

For the higher standard we have tabulated in Fig. 197 the relevant string-run information (the dot indicates on which bight-boundary braiding starts).

FIG. 195		FIG. 196	
GRID-DIAGRAM	LEFT BIGHT-BOUNDARIES	GRID-DIAGRAM	LEFT BIGHT-BOUNDARIES
1	↓ → 1̇,2,3	1	↑ → 1̇,2,3
2		2	
3	1,2,3̇ ← ↑	3	1,2,3̇ ← ↓
4		4	
5	1 ← ↑ ↓ → 2̇,3	5	1 ← ↓ ↑ → 2̇,3
6		6	
7	1,2 ← ↑ ↓ → 3̇	7	1,2 ← ↓ ↑ → 3̇
8		8	
9	1 ← ↑ ⇕ ↓ → 3 (2)	9	1 ← ↓ ⇕ ↑ → 3 (2)
10		10	

Fig. 197 — The relevant string-run information for the higher standard.

Let's now examine what has been said in that above referred to article in *The Australian Whipmaker* No. 41 (December 1995), pp. 764–765. The construction procedure described there is in accordance with the grid-diagram of Fig. 198.

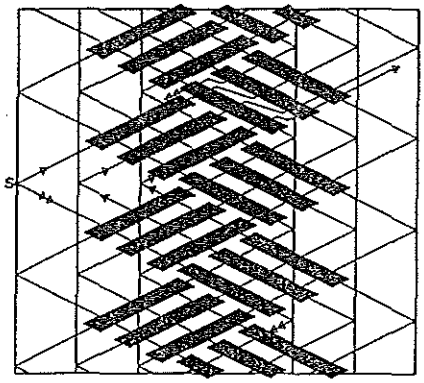


Fig. 198 — The construction procedure described in *The Australian Whipmaker* No. 41, pp. 764–765.

It should immediately be noticed that this is a construction procedure of a low quality standard on two accounts: firstly, the string-ends are, without any justification, worked away near the top side of the knob (hence the right-hand side in the diagram); secondly, one string-end emerges from the outermost bight-boundary after its passage under the last coding-block set. Obviously this construction method should never be used. It should also be obvious that this inferior construction method can readily be changed, with a very small alteration to the string-run, into a construction procedure of

the higher standard. In fact all that has to be done is a small, but very important change at the end of the foundation knot construction phase (the foundation knot is in this case associated with the string-run between the left-hand bight-boundary 1 and the right-hand bight-boundary 3), which will result in obtaining the construction process depicted in grid-diagram 5 of Fig. 196. Hence if we want to retain the braiding directions and the sequence of interbraiding, then the construction process as depicted in grid-diagram 5 of Fig. 196 should be used. Note how this small change affects the position of point S.

The construction process for the foundation 3-parts/4-bights Casa Knot in the construction procedures depicted in the grid-diagrams $1, 2, 7, 8$ of Fig. 195 is illustrated in the lower-left diagrams of Fig. 199, while its construction process in the construction procedures depicted in the grid-diagrams $3, 4, 5, 6$ of Fig. 195 is illustrated in the upper-left diagrams of Fig. 199.

The construction process for the foundation 3-parts/4-bights Casa Knot is illustrated in the upper-right diagrams of Fig. 199 for the construction procedures depicted in the grid-diagrams $1, 2, 7, 8$ of Fig. 196, and in the lower-right diagrams of Fig. 199 for the construction procedures depicted in the grid-diagrams $3, 4, 5, 6$ of Fig. 196.

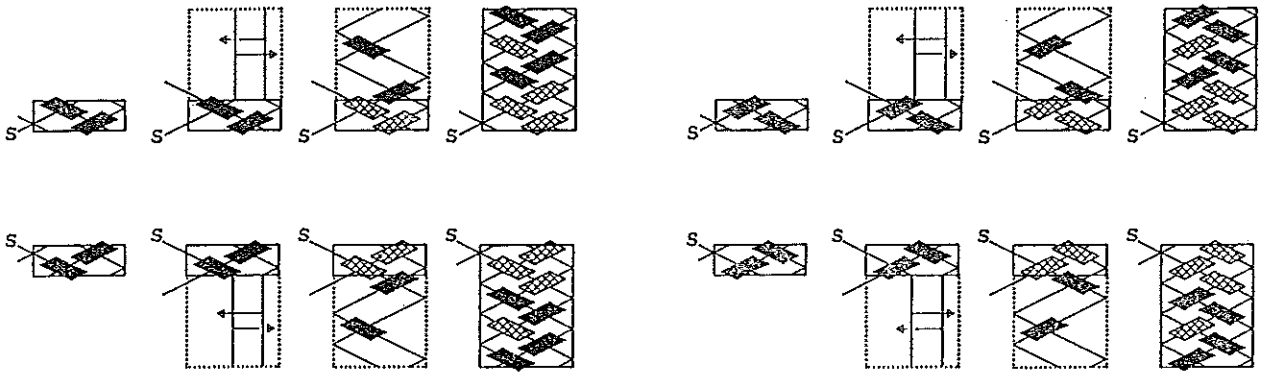


Fig. 199 — The construction procedures for a foundation 3/4 Casa Knot.

The construction process for the foundation 3-parts/4-bights Casa Knot in the construction procedures depicted in the grid-diagrams $9, 10$ of Fig. 195 is illustrated in the uppermost diagrams of Fig. 200, while its construction process in the construction procedures depicted in the grid-diagrams $9, 10$ of Fig. 196 is illustrated in the lowermost diagrams of Fig. 200.

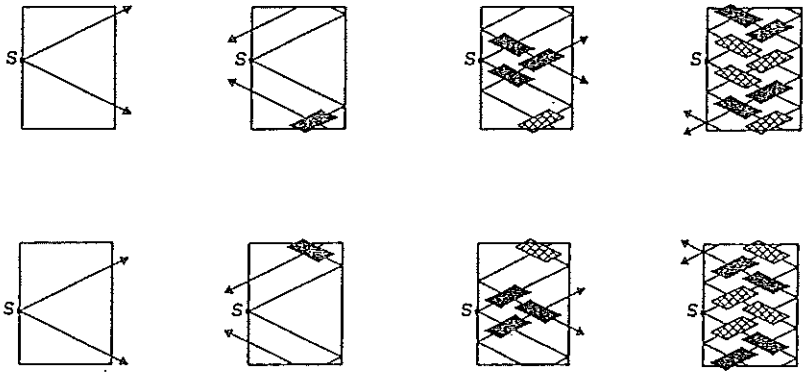


Fig. 200 — The construction procedures for a foundation 3/4 Casa Knot.

The reader has met here not only an example which shows the important function of grid-diagrams in the determination of the various construction procedures, but also an

example which illustrates that construction procedure, quality-level, braiding direction, and coding arrangement may be interdependent.

The article in *The Australian Whipmaker* No. 41, pp. 764 – 765, referred to above, states with respect to the construction method described, and depicted here in Fig. 198: "The advantage of this method is that the second and third layers go above rather than below the foundation knot. If you try out the common method of starting with one long strand and feeding the whole lot through at every pass you will discover that the second and third layers go below the foundation knot." This statement is total nonsense (see pg. 213), and clearly demonstrates the extremely limited braiding skill possessed by whipmakers in general. It is especially important to draw the attention of novices to such erroneous statements in order keep them on the path towards braiding artisans.

The construction method described in *The Australian Whipmaker* No. 41, pg. 766, for a single-string imitation 2-ply 5-parts/4-bights Casa Knot is depicted in the leftmost grid-diagram of Fig. 201. Once again the method is that of a lower quality standard since one of the string-ends emerges from an outermost bight-boundary without a justifiable reason. With the coding arrangement used, and starting at the 1/2 string-length position†, the braiding procedure should have been as depicted in the central grid-diagram of Fig. 201.

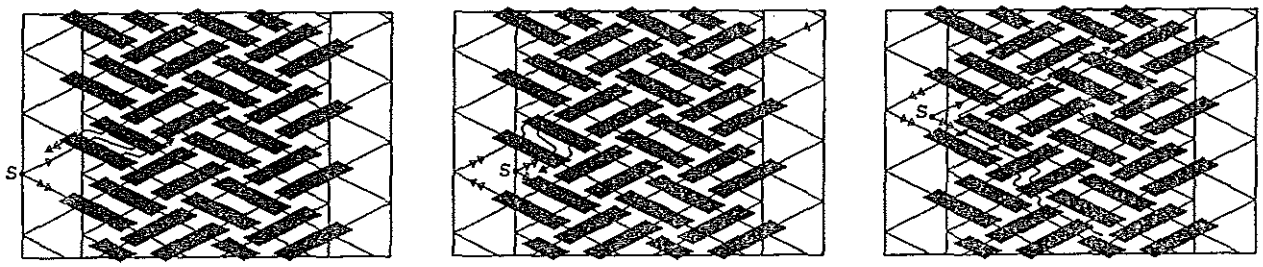


Fig. 201 — The single-string imitation 2-ply 5/4 Casa Knot.

Since we are dealing with a single string imitation 2-ply Casa Knot, a better construction method is depicted by the rightmost grid-diagram in Fig. 201 (refer to *The Braider* No. 3, pp. 55 – 57; No. 4, pp. 66 – 68; and No. 5, pp. 85 – 86). This construction method also provides a much better and more secure cover for the string-ends.

It should be stressed that single flat-string imitation multi-ply "regular" knots belong to the bush knots and hence should not be used in high quality braidwork. The single-string imitation 3-ply or 4-ply 5-parts/4-bights Casa knot is the best bush knot for whip handles, and was used by all good whip handle makers around a century ago. The single-string imitation 3-ply or 4-ply 3-parts/4-bights Casa knot is used by many professional whipmakers since it is quick to tie, but is inferior to the single-string imitation 3-ply or 4-ply 5-parts/4-bights Casa knot. It is interesting to note that both these imitation knots are nearly always tied by whipmakers in the lower quality-standard way.

---

† Not the 1/3 string-length position as stated in the accompanying text since we are dealing with a single-string imitation 2-ply knot and not with a single-string imitation 3-ply knot. In the accompanying pictorial drawing # 10, the dashed line should make an *over*-movement instead of the indicated *under*-movement. The pictorial drawings show an incorrect sequential position of the braid-pattern on the whip-handle. For orientation purpose, this sequential position should be correctly displayed, especially for the novice.

Braiding Instructions
and their
Presentation

The round braids in Figs. 202 and 203 are depicted in the way described in *The Braider* issue No. 7, pp. 146–153.

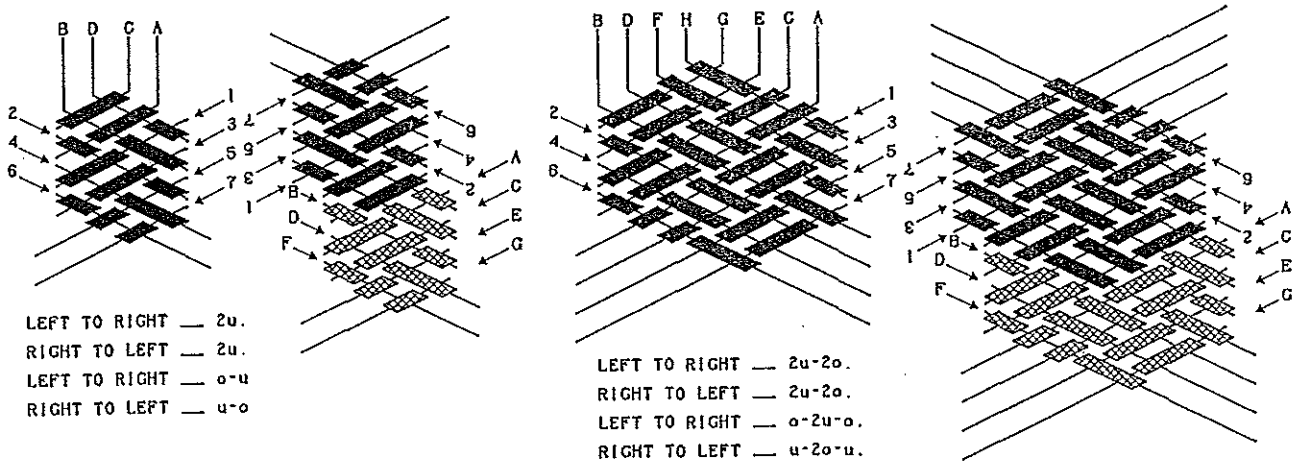

LEFT TO RIGHT — 2u.
RIGHT TO LEFT — 2u.
LEFT TO RIGHT — o-u
RIGHT TO LEFT — u-o
LEFT TO RIGHT — 2u-2o.
RIGHT TO LEFT — 2u-2o.
LEFT TO RIGHT — o-2u-o.
RIGHT TO LEFT — u-2o-u.

Fig. 202 — Round braids with a chequered coding-pattern.

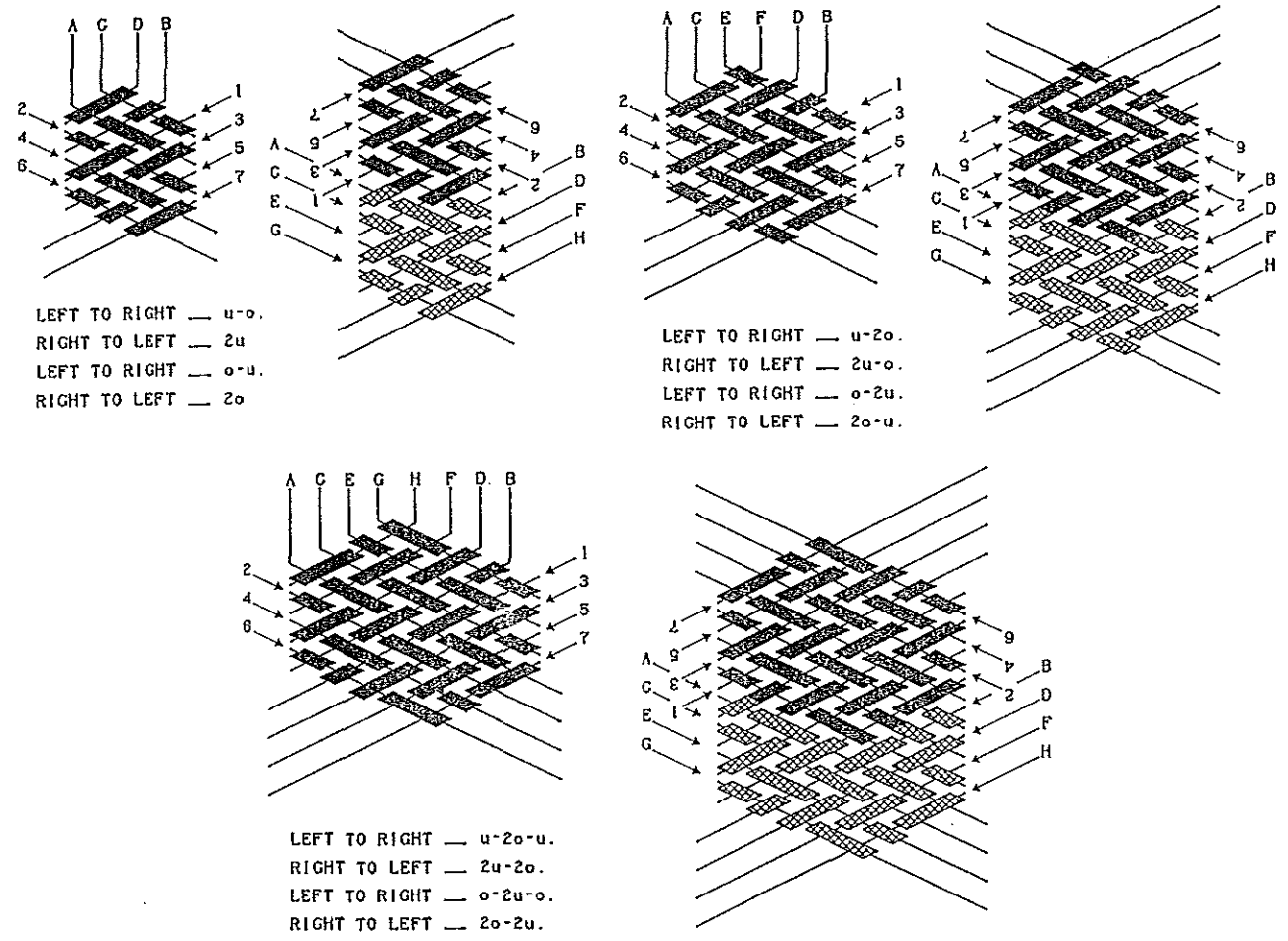

LEFT TO RIGHT — u-o.
RIGHT TO LEFT — 2u
LEFT TO RIGHT — o-u.
RIGHT TO LEFT — 2o
LEFT TO RIGHT — u-2o.
RIGHT TO LEFT — 2u-o.
LEFT TO RIGHT — o-2u.
RIGHT TO LEFT — 2o-u.
LEFT TO RIGHT — u-2o-u.
RIGHT TO LEFT — 2u-2o.
LEFT TO RIGHT — o-2u-o.
RIGHT TO LEFT — 2o-2u.

Fig. 203 — Round braids with a Gaucho coding-pattern.

However, braiding these braids in the way depicted is not the easiest way, due to the sequences which start with **over**-crossings. Recall (see pg. 148) that every time a string is laid down along the front of the braid, its crossing-movements should start with one or more **unders** if possible. This was not only important for an easier braiding operation, but it also facilitates in obtaining a neat braid. Since a small change in the usual braiding procedure makes 'under-starts' also possible for these coding-patterns, we depict and braid these round braids as shown in Figs. 204 and 205. Note that the consecutive braiding sequences have become much simpler, especially in the case of the chequered coding-pattern.

In *The Braider* issue No. 6, pp. 127 – 132 we discussed *orientation and similarity* and gave some examples of knots where these coding-patterns were used in combination with a herringbone-coding. In a similar manner can we combine in round braids the chequered and Gaucho coding-patterns with a herringbone coding-pattern. Some examples of round braids in which this has been done are shown in Figs. 206, 207 and 208, 209 respectively.

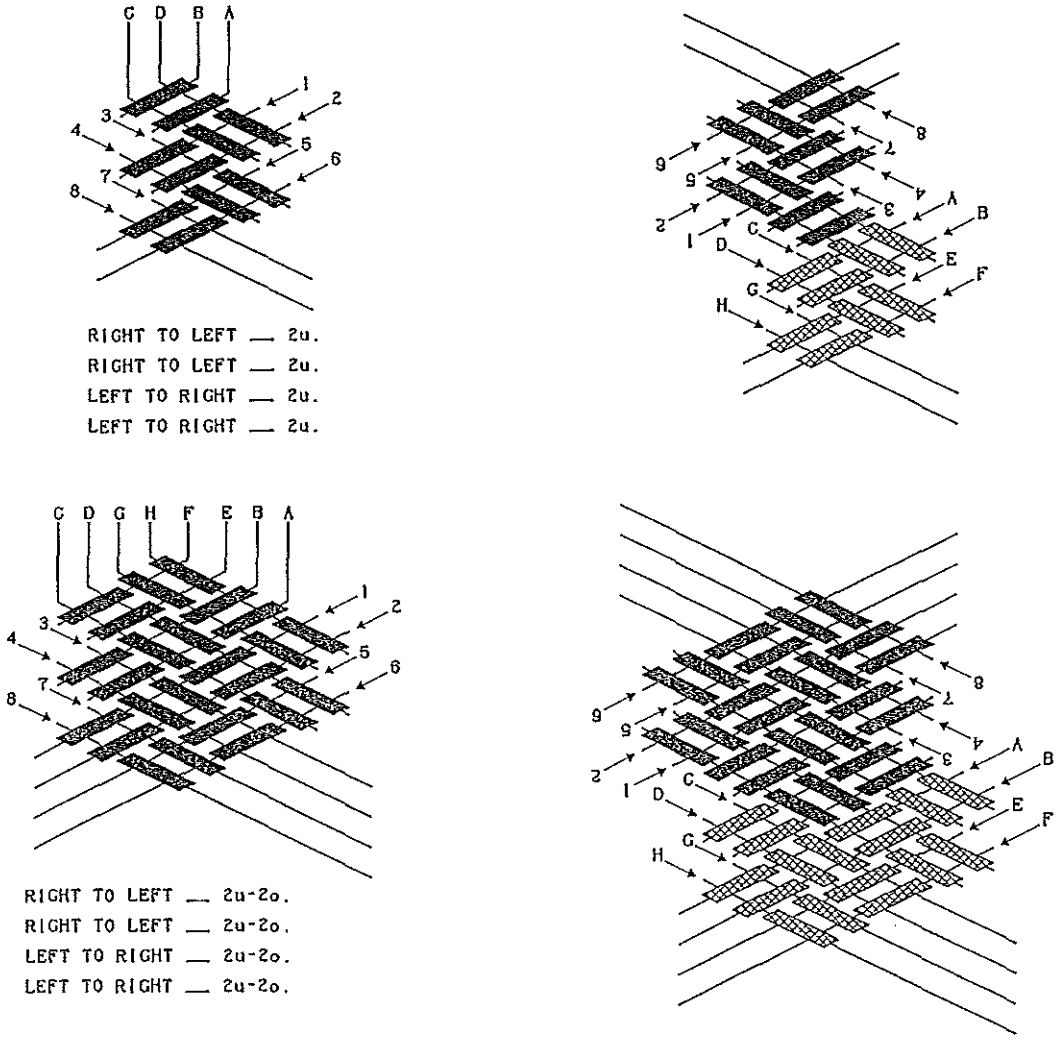
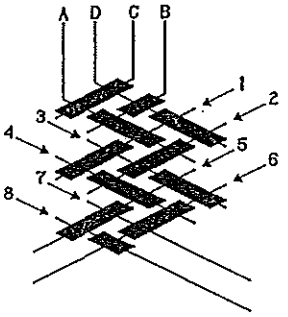
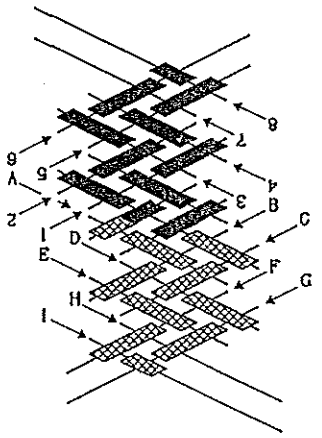
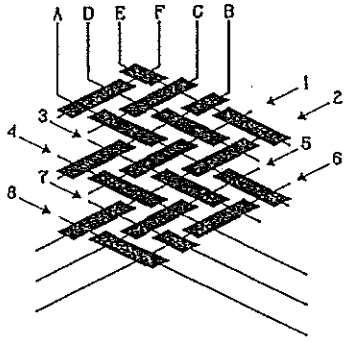

C D B A
RIGHT TO LEFT — 2u.
RIGHT TO LEFT — 2u.
LEFT TO RIGHT — 2u.
LEFT TO RIGHT — 2u.
C D G H F E B A
RIGHT TO LEFT — 2u-2o.
RIGHT TO LEFT — 2u-2o.
LEFT TO RIGHT — 2u-2o.
LEFT TO RIGHT — 2u-2o.

Fig. 204 — The grid-diagrams and braiding procedure for round braids with a chequered coding-pattern.

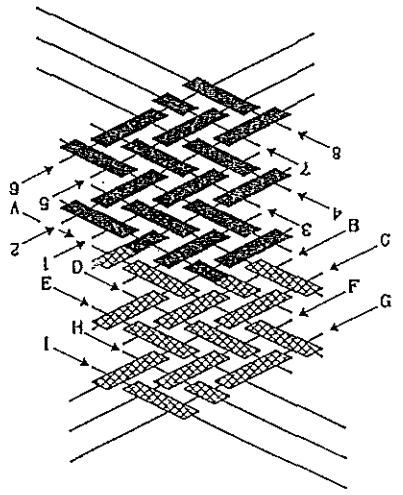


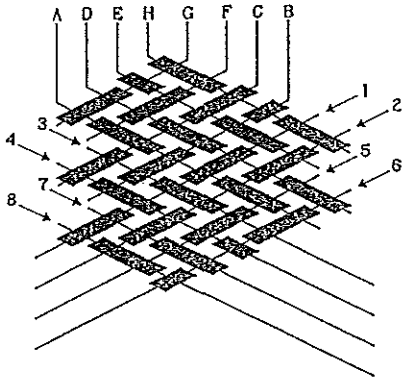
RIGHT TO LEFT — 2u.
RIGHT TO LEFT — u-o.
LEFT TO RIGHT — 2u.
LEFT TO RIGHT — u-o.





RIGHT TO LEFT — 2u-o.
RIGHT TO LEFT — u-2o.
LEFT TO RIGHT — 2u-o.
LEFT TO RIGHT — u-2o.





RIGHT TO LEFT — 2u-2o.
RIGHT TO LEFT — u-2o-u.
LEFT TO RIGHT — 2u-2o.
LEFT TO RIGHT — u-2o-u.

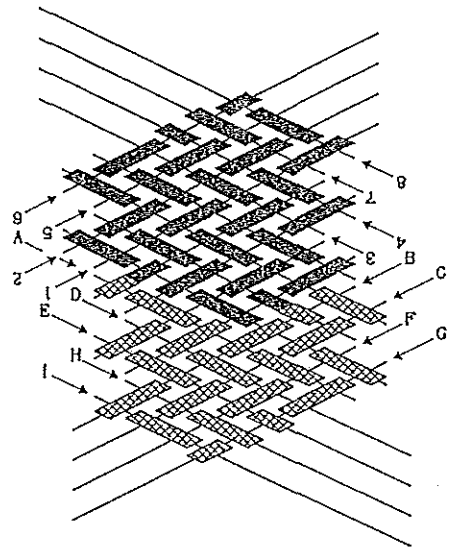


Fig. 205 — The grid-diagrams and braiding procedure for round braids with a Gaucho coding-pattern.

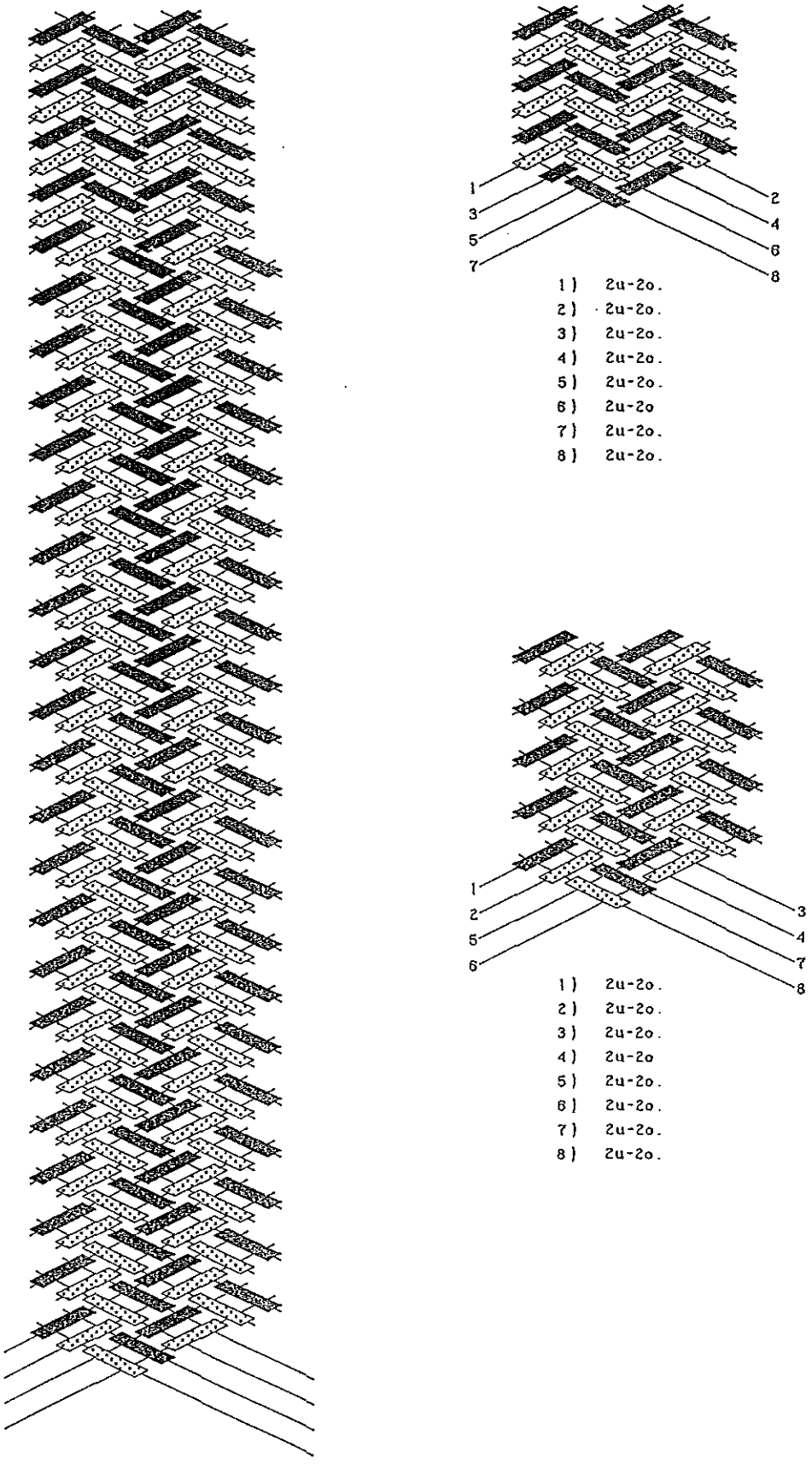


Fig. 206 — Round braid with herringbone and chequered coding-pattern.

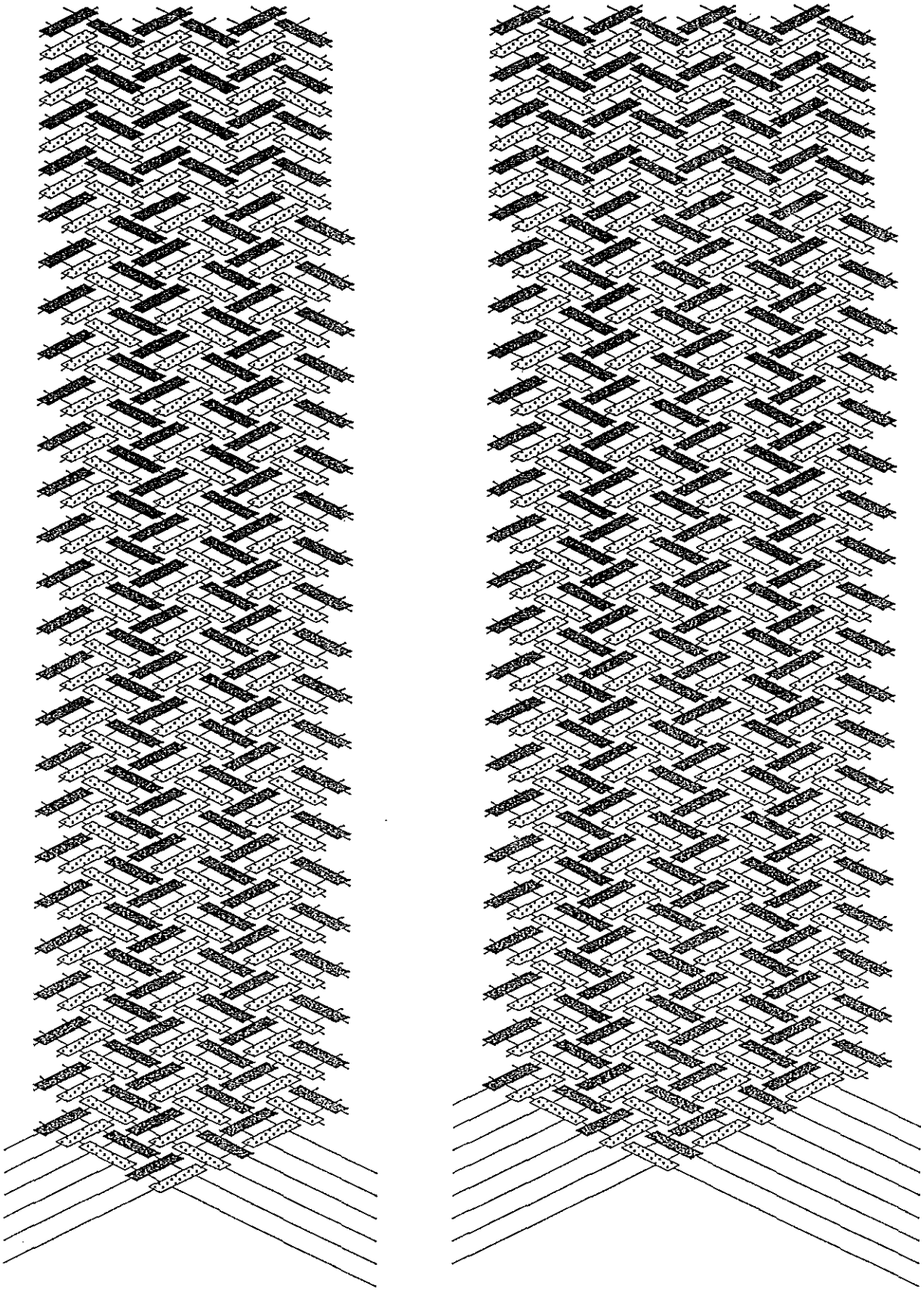
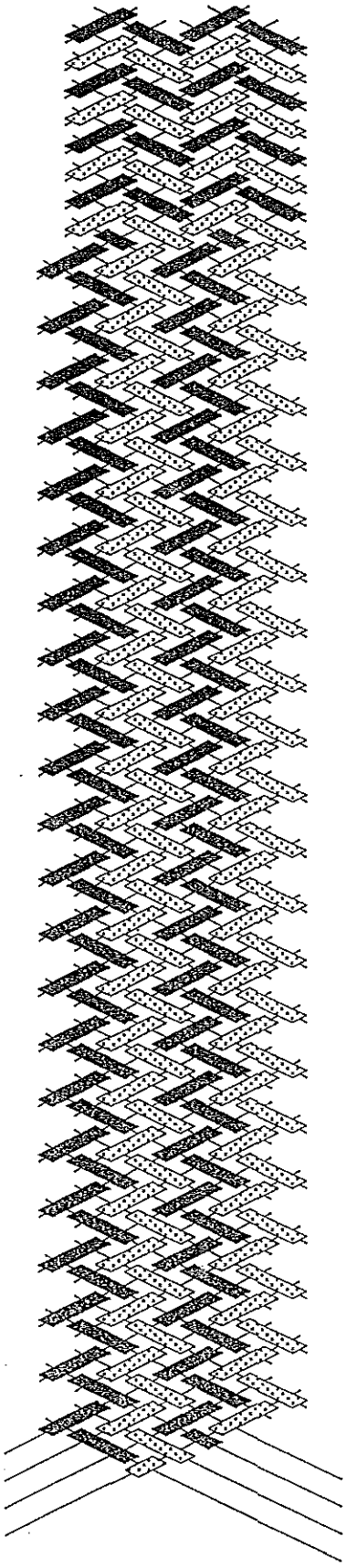
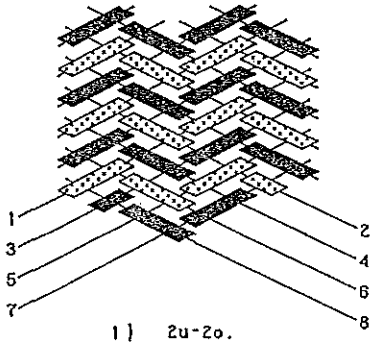
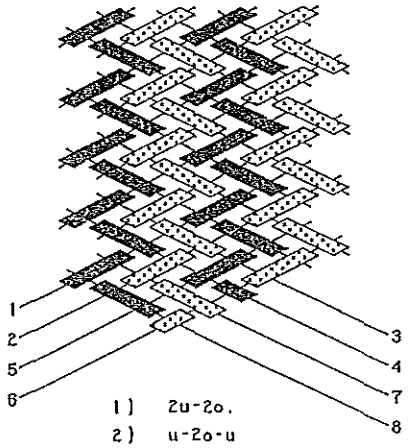


Fig. 207 — Round braid with herringbone and chequered coding-pattern.





1) 2u-2o.
2) 2u-2o.
3) 2u-2o.
4) 2u-2o.
5) 2u-2o.
6) 2u-2o.
7) 2u-2o.
8) 2u-2o.



1) 2u-2o.
2) u-2o-u
3) 2u-2o.
4) u-2o-u.
5) 2u-2o.
6) u-2o-u.
7) 2u-2o.
8) u-2o-u.

Fig. 208 — Round braid with herringbone and Gaucho coding-pattern.

THE KNOTTER

No.11 **AUGUST 1997.**

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A quarterly publication for the braiding artisan

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Solution to the Question in Issue No. 10

Question on pg. 208.

The mechanism by which we can determine whether an enlargement process is of the 1st-order or of the 2nd-order is in essence revealed by the diagrams in Figs. 185 and 186. It will be observed that in Fig. 185 the string-run associated with the enlargement phase, when returning **adjacent** to the Standing End half-cycle, increases the number of bights with **less** than the increase which is associated with the **fundamental** enlargement process concerned. In Fig. 186 the string-run associated with the enlargement phase increases the number of bights with the increase which is associated with the **fundamental** enlargement process concerned, **before** returning **adjacent** to the Standing End half-cycle. In order to express these relationships in a useful manner, we employ a little mathematics.

In Fig. 210 are depicted the essential parts of the string-runs associated with a Method I and a Method II enlargement.

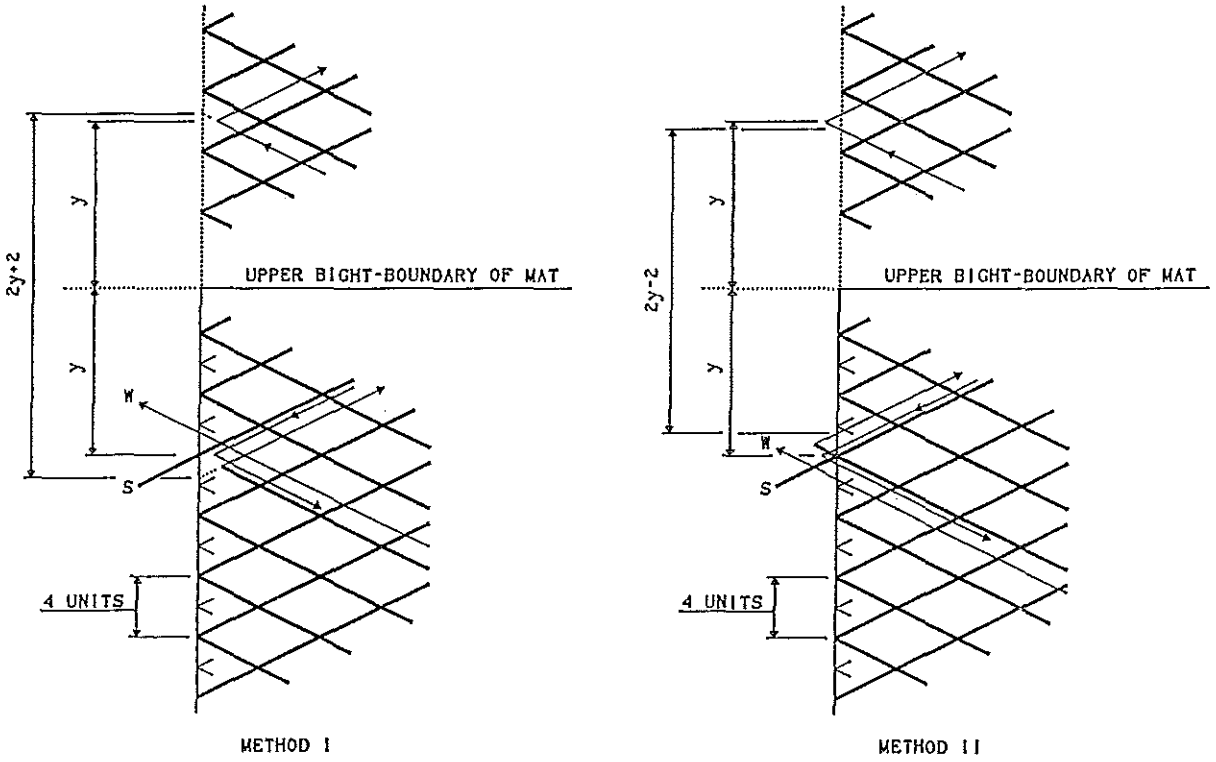


Fig. 210 — Essential string-run parts associated with Methods I and II enlargements.

The thick lines in these diagrams are associated with the string-run of the Regular Knot. The thin $<$ bights are the bights of the Regular Rectangular Mat which correspond to those bights of the Regular Knot which are above the upper bight-boundary of the mat; thus the thick lines below the upper bight-boundary of the mat and the thin $<$ belong to the string-run of the Regular Rectangular Mat.

Method I Enlargement:

Let c_I cycles be required from the beginning of the enlargement phase to the return adjacent to the Standing End. One cycle in the Regular Knot gives a distance of

$|4b_h|_{4b_v}$ along its bight-boundary, hence c_I cycles give a distance of $|c_I \cdot 4b_h|_{4b_v}$ along its bight-boundary. This distance is also equal to $(2y+2)$. Thus:

$$|c_I \cdot 4b_h|_{4b_v} = 2y+2$$
$$4|c_I \cdot b_h|_{b_v} = 2y+2$$
$$|c_I \cdot b_h|_{b_v} = \frac{y+1}{2}.$$

If we denote the increase in bights due to the **fundamental** Method I enlargement by $\Delta_{I,1}b_v$, then we have the following three cases†:

$c_I < \Delta_{I,1}b_v \longrightarrow 1^{st}$-order (fundamental) Method I enlargement.
$c_I = \Delta_{I,1}b_v \longrightarrow 1^{st}$-order (fundamental) Method I enlargement. See below.
$c_I > \Delta_{I,1}b_v \longrightarrow 2^{nd}$-order Method I enlargement.

Method II Enlargement:

Let c_{II} cycles be required from the beginning of the enlargement phase to the return adjacent to the Standing End. One cycle in the Regular Knot gives a distance of $|4b_h|_{4b_v}$ along its bight-boundary, hence c_{II} cycles give a distance of $|c_{II} \cdot 4b_h|_{4b_v}$ along its bight-boundary. This distance is also equal to $(2y-2)$. Thus:

$$|c_{II} \cdot 4b_h|_{4b_v} = 2y-2$$
$$4|c_{II} \cdot b_h|_{b_v} = 2y-2$$
$$|c_{II} \cdot b_h|_{b_v} = \frac{y-1}{2}.$$

If we denote the increase in bights due to the **fundamental** Method II enlargement by $\Delta_{II,1}b_v$, then we have the following three cases:

$c_{II} < \Delta_{II,1}b_v \longrightarrow 1^{st}$-order (fundamental) Method II enlargement.
$c_{II} = \Delta_{II,1}b_v \longrightarrow 1^{st}$-order (fundamental) Method II enlargement. See below.
$c_{II} > \Delta_{II,1}b_v \longrightarrow 2^{nd}$-order Method II enlargement.

The case under Method I where $c_I = \Delta_{I,1}b_v$, is the consistent 1st-order Method I enlargement. For the braiding direction indicated it is therefore always associated with the leftmost upper bight-point of the Regular Rectangular Mat. A simple geometric proof of this special case is depicted in the upper row of diagrams of Fig. 211. Here, the thick lines are the string-runs in the mat. The dotted horizontal line, which will form the upper bight-boundary of the mat, must be placed in such a position that it will enable the end of half-cycle 2 to complete the enlargement.

The case under Method II where $c_{II} = \Delta_{II,1}b_v$, is the consistent 1st-order Method II enlargement. For the braiding direction indicated it is therefore always associated with the leftmost lower bight-point of the Regular Rectangular Mat. A simple geometric proof of this special case is depicted in the lower row of diagrams of Fig. 211. Here again, the thick lines are the string-runs in the mat. The dotted horizontal line, which will form the lower bight-boundary of the mat, must be placed in such a position that it will enable the end of half-cycle 2 to complete the enlargement.

† In the expressions the symbol $<$ stands for *smaller than*, and the symbol $>$ stands for *greater than*.

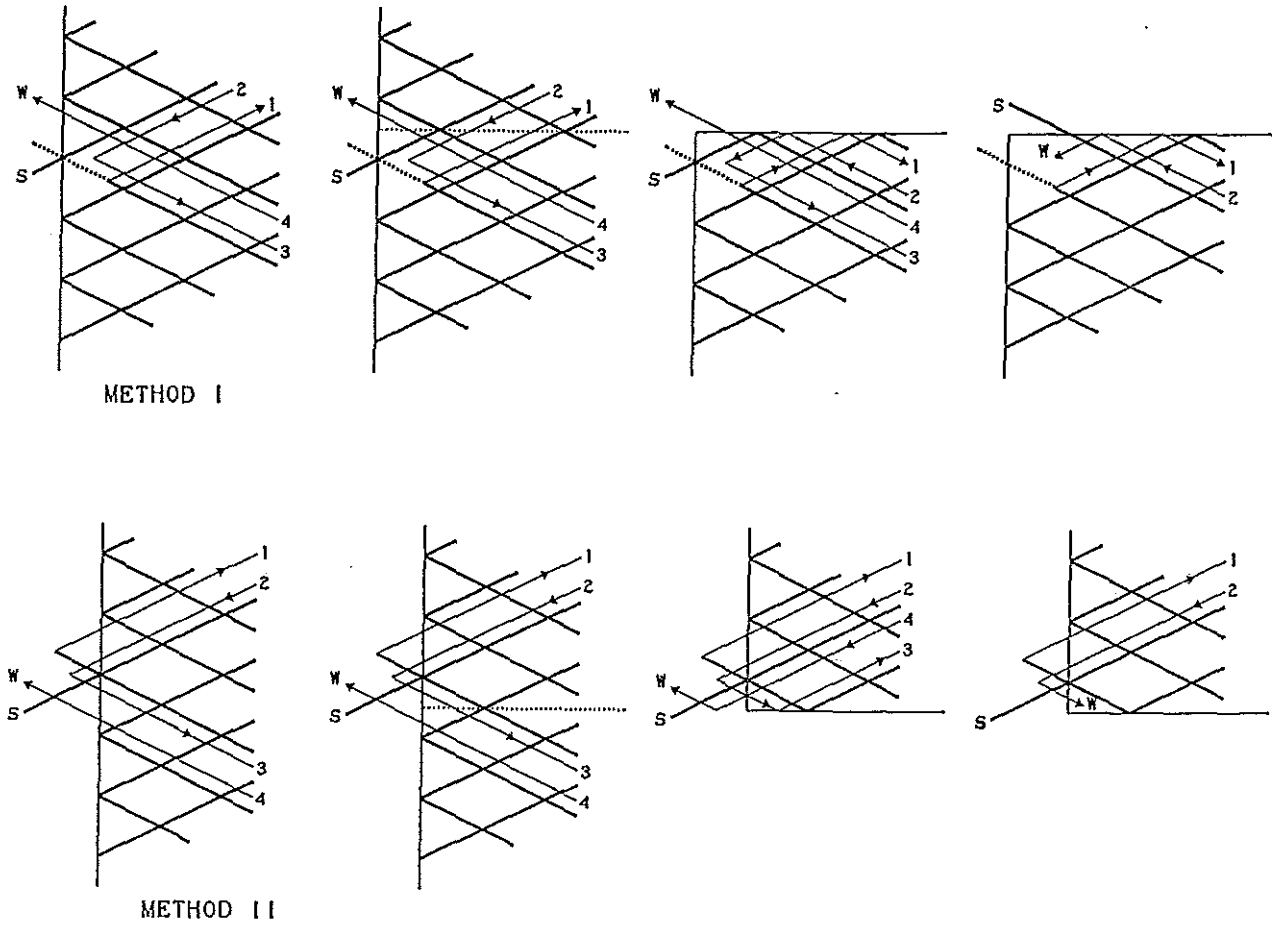


Fig. 211 — The cases $c_I = \Delta_{I,1} b_v$ and $c_{II} = \Delta_{II,1} b_v$.

Let's now continue with the enlargememt from the end of the return adjacent to the Standing End half-cycle to W, and let the number of cycles required for this section of the enlargement be equal to c_I^* for Method I, and be equal to c_{II}^* for Method II. Then with the aid of the diagrams in Fig. 212 we derive the following:

Method I Enlargement:

$$\left|c_I^* \cdot 4b_h\right|_{4b_v} = 2b_v - y + \frac{1}{2} + 2b_v - y + \frac{3}{2} = 4b_v - 2y + 2$$

$$4\left|c_I^* \cdot b_h\right|_{b_v} = 4b_v - 2y + 2$$

$$\left|c_I^* \cdot b_h\right|_{b_v} = b_v - \frac{y-1}{2}.$$

Hence $c_I^* = b_v - c_{II}$ since b_v and b_h are coprime.

For the complete enlargement from S to W we obtain:

$$\left|(c_I + c_I^*) b_h\right|_{b_v} = |b_v + 1|_{b_v} = 1 \quad \text{hence}$$

$$c_I + c_I^* = \frac{m_I b_v + 1}{b_h}.$$

Since b_h and b_v of the Regular Rectangular Mat are equivalent to respectively p and b of its associated Regular Knot, it follows that $(c_I + c_I^*) = \Delta_I b_v$ and m_I of the Regular Rectangular Mat are equivalent to respectively $\Delta_I b$ and $\Delta_I p$ of its associated Regular Knot. Hence $m_I = \Delta_I b_h$.

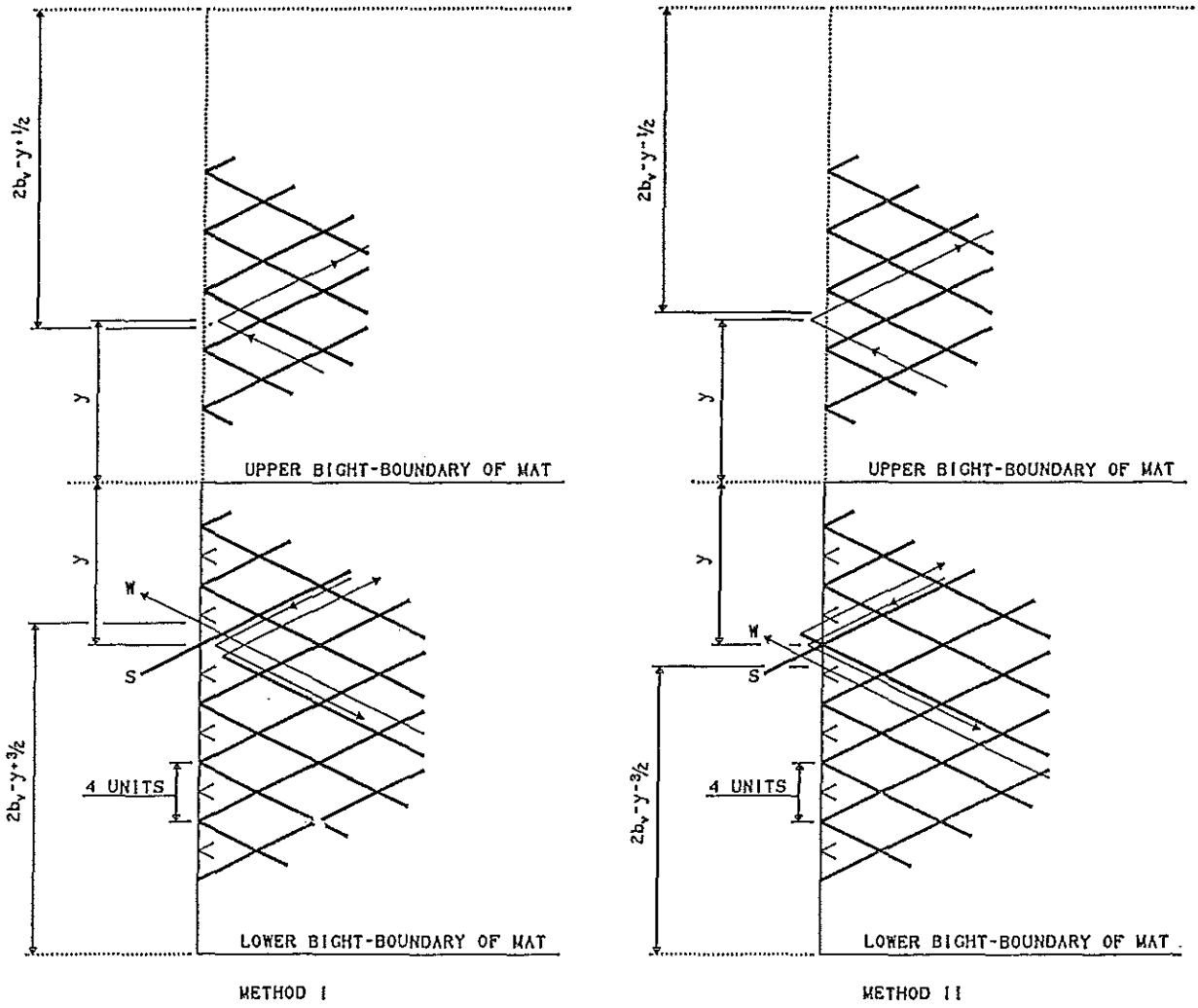


Fig. 212 — Essential string-run parts associated with Methods I and II enlargements.

Method II Enlargement:

$$\left|c_{II}^* \cdot 4b_h\right|_{4b_v} = 2b_v - y - \frac{1}{2} + 2b_v - y - \frac{3}{2} = 4b_v - 2y - 2$$

$$4\left|c_{II}^* \cdot b_h\right|_{b_v} = 4b_v - 2y - 2$$

$$\left|c_{II}^* \cdot b_h\right|_{b_v} = b_v - \frac{y+1}{2}.$$

Hence $c_{II}^* = b_v - c_I$ since b_v and b_h are coprime.

For the complete enlargement from S to W we obtain:

$$\left|(c_{II} + c_{II}^*)b_h\right|_{b_v} = b_v - 1 \quad \text{hence}$$

$$c_{II} + c_{II}^* = \frac{m_{II}b_v - 1}{b_h}.$$

Since b_h and b_v of the Regular Rectangular Mat are equivalent to respectively p and b of its associated Regular Knot, it follows that $(c_{II} + c^*_{II}) = \Delta_{II} b_v$ and m_{II} of the Regular Rectangular Mat are equivalent to respectively $\Delta_{II} b$ and $\Delta_{II} p$ of its associated Regular Knot. Hence $m_{II} = \Delta_{II} b_h$.

The relationships $c^*_I = b_v - c_{II}$ and $c^*_{II} = b_v - c_I$ are **complementary** relationships **between** the enlargement methods I and II.

Since $(c_I + c^*_I) = \Delta_I b_v$ it follows that $\Delta_I b_v = \Delta_{I,1} b_v$ when $c_I < \Delta_{I,1} b_v$.
Since $c_I = b_v - c^*_{II}$ and $c^*_I = b_v - c_{II}$ it follows that $(c_I + c^*_I) = 2b_v - (c_{II} + c^*_{II})$, and thus $(c_{II} + c^*_{II}) = 2b_v - \Delta_I b_v$. Consequently $(c_{II} + c^*_{II}) = \Delta_{II} b_v$, and hence $\Delta_{II} b_v = 2b_v - \Delta_I b_v$. When $\Delta_I b_v = \Delta_{I,1} b_v$, then $b_v - \Delta_{I,1} b_v = \Delta_{II,1} b_v$ (see Fig. 213), and hence $\Delta_{II} b_v = 2b_v - \Delta_I b_v = b_v + [b_v - \Delta_{I,1} b_v] = b_v + \Delta_{II,1} b_v = \Delta_{II,2} b_v$. When $\Delta_I b_v = \Delta_{I,2} b_v = b_v + \Delta_{I,1} b_v$, then $\Delta_{II} b_v = 2b_v - \Delta_I b_v = 2b_v - [b_v + \Delta_{I,1} b_v] = b_v - \Delta_{I,1} b_v = \Delta_{II,1} b_v$.

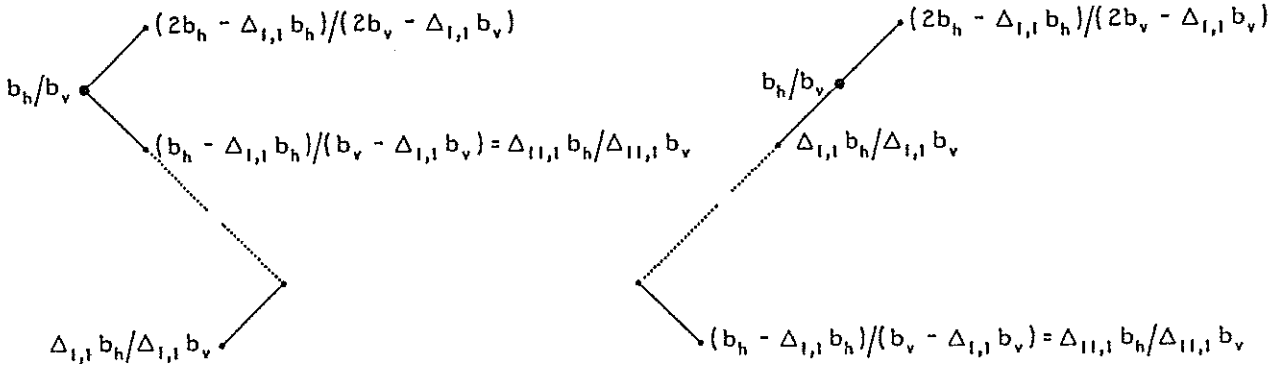

(2b_h - Δ_{I,1} b_h)/(2b_v - Δ_{I,1} b_v)
b_h/b_v
(b_h - Δ_{I,1} b_h)/(b_v - Δ_{I,1} b_v) = Δ_{II,1} b_h/Δ_{II,1} b_v
Δ_{I,1} b_h/Δ_{I,1} b_v
(2b_h - Δ_{I,1} b_h)/(2b_v - Δ_{I,1} b_v)
b_h/b_v
Δ_{I,1} b_h/Δ_{I,1} b_v
(b_h - Δ_{I,1} b_h)/(b_v - Δ_{I,1} b_v) = Δ_{II,1} b_h/Δ_{II,1} b_v

Fig. 213 — Enlargement paths in the RKT.

The relationships "when $\Delta_I b_v = \Delta_{I,1} b_v$, then $\Delta_{II} b_v = \Delta_{II,2} b_v$" (and vice versa), and "when $\Delta_I b_v = \Delta_{I,2} b_v$, then $\Delta_{II} b_v = \Delta_{II,1} b_v$" (and vice versa) form another set of **complementary** relationships **between** the enlargement methods I and II.

Let's now have a look at a **complementary** relationship which operates **within** each enlargement method. We shall investigate identical Method enlargements associated with bight-points on the left bight-boundary which are at equal distance from the horizontal centre line of the Regular Rectangular Mat. We shall number the bight-points on the left bight-boundary in accordance with the convention depicted in Fig. 214.

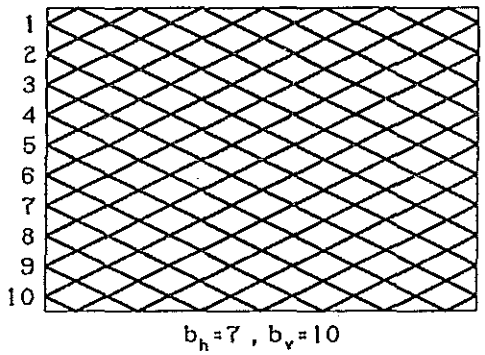


Fig. 214 — Numbering convention for the bight-points on left bight-boundary.

Method I Enlargement:

For bight-point x, hence $y = 2x - 1$, we obtain:

$$\left|c_{I_x} \cdot b_h\right|_{b_v} = \frac{y+1}{2} = \frac{2x-1+1}{2} = x\,.$$

For bight-point $(b_v + 1 - x)$, hence $y = 2b_v + 2 - 2x - 1 = 2b_v + 1 - 2x$, we obtain:

$$\left|c_{I_{b_v+1-x}} \cdot b_h\right|_{b_v} = \frac{y+1}{2} = \frac{2b_v+1-2x+1}{2} = b_v + 1 - x\,.$$

Hence:

$$\left|(c_{I_{b_v+1-x}} + c_{I_x}) \cdot b_h\right|_{b_v} = |b_v + 1|_{b_v} = 1\,.$$

$$c_{I_{b_v+1-x}} + c_{I_x} = \frac{mb_v + 1}{b_h}\,.$$

$$c_{I_{b_v+1-x}} + c_{I_x} = \Delta_I b_v\,.$$

From this complementary relationship we can derive the relationship concerning the order of the Method I enlargement associated with bight-point x and bight-point $(b_v + 1 - x)$ respectively:

When $c_{I_x} < \Delta_{I,1} b_v$, then the lowest possible value for $\Delta_I b_v$ is equal to $\Delta_{I,1} b_v$, and consequently the lowest possible value for $c_{I_{b_v+1-x}}$ is less than $\Delta_{I,1} b_v$. Thus when the enlargement Method I associated with bight-point x is of the 1st-order, then also the enlargement Method I associated with bight-point $(b_v + 1 - x)$ is of the 1st-order (and vice versa). Consequently when $c_{I_x} > \Delta_{I,1} b_v$, hence when the enlargement Method I associated with bight-point x is of the 2nd-order, then also the enlargement Method I associated with bight-point $(b_v + 1 - x)$ is of the 2nd-order (and vice versa).

When $c_{I_x} = \Delta_{I,1} b_v$, then $\Delta_I b_v$ must be equal to $\Delta_{I,2} b_v = b_v + \Delta_{I,1} b_v$, since $c_{I_{b_v+1-x}}$ must be greater than zero. Consequently $c_{I_{b_v+1-x}} = b_v$. Since $b_v > \Delta_{I,1} b_v$, hence $c_{I_{b_v+1-x}} > \Delta_{I,1} b_v$, it follows that the Method I enlargement associated with bight-point $(b_v + 1 - x)$ is of the 2nd-order. The Method I enlargement associated with bight-point x is of the 1st-order. This case arises only when $x = 1$, as we have already seen earlier.

Method II Enlargement:

For bight-point x, hence $y = 2x - 1$, we obtain:

$$\left|c_{II_x} \cdot b_h\right|_{b_v} = \frac{y-1}{2} = \frac{2x-1-1}{2} = x - 1\,.$$

For bight-point $(b_v + 1 - x)$, hence $y = 2b_v + 2 - 2x - 1 = 2b_v + 1 - 2x$, we obtain:

$$\left|c_{II_{b_v+1-x}} \cdot b_h\right|_{b_v} = \frac{y-1}{2} = \frac{2b_v+1-2x-1}{2} = b_v - x\,.$$

Hence:

$$\left|(c_{II_{b_v+1-x}} + c_{II_x}) \cdot b_h\right|_{b_v} = b_v - 1\,.$$

$$c_{II_{b_v+1-x}} + c_{II_x} = \frac{mb_v - 1}{b_h}\,.$$

$$c_{II_{b_v+1-x}} + c_{II_x} = \Delta_{II} b_v\,.$$

From this complementary relationship we can derive the relationship concerning the order of the Method II enlargement associated with bight-point x and bight-point

$(b_v + 1 - x)$ in exactly the same manner as for the Method I enlargements. Hence when the enlargement Method II associated with bight-point x is of the 1st-order, then also the enlargement Method II associated with bight-point $(b_v + 1 - x)$ is of the 1st-order (and vice versa). Consequently when $c_{I_x} > \Delta_{I,1} b_v$, hence when the enlargement Method II associated with bight-point x is of the 2nd-order, then also the enlargement Method II associated with bight-point $(b_v + 1 - x)$ is of the 2nd-order (and vice versa).

When $c_{II_x} = \Delta_{II,1} b_v$, then $\Delta_{II} b_v$ must be equal to $\Delta_{II,2} b_v = b_v + \Delta_{II,1} b_v$, since $c_{II_{b_v+1-x}}$ must be greater than zero. Consequently $c_{II_{b_v+1-x}} = b_v$. Since $b_v > \Delta_{II,1} b_v$, hence $c_{II_{b_v+1-x}} > \Delta_{II,1} b_v$, it follows that the Method II enlargement associated with bight-point $(b_v + 1 - x)$ is of the 2nd-order. The Method II enlargement associated with bight-point x is of the 1st-order. This case arises only when $x = b_v$, as we have already seen earlier.

The bight-points $x = 1$ and $x = b_v$ are thus very special. Not only are these bight-points associated with the consistent enlargements (in which case the braiding directions for the respective enlargement Methods I and II associated with the same bight-point are opposite), but for the same braiding direction one of the enlargement methods is of the 1st-order while the other is of the 2nd-order. The 2nd-order enlargement is quite special in so far that $c = b_v$, hence that first b_v cycles are laid down followed by the cycles of the fundamental enlargement.

By now the reader should be well conversant with the role which the 1st-order and the 2nd-order Method I and Method II enlargements play. Although the notations:

$\Delta_{I,1} b_v$ for the 1st-order Method I Enlargement,

$\Delta_{II,1} b_v$ for the 1st-order Method II Enlargement,

$\Delta_{I,2} b_v$ for the 2nd-order Method I Enlargement, and

$\Delta_{II,2} b_v$ for the 2nd-order Method II Enlargement,

clearly indicate in the various expressions the role which these enlargements play, the notation is somewhat cumbersome for practical use. We have already met another notation for these 1st-order enlargements which is much more convenient for practical use, and consequently a more convenient notation for these 2nd-order enlargements is available also. Recall from Issue No. 6, pp. 114-115 the "Δ" and the "Δ^*". Some readers might have noticed that for Regular Knots $\Delta = \Delta_{I,1} b$ and $\Delta^* = \Delta_{II,1} b$.

$\star\star$ *Prove these relationships.*

Thus for Regular Rectangular Mats we obtain:

$\Delta = \Delta_{I,1} b_v$,

$\Delta = \dfrac{n b_v + 1}{|b_h|_{b_v}}$ where n is the smallest positive integer such that Δ is an integer.

$\Delta^* = \Delta_{II,1} b_v$,

$\Delta^* = \dfrac{n^* b_v - 1}{|b_h|_{b_v}}$ where n^* is the smallest positive integer such that Δ^* is an integer.

Consequently:

$$\Delta_{I,2} b_v = \Delta + b_v .$$

$$\Delta_{II,2} b_v = \Delta^* + b_v .$$

There is of course no need to calculate in this way both Δ and Δ^* since $\Delta + \Delta^* = b_v$, as we have seen in Issue No. 7, pg. 133. See also above, pg. 227.

Let's look at four more important relationships:

$$\left|c_{I_x} \cdot b_h\right|_{b_v} = \frac{y+1}{2}.$$

$$\left|c_{II_x} \cdot b_h\right|_{b_v} = \frac{y-1}{2}.$$

Hence

$$\left|c_{I_x} - c_{II_x}\right|_{b_v} = \Delta.$$

$$\left|c_{II_x} - c_{I_x}\right|_{b_v} = \Delta^*.$$

These last two relationships follow also directly from the following relationships on pp. 226 and 227: $(c_I + c_I^*) = \Delta_I b_v$ and $(c_{II} + c_{II}^*) = \Delta_{II} b_v$

Method I Enlargement:

For bight-point x we have $y_x = 2x - 1$, and hence

$$\left|c_{I_x} \cdot b_h\right|_{b_v} = \frac{2x-1+1}{2} = x.$$

For bight-point $(x+1)$ we have $y_{x+1} = 2x + 1$, and hence

$$\left|c_{I_{x+1}} \cdot b_h\right|_{b_v} = \frac{2x+1+1}{2} = x+1.$$

Thus

$$c_{I_{x+1}} = \left|c_{I_x} + \Delta\right|_{b_v}.$$

Method II Enlargement:

For bight-point x we have $y_x = 2x - 1$, and hence

$$\left|c_{II_x} \cdot b_h\right|_{b_v} = \frac{2x-1-1}{2} = x-1.$$

For bight-point $(x+1)$ we have $y_{x+1} = 2x + 1$, and hence

$$\left|c_{II_{x+1}} \cdot b_h\right|_{b_v} = \frac{2x+1-1}{2} = x.$$

Thus

$$c_{II_{x+1}} = \left|c_{II_x} - \Delta^*\right|_{b_v}.$$

For the Methods I and II enlargements we can now readily calculate with these formulae the c-values for the consecutive bight-points, since we know that bight-point 1 is always associated with the following c-values:

For Method I: $c_{I_1} = \Delta$.
For Method II: $c_{II_1} = b_v$.

Example 1:

Let $b_h = 11$ and $b_v = 8$. Then:

$$\Delta = \frac{nb_v + 1}{|b_h|_{b_v}} = \frac{8n+1}{|11|_8} = \frac{8n+1}{3} = 3 \qquad (n = 1).$$

$$\Delta^* = \frac{n^* b_v - 1}{|b_h|_{b_v}} = \frac{8n^* - 1}{|11|_8} = \frac{8n^* - 1}{3} = 5 \qquad (n^* = 2).$$

- bight-point 1 $\longrightarrow$ $c_{I_1} = 3$.
  bight-point 2 $\longrightarrow$ $c_{I_2} = 6$.
- bight-point 3 $\longrightarrow$ $c_{I_3} = 1$.

  bight-point 1 $\longrightarrow$ $c_{II_1} = 8$.
- bight-point 2 $\longrightarrow$ $c_{II_2} = 3$.
  bight-point 3 $\longrightarrow$ $c_{II_3} = 6$.

bight-point 4 $\longrightarrow$	$c_{I_4} = 4$.	• bight-point 4 $\longrightarrow$	$c_{II_4} = 1$.
bight-point 5 $\longrightarrow$	$c_{I_5} = 7$.	• bight-point 5 $\longrightarrow$	$c_{II_5} = 4$.
• bight-point 6 $\longrightarrow$	$c_{I_6} = 2$.	bight-point 6 $\longrightarrow$	$c_{II_6} = 7$.
bight-point 7 $\longrightarrow$	$c_{I_7} = 5$.	• bight-point 7 $\longrightarrow$	$c_{II_7} = 2$.
bight-point 8 $\longrightarrow$	$c_{I_8} = 8$.	• bight-point 8 $\longrightarrow$	$c_{II_8} = 5$.

The entries preceded by a bullet are 1st-order enlargements, the others are 2nd-order enlargements. A 1st-order Method I enlargement leads to $b_h = 15$, $b_v = 11$. A 2nd-order Method I enlargement leads to $b_h = 26$, $b_v = 19$. A 1st-order Method II enlargement leads to $b_h = 18$, $b_v = 13$. A 2nd-order Method II enlargement leads to $b_h = 29$, $b_v = 21$.

Example 2:

Let $b_h = 7$ and $b_v = 9$. Then:

$$\Delta = \frac{nb_v + 1}{|b_h|_{b_v}} = \frac{9n+1}{|7|_9} = \frac{9n+1}{7} = 4 \qquad (n = 3).$$

$$\Delta^* = \frac{n^* b_v - 1}{|b_h|_{b_v}} = \frac{9n^* - 1}{|7|_9} = \frac{9n^* - 1}{7} = 5 \qquad (n^* = 4).$$

• bight-point 1 $\longrightarrow$	$c_{I_1} = 4$.	bight-point 1 $\longrightarrow$	$c_{II_1} = 9$.
bight-point 2 $\longrightarrow$	$c_{I_2} = 8$.	• bight-point 2 $\longrightarrow$	$c_{II_2} = 4$.
• bight-point 3 $\longrightarrow$	$c_{I_3} = 3$.	bight-point 3 $\longrightarrow$	$c_{II_3} = 8$.
bight-point 4 $\longrightarrow$	$c_{I_4} = 7$.	• bight-point 4 $\longrightarrow$	$c_{II_4} = 3$.
• bight-point 5 $\longrightarrow$	$c_{I_5} = 2$.	bight-point 5 $\longrightarrow$	$c_{II_5} = 7$.
bight-point 6 $\longrightarrow$	$c_{I_6} = 6$.	• bight-point 6 $\longrightarrow$	$c_{II_6} = 2$.
• bight-point 7 $\longrightarrow$	$c_{I_7} = 1$.	bight-point 7 $\longrightarrow$	$c_{II_7} = 6$.
bight-point 8 $\longrightarrow$	$c_{I_8} = 5$.	• bight-point 8 $\longrightarrow$	$c_{II_8} = 1$.
bight-point 9 $\longrightarrow$	$c_{I_9} = 9$.	• bight-point 9 $\longrightarrow$	$c_{II_9} = 5$.

The entries preceded by a bullet are again 1st-order enlargements, the others are 2nd-order enlargements. A 1st-order Method I enlargement leads to $b_h = 10$, $b_v = 13$. A 2nd-order Method I enlargement leads to $b_h = 17$, $b_v = 22$. A 1st-order Method II enlargement leads to $b_h = 11$, $b_v = 14$. A 2nd-order Method II enlargement leads to $b_h = 18$, $b_v = 23$.

Example 3:

Let $b_h = 7$ and $b_v = 6$. Then:

$$\Delta = \frac{nb_v + 1}{|b_h|_{b_v}} = \frac{6n+1}{|7|_6} = \frac{6n+1}{1} = 1 \qquad (n = 0).$$

$$\Delta^* = \frac{n^* b_v - 1}{|b_h|_{b_v}} = \frac{6n^* - 1}{|7|_6} = \frac{6n^* - 1}{1} = 5 \qquad (n^* = 1).$$

• bight-point 1 $\longrightarrow$	$c_{I_1} = 1$.	bight-point 1 $\longrightarrow$	$c_{II_1} = 6$.
bight-point 2 $\longrightarrow$	$c_{I_2} = 2$.	• bight-point 2 $\longrightarrow$	$c_{II_2} = 1$.
bight-point 3 $\longrightarrow$	$c_{I_3} = 3$.	• bight-point 3 $\longrightarrow$	$c_{II_3} = 2$.
bight-point 4 $\longrightarrow$	$c_{I_4} = 4$.	• bight-point 4 $\longrightarrow$	$c_{II_4} = 3$.
bight-point 5 $\longrightarrow$	$c_{I_5} = 5$.	• bight-point 5 $\longrightarrow$	$c_{II_5} = 4$.
bight-point 6 $\longrightarrow$	$c_{I_6} = 6$.	• bight-point 6 $\longrightarrow$	$c_{II_6} = 5$.

The entries preceded by a bullet are 1st-order enlargements, the others are 2nd-order enlargements. A 1st-order Method I enlargement leads to $b_h = 8$, $b_v = 7$. A

2nd-order Method I enlargement leads to $b_h = 15$, $b_v = 13$. A 1st-order Method II enlargement leads to $b_h = 13$, $b_v = 11$. A 2nd-order Method II enlargement leads to $b_h = 20$, $b_v = 17$.

$\star\star$ *Calculate the paths of the enlargements for Examples 1,2 and 3 (see Issue No. 6, pg. 110 and Issue No. 7, pg. 135).*

Not only have we seen so far the close association and the "intertwined" relationship between Regular Knots and Regular Rectangular Mats, but we have "discovered" important relationships in the life of Regular Rectangular Mats. These relationships provided us with a simple practical method by which we can determine whether a smallest possible enlargement associated with a given bight-point is of the 1st-order or of the 2nd-order. However we are not able yet to answer some other very important questions regarding enlargements. Since self-discovery is the best way of learning, let's pose these "unsolved" questions in order to encourage the reader along the road of discovery.

$\star\star$ *Say we give a Regular Rectangular Mat an enlargement which, for the Enlargement Method employed, is the smallest possible for the bight-point concerned. Let's give the resulting enlarged mat a further enlargement (not necessarily of the same Method). What can in general be said with regards the order of this further enlargement. When we want to braid a Regular Rectangular Mat, what can be said about braiding it by means of enlargements?*†

(Regular) Band Knots

There are many occasions where single string large diameter knots are required or preferred. Such knots have essentially very many bights and relatively few parts. They can, for instance, be used as woven hat-bands.

To braid such large diameter knots is generally quite cumbersome since it is often difficult to maintain unaided their form during the early braiding stages. There are, however, two sets of Regular Knots which are easy to braid in large diameters. One is based on the 3-parts Casa-coded Regular Knots (single string), and the other on the 2-parts Column-coded Regular Knots (single string).

Large diameter 2-parts (single string) Column-coded Regular Knots are nothing but "closed" overhand knots which can readily, and most conveniently, be enlarged to 4-parts Casa-coded Regular Knots by means of the so-called **Nilsen** or **Masurel algorithm** (see Issue No. 8, pg.166).

Large diameter 3-parts single string Casa Knots can readily be braided by using the method of *"Raising the number of bights without raising the number of parts"* (see issue No. 8, pg. 156; the 1996 Appendix, pg. *i*; Fig. 215 below). The upper row of diagrams in Fig. 215 depicts the bights raising process for the 3-parts Casa-coded Regular Knots with $(3n+1)$ bights $(n = 1, 2, 3, \cdots)$, and the lower row of diagrams in Fig. 215 depicts the bights raising process for the 3-parts Casa-coded Regular Knots with $(3n+2)$ bights $(n = 1, 2, 3, \cdots)$.

In this article we shall limit our attention to a very select set of the **Regular Knots** which are obtained from the 3-parts Casa-coded Regular Knots and which have either a **Spanish ring** or **Gaucho** coding.

† Most conclusions can now be arrived at without the aid of any mathematics!!!

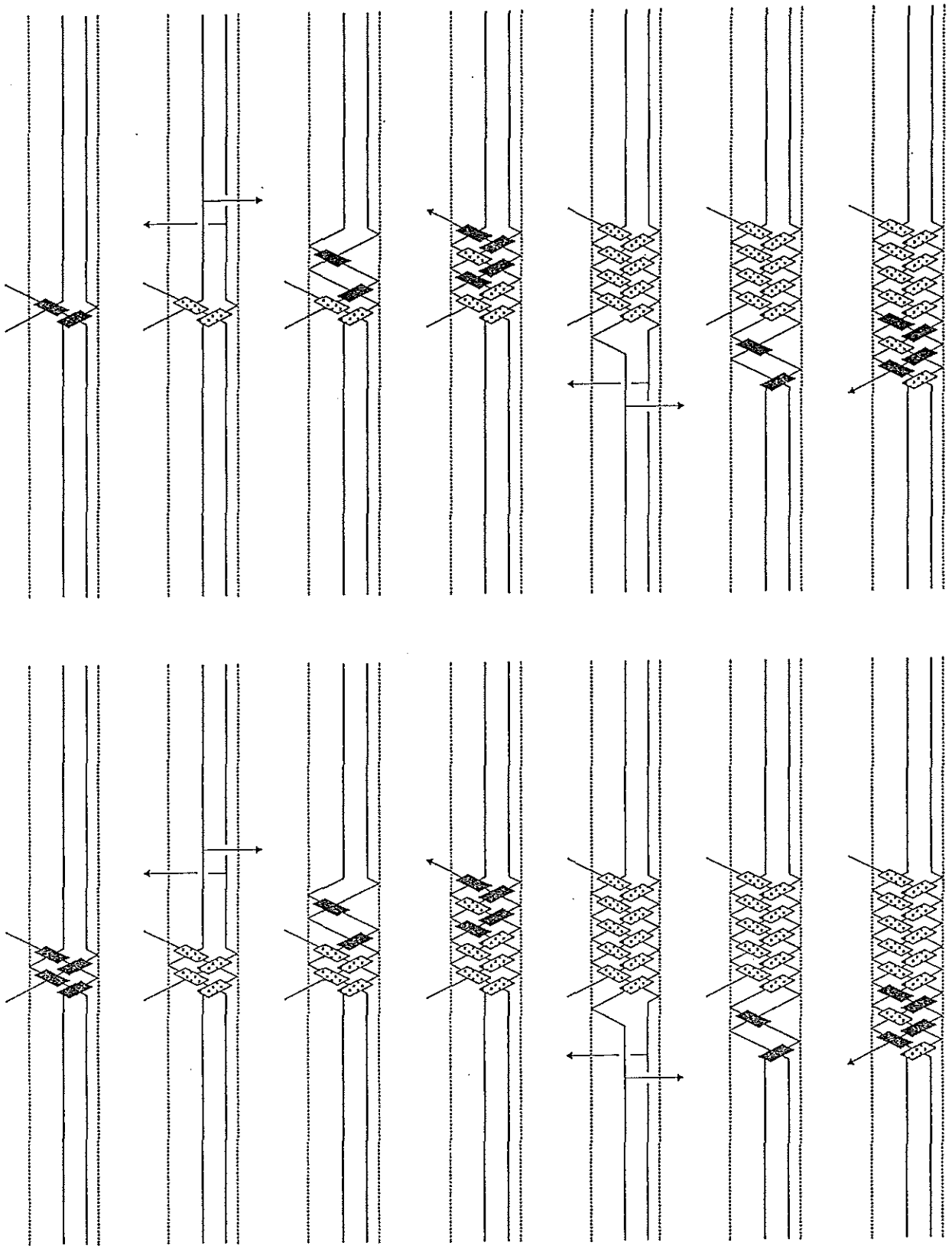


Fig. 215 — Increasing the bights in 3-parts Casa-coded Regular Knots.

First let's define the terms we shall be using.

A section of a **4-pass Gaucho knot** has been depicted in Fig. 216. The **basic unit** of columns consists of two sets of columns, each set having the **same number** of **identically coded** columns; this number we shall indicate by α, which can be any **natural number**†. Hence $\alpha = 4$ in Fig. 216. The number of basic units is indicated by m, which can also be any **natural number**. Hence $m = 3$ in Fig. 216. Thus the number of parts of a Gaucho knot is equal to $(2m \cdot \alpha + 1)$. The reader should observe that the smallest possible value for α is equal to 1. Such knots are normally not called Gaucho knots since they belong to the Casa Knots; it are the Casa-coded Regular Knots with an odd number of parts. Observe that when $\alpha = 1$ and $m = 1$ we have the 3-parts Casa-coded Regular Knots. The smallest possible value for m is 1. Gaucho knots for which $m = 1$ are called **Spanish Ring Knots**. Thus the term **Spanish Ring Knot** is normally restricted to knots with $\alpha \geq 2$ and $m = 1$, while the term **Gaucho Knot** is normally restricted to knots with $\alpha \geq 2$ and $m \geq 2$.

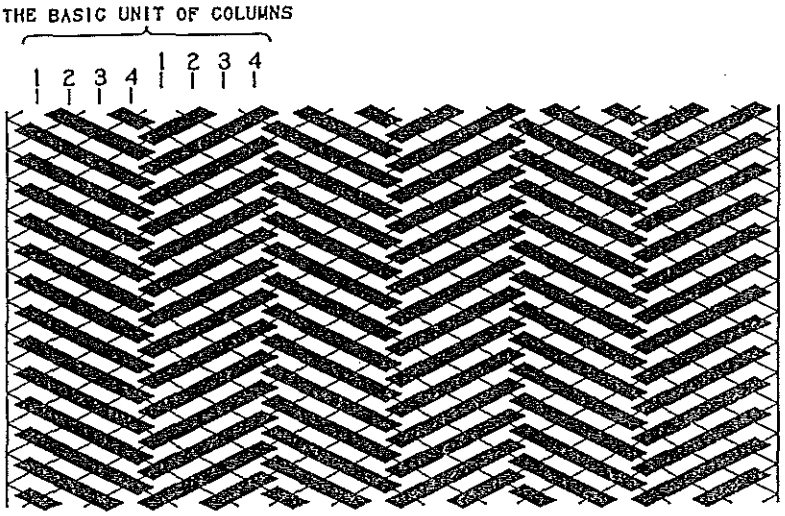


Fig. 216 — A section of a Gaucho Knot with $\alpha = 4$ and $m = 3$.

The very select set of **Regular Knots** which we shall discuss in this article and which are obtained from 3-parts Casa-coded Regular Knots, have either an α-pass **Spanish ring** or an α-pass **Gaucho** coding that can readily be enlarged to an $(\alpha + 1)$-pass. We can divide this select set of Regular Knots into two sub-sets:

One sub-set whose α-pass members have a number of parts equal to $[2m\alpha + 1]$, and a number of bights equal to $[(2m\alpha + 1)N + \alpha]$, while the associated $(\alpha + 1)$-pass members have a number of parts equal to $[2m(\alpha + 1) + 1]$, and a number of bights equal to $[\{2m(\alpha + 1) + 1\}N + (\alpha + 1)]$, where $\alpha \geq 1$, $m \geq 1$, $N \geq 1$. This sub-set is associated with a Method I Enlargement between the α-pass and $(\alpha + 1)$-pass knots. Note that for $\alpha = 1$ the α-pass members are Casa-coded Regular Knots with $[2m + 1]$ parts and $[(2m + 1)N + 1]$ bights; thus for $m = 1$ the Casa-coded Reguar Knots have 3 parts and $(3N + 1)$ bights. Their path in the RKT is as depicted by the left-hand diagram in Fig. 217. If such a 3-parts/$(3N+1)$-bights Casa-coded Regular Knot receives $2(m-1) = (2m-2)$ consecutive Method II enlargements, we can maintain a Casa-coding and the final result will thus be a Casa-coded Regular Knot with $[3+(2m-2)] = [2m+1]$ parts and $[3N + 1 + (2m - 2)N] = [(2m + 1)N + 1]$ bights. The path in the RKT is

† The natual numbers are the numbers $1, 2, 3, \cdots$.

depicted by the right-hand diagram in Fig. 217.

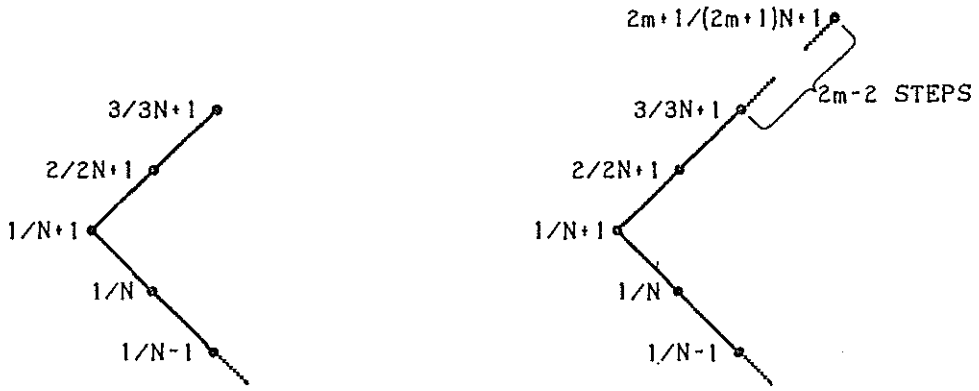


Fig. 217 — Paths in the RKT. Method II enlargements along final branch.

The other sub-set whose α-pass members have a number of parts equal to $[2m\alpha+1]$, and a number of bights equal to $[(2m\alpha+1)N-\alpha]$, while the associated $(\alpha+1)$-pass members have a number of parts equal to $[2m(\alpha+1)+1]$, and a number of bights equal to $[\{2m(\alpha+1)+1\}N-(\alpha+1)]$, where $\alpha \geq 1$, $m \geq 1$, $N \geq 1$. This sub-set is associated with a Method II Enlargement between the α-pass and $(\alpha+1)$-pass knots. Note that for $\alpha = 1$ the α-pass members are Casa-coded Regular Knots with $[2m+1]$ parts and $[(2m+1)N-1]$ bights; thus for $m = 1$ the Casa-coded Reguar Knots have 3 parts and $(3N-1)$ bights. Their path in the RKT is as depicted by the left-hand diagram in Fig. 218. If such a 3-parts/$(3N-1)$-bights Casa-coded Regular Knot receives $2(m-1) = (2m-2)$ consecutive Method I enlargements, we can maintain a Casa-coding and the final result will thus be a Casa-coded Regular Knot with $[3+(2m-2)] = [2m+1]$ parts and $[3N-1+(2m-2)N] = [(2m+1)N-1]$ bights. The path in the RKT is depicted by the right-hand diagram in Fig. 218.

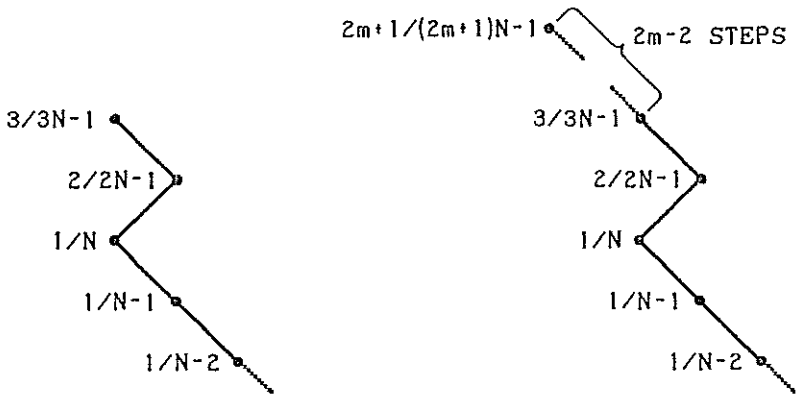


Fig. 218 — Paths in the RKT. Method I enlargements along final branch.

In the *Encyclopedia of Rawhide and Leather Braiding*, Bruce Grant touched upon this select set of knots. On pp. 390-391 he discussed a 2-pass Spanish ring knot, on pp. 392-393 a 3-pass Spanish ring knot, and on pp. 402-403 a 2-pass and 3-pass Gaucho knot. All these knots are however of a small diameter, and braiding a large diameter knot from one end of the string is not a practical proposition. Not only would this require, especially in the earlier stages, an unnecessary amount of time due to feeding a long length of braiding material through the braid at every half-cycle, but it would also expose this braiding material to unnecessary wear. In order to alleviate these problems

we braid large diameter knots from the centre of the string-length by the alternating use of the two string-ends after each revolution of the circumference of the knot. This gradually shortens both string-ends and makes the braiding operation much easier and quicker.

This enlargement process with the alternating use of the string-ends has already been shown in Fig. 215 for the 3-parts Casa-coded Regular Knots. A subsequent enlargement to a 5-parts Casa-coded Regular Knot is shown in Fig. 219: the two left-hand diagrams depict the two consecutive Method I enlargements, and the two right-hand diagrams depict the two consecutive Method II enlargements.

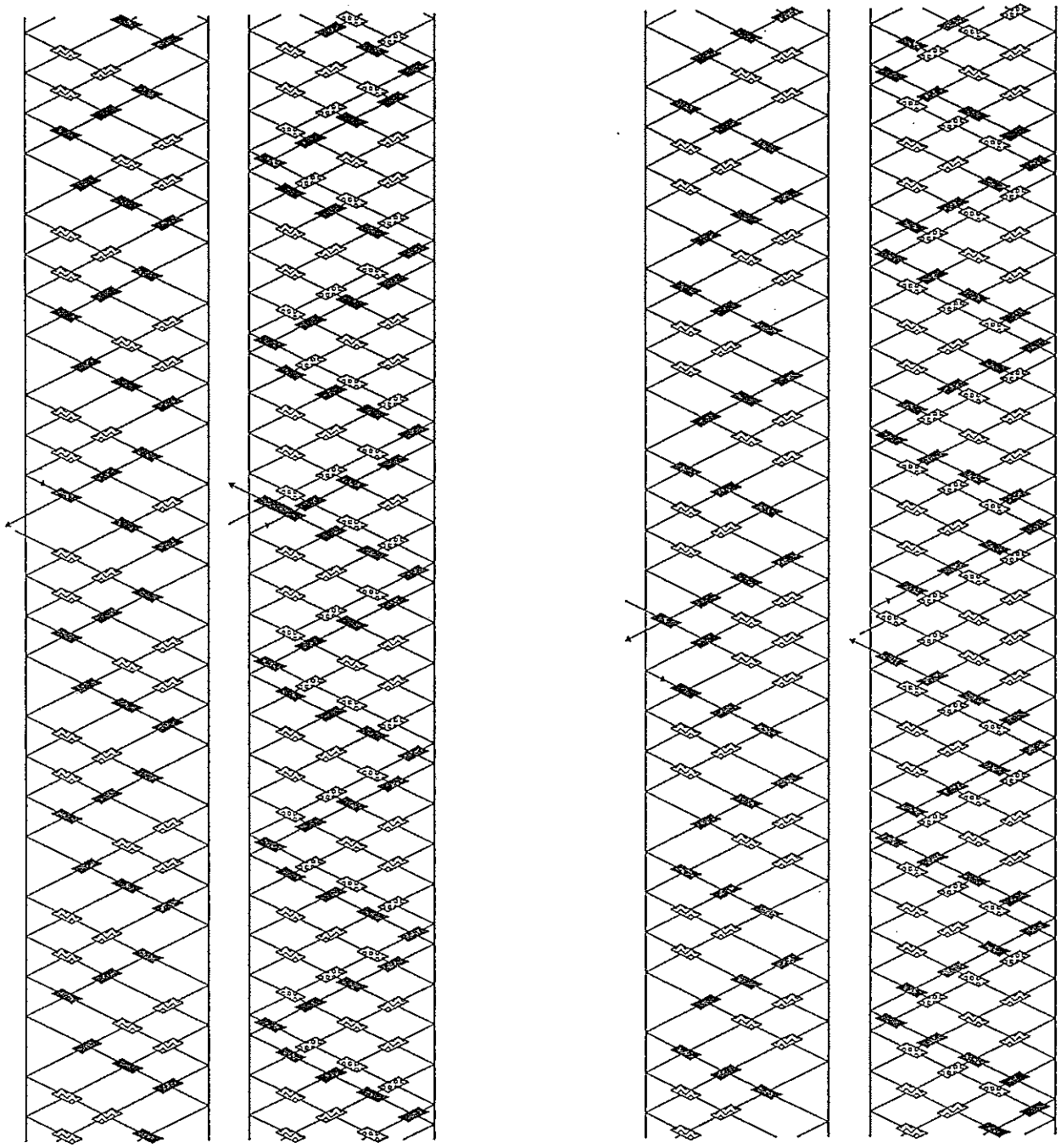


Fig. 219 — Method I enlargement (left) and Method II enlargement (right) from a 3-parts to a 5-parts Casa-coded Regular Knot.

Thus the 5-parts Casa-coded Regular Knot depicted by the second diagram from the left in Fig. 219 has $(5N-1)$ bights, and the 5-parts Casa-coded Regular Knot depicted by the fourth diagram from the left in Fig. 219 has $(5N+1)$ bights.

Fig. 220 depicts the evolution of a 5-parts Casa-coded Regular Knot with $(5N+1)$ bights to a 2-pass 9-parts Gaucho-coded Regular Knot with $(9N+2)$ bights (note that $\alpha = 1$, hence $(\alpha+1) = 2$, and $m = 2$).

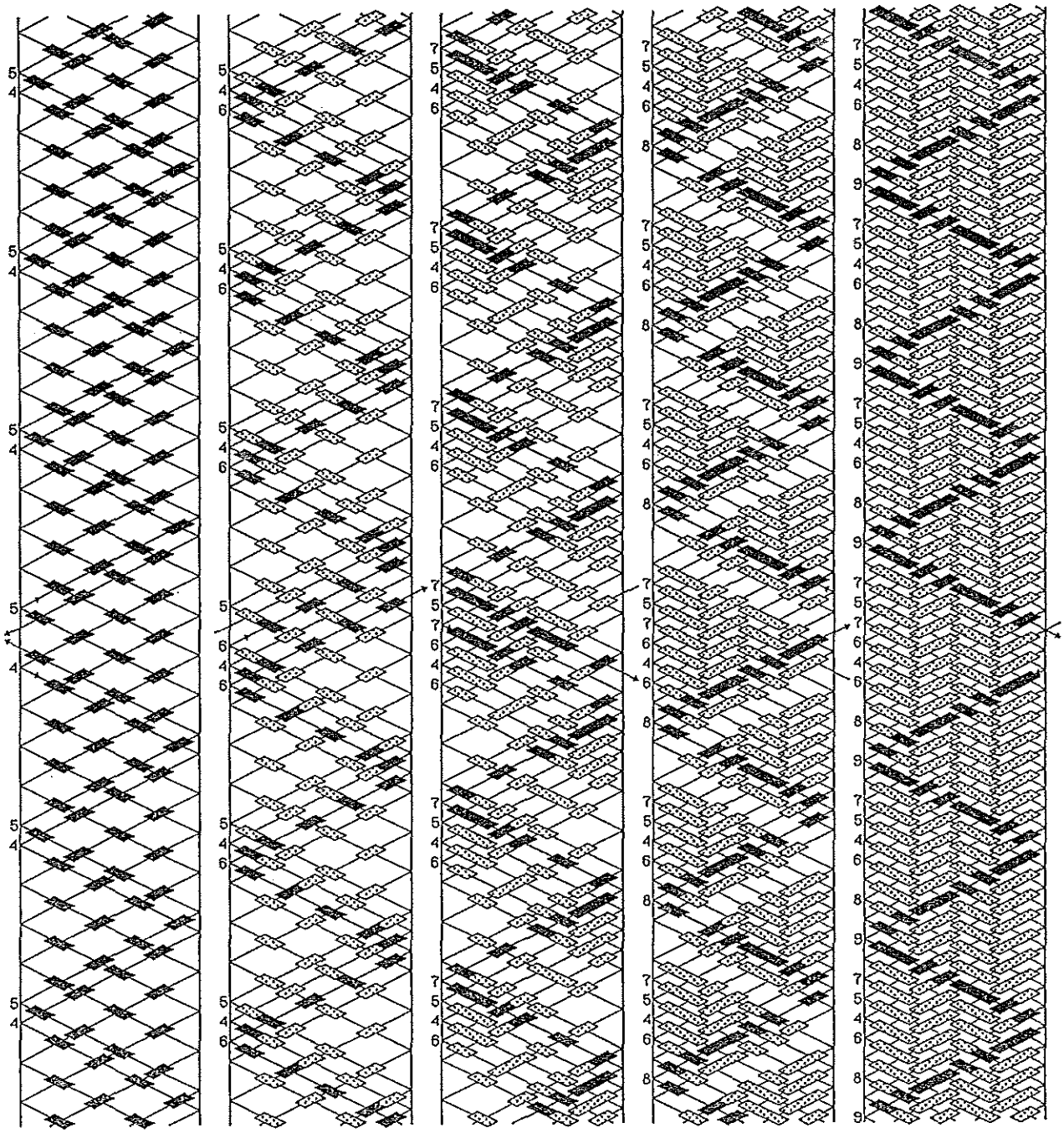


Fig. 220 — Evolution by Method I of a 5-parts/$(5N+1)$-bights Casa Knot to a 2-pass 9-parts/$(9N+2)$-bights Gaucho Knot.

Since this process involves an increase in parts that is equal to $2m = 4$, we have to braid each string-end m times, hence twice around the circumference of the knot.

Let's first look at that part of the string-run, and in particular at the consecutive revolutions from one string-end to the other, of an $(\alpha+1)$-pass $[2m(\alpha+1)+1]$-parts, $[\{2m(\alpha+1)+1\}N+(\alpha+1)]$-bights Gaucho Knot which constitutes the string-run of an α-pass $[2m\alpha+1]$-parts, $[(2m\alpha+1)N+\alpha]$-bights Gaucho Knot in order to see how the additional revolutions associated with the evolution of this knot to the $(\alpha+1)$-pass $[2m(\alpha+1)+1]$-parts, $[\{2m(\alpha+1)+1\}N+(\alpha+1)]$-bights Gaucho Knot will be laid down (see Fig. 221).

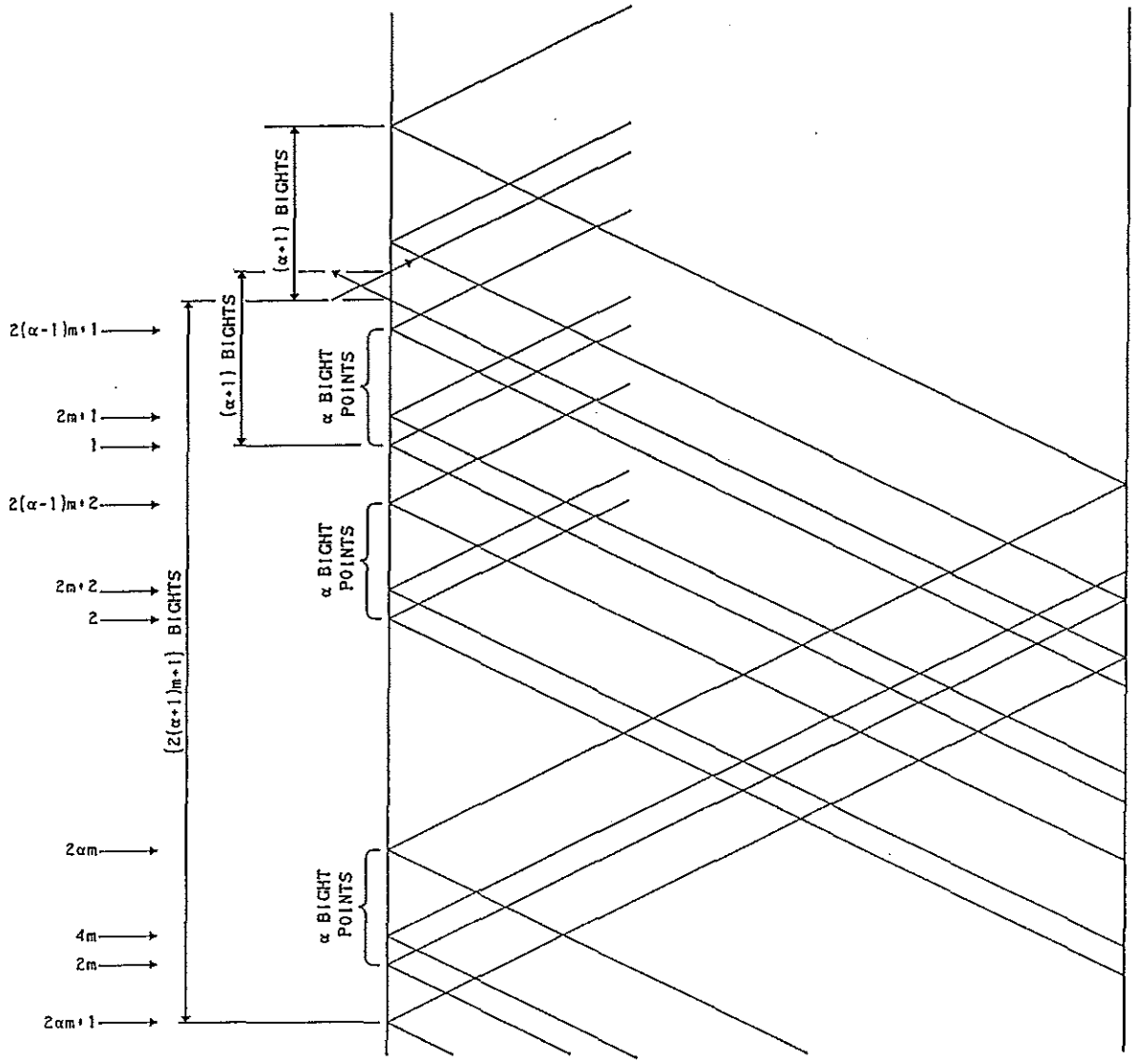


Fig. 221 — A section of the string-run of an α-pass $[2m\alpha+1]$-parts/$[(2m\alpha+1)N+\alpha]$-bights Gaucho Knot.

The first revolution is a distance of $(\alpha+1)$ bights more than N cycles (the end-point of N cycles is indicated by 1). The second revolution is a distance of $2(\alpha+1)$ bights more than $2N$ cycles (the end-point of $2N$ cycles is indicated by 2). And so on. From this string-run a clear picture emerges of the way in which the additional $2m$ revolutions of the string-run associated with the evolution of an α-pass $[2m\alpha+1]$-parts, $[(2m\alpha+1)N+\alpha]$-bights Gaucho Knot to an $(\alpha+1)$-pass $[2m(\alpha+1)+1]$-parts, $[\{2m(\alpha+1)+1\}N+(\alpha+1)]$-bights Gaucho Knot are being laid down. Fig. 222 depicts a section of the string-run of such a Gaucho Knot. The solid lines belong to the α-pass

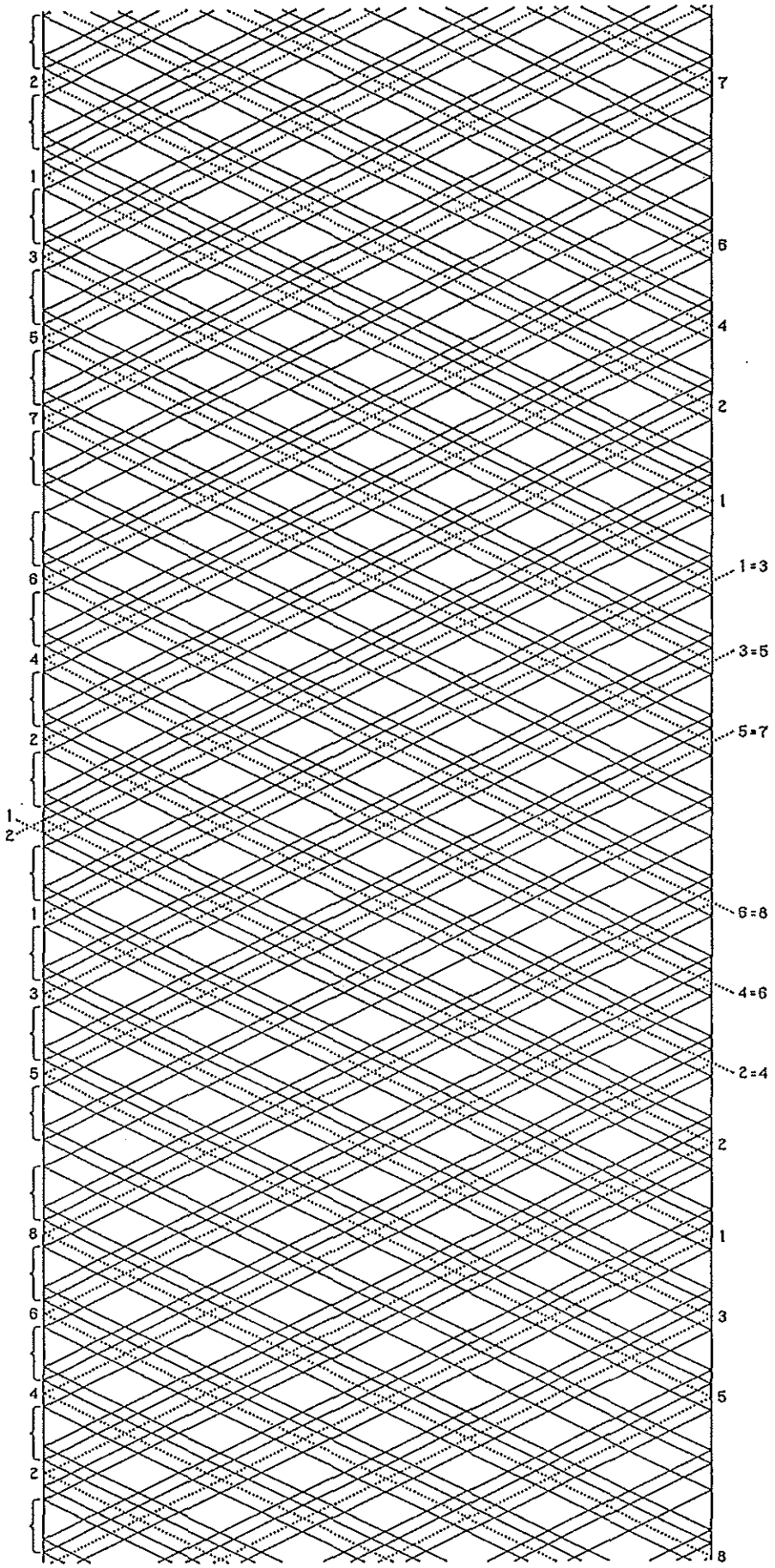


Fig. 222 — A section of a knot as in Fig. 221.

knot, and the dotted lines belong to the additional $2m$ revolutions associated with the evolution of the α-pass knot to the $(\alpha+1)$-pass knot. The numbers indicate the revolutions to which the additional half-cycles belong. Note that an additional $2(2mN+1) = (4mN+2)$ half-cycles are associated with an evolution of an α-pass to an $(\alpha+1)$-pass Gaucho Knot, and that each string-end lays down $(2mN+1)$ half-cycles.

First we lay down the $(2N+1)$ half-cycles marked 1. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down the $(2N+1)$ half-cycles marked 2. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down $2N$ half-cycles marked 3. Next the $2N$ half-cycles marked 4. Then the $2N$ half-cycles marked 5. Then the $2N$ half-cycles marked 6. And so on. Finally we lay down the $2N$ half-cycles marked $2m$ (not shown). Thus we end on the right-hand bight-boundary. For the next evolution from the $(\alpha+1)$-pass to the $(\alpha+2)$-pass we can turn the $(\alpha+1)$-pass knot through 180° in order to get the string-ends on the left-hand bight-boundary again.

If the coding of the first intersection column from the left is \, then the half-cycle algorithms associated with the evolution are:

First half-cycle **1**, from **Left** to **Right**: [repeat $\{\alpha u - \alpha o\}$].

Half-cycles **1**, from **Left** to **Right**: $(\alpha+1)u - \alpha o -$ [repeat $\{\alpha u - \alpha o\}$].

Half-cycles **1**, from **Right** to **Left**: $(\alpha+1)u - \alpha o -$ [repeat $\{\alpha u - \alpha o\}$].

Half-cycles **2**, from **Left** to **Right**: $(\alpha+1)o - (\alpha+1)u -$ [repeat $\{\alpha o - \alpha u\}$].

Last half-cycle **2**, from **Left** to **Right**: $(\alpha+1)o - (\alpha+1)u - (\alpha+1)o - \alpha u -$ [repeat $\{\alpha o - \alpha u\}$].

Half-cycles **2**, from **Right** to **Left**: $(\alpha+1)o - (\alpha+1)u -$ [repeat $\{\alpha o - \alpha u\}$].

Half-cycles **3**, from **Right** to **Left**: $1 \times \{(\alpha+1)u - (\alpha+1)o\} - (\alpha+1)u - \alpha o -$ [repeat $\{\alpha u - \alpha o\}$].

Half-cycles **3**, from **Left** to **Right**: $1 \times \{(\alpha+1)u - (\alpha+1)o\} - (\alpha+1)u - \alpha o -$ [repeat $\{\alpha u - \alpha o\}$].

Last half-cycle **3**, from **Left** to **Right**: $2 \times \{(\alpha+1)u - (\alpha+1)o\} -$ [repeat $\{\alpha u - \alpha o\}$].

Half-cycles **4**, from **Right** to **Left**: $2 \times \{(\alpha+1)o - (\alpha+1)u\} -$ [repeat $\{\alpha o - \alpha u\}$].

Half-cycles **4**, from **Left** to **Right**: $2 \times \{(\alpha+1)o - (\alpha+1)u\} -$ [repeat $\{\alpha o - \alpha u\}$].

Last half-cycle **4**, from **Left** to **Right**: $2 \times \{(\alpha+1)o - (\alpha+1)u\} - (\alpha+1)o - \alpha u -$ [repeat $\{\alpha o - \alpha u\}$].

Half-cycles **5**, from **Right** to **Left**: $2 \times \{(\alpha+1)u - (\alpha+1)o\} - (\alpha+1)u - \alpha o -$ [repeat $\{\alpha u - \alpha o\}$].

Half-cycles **5**, from **Left** to **Right**: $2 \times \{(\alpha+1)u - (\alpha+1)o\} - (\alpha+1)u - \alpha o -$ [repeat $\{\alpha u - \alpha o\}$].

Last half-cycle **5**, from **Left** to **Right**: $3 \times \{(\alpha+1)u - (\alpha+1)o\} -$ [repeat $\{\alpha u - \alpha o\}$].

Half-cycles **6**, from **Right** to **Left**: $3 \times \{(\alpha+1)o - (\alpha+1)u\} -$ [repeat $\{\alpha o - \alpha u\}$].

Half-cycles **6**, from **Left** to **Right**: $3 \times \{(\alpha+1)o - (\alpha+1)u\} -$ [repeat $\{\alpha o - \alpha u\}$].

Last half-cycle **6**, from **Left** to **Right**: $3 \times \{(\alpha+1)o - (\alpha+1)u\} - (\alpha+1)o - \alpha u -$ [repeat $\{\alpha o - \alpha u\}$].

And so on.

If the coding of the first intersection column from the left is /, then replace in the above half-cycle algorithms "o" by "u" and vice versa.

Fig. 223 depicts the evolution of a 5-parts Casa-coded Regular Knot with $(5N-1)$ bights to a 2-pass 9-parts Gaucho-coded Regular Knot with $(9N-2)$ bights (note again that $\alpha = 1$, hence $(\alpha+1) = 2$, and $m = 2$).

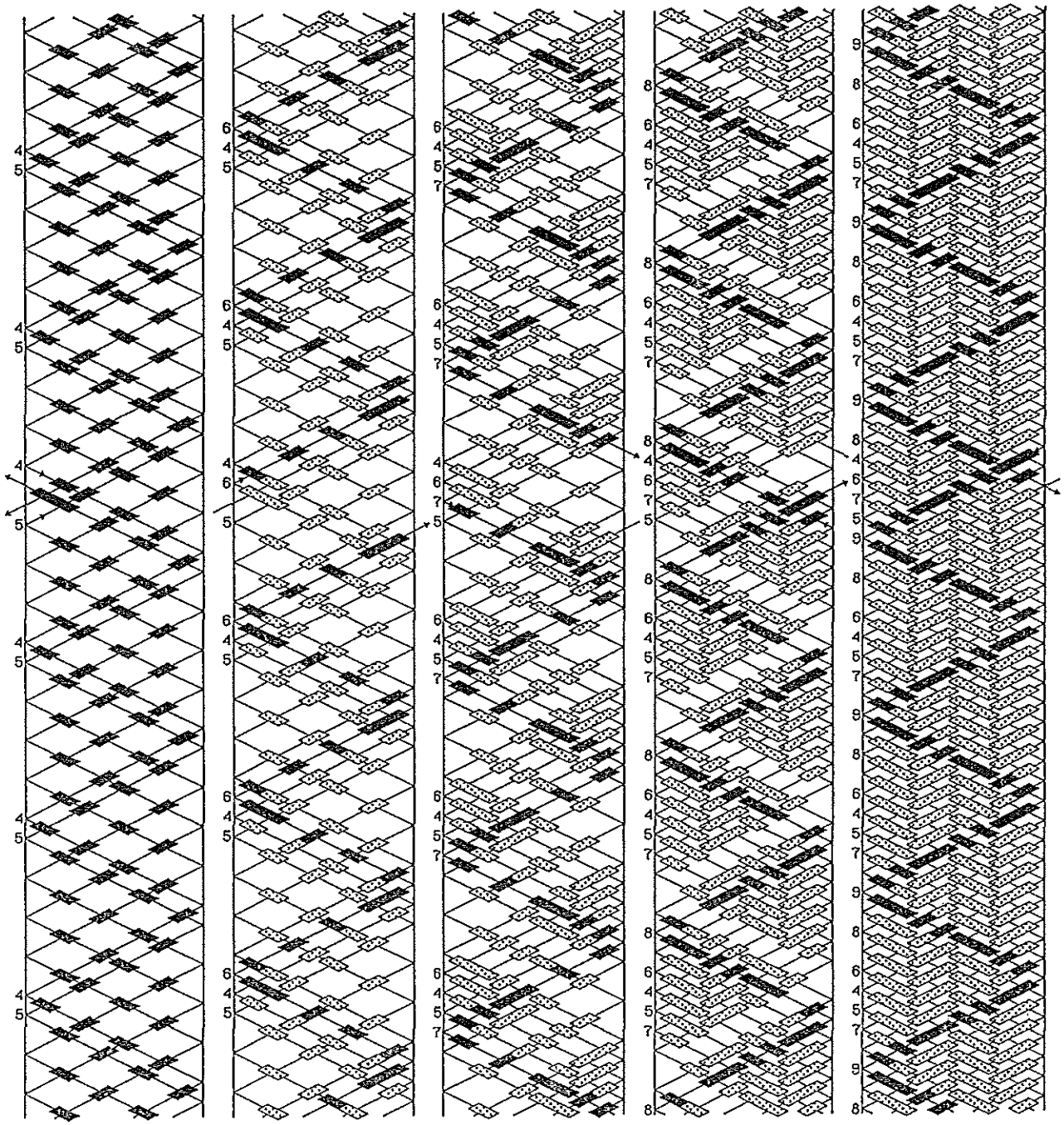


Fig. 223 — Evolution by Method II of a 5-parts/$(5N-1)$-bights Casa Knot to a 2-pass 9-parts/$(9N-2)$-bights Gaucho Knot.

Since this process involves an increase in parts that is equal to $2m = 4$, we have to braid each string-end m times, hence twice around the circumference of the knot.

In order to see how the additional revolutions associated with the evolution of an α-pass $[2m\alpha + 1]$-parts, $[(2m\alpha + 1)N - \alpha]$-bights Gaucho Knot to an $(\alpha + 1)$-pass $[2m(\alpha+1)+1]$-parts, $[\{2m(\alpha+1)+1\}N-(\alpha+1)]$-bights Gaucho Knot are laid down, a part of the string-run of these knots is depicted in Fig. 224.

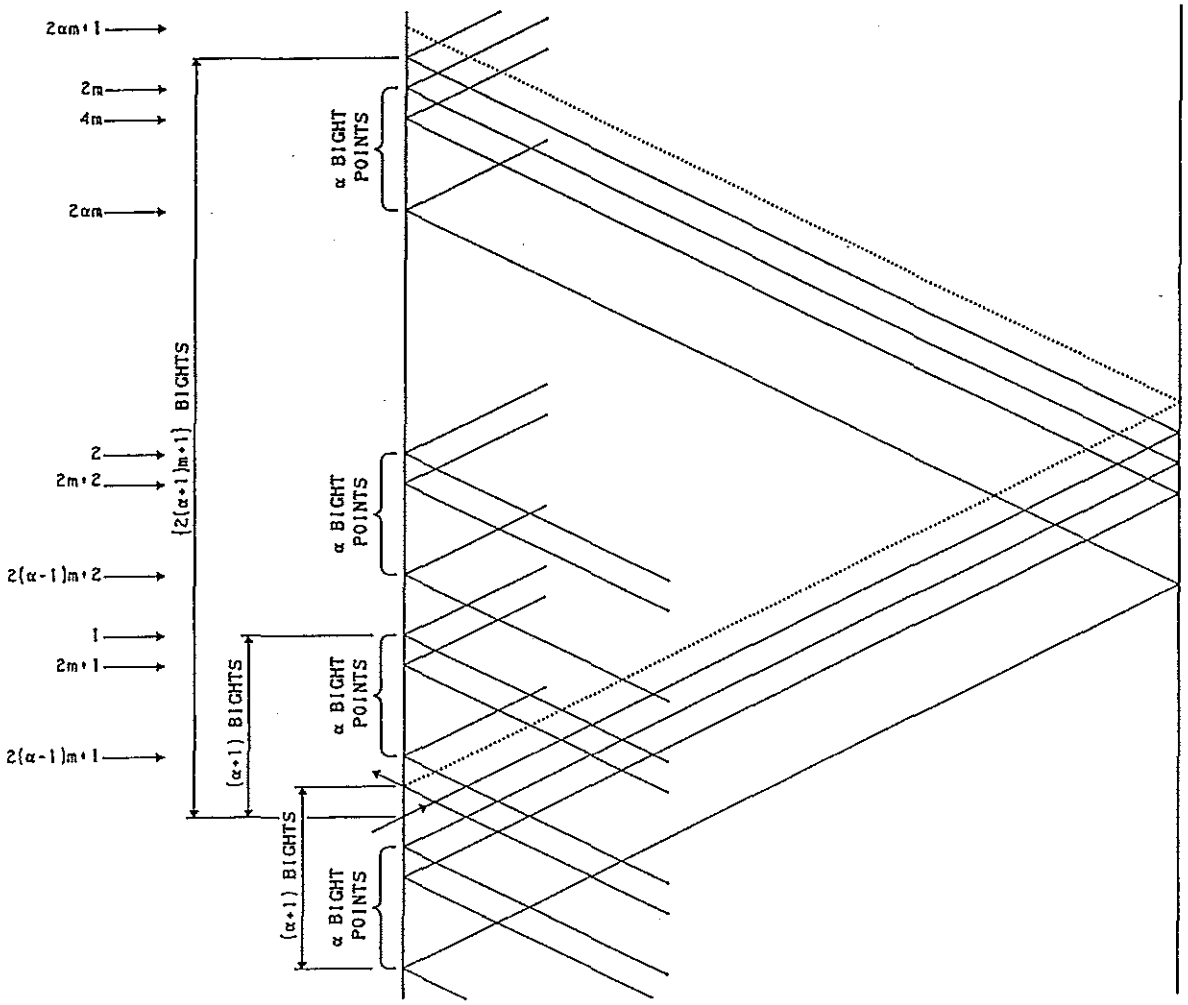


Fig. 224 — A section of the string-run of an α-pass $[2m\alpha + 1]$-parts/$[(2m\alpha + 1)N - \alpha]$-bights Gaucho Knot.

The first revolution is a distance of $(\alpha+1)$ bights less than N cycles (the end-point of N cycles is indicated by 1). The second revolution is a distance of $2(\alpha + 1)$ bights less than $2N$ cycles (the end-point of $2N$ cycles is indicated by 2). And so on. The string-run clearly shows the way in which the additional $2m$ revolutions of the string-run associated with the evolution of an α-pass $[2m\alpha+1]$-parts, $[(2m\alpha+1)N-\alpha]$-bights Gaucho Knot to an $(\alpha+1)$-pass $[2m(\alpha+1)+1]$-parts, $[\{2m(\alpha+1)+1\}N-(\alpha+1)]$-bights Gaucho Knot are being laid down. Fig. 225 depicts a section of the string-run of such a Gaucho Knot. The solid lines belong to the α-pass knot, and the dotted lines belong to the additional $2m$ revolutions associated with the evolution of the α-pass knot to the $(\alpha + 1)$-pass knot. The numbers indicate the revolutions to which the additional half-cycles belong.

Note that an additional $2(2mN - 1) = (4mN - 2)$ half-cycles are associated with an evolution of an α-pass to an $(\alpha + 1)$-pass Gaucho Knot, and that each string-end lays down $(2mN - 1)$ half-cycles.

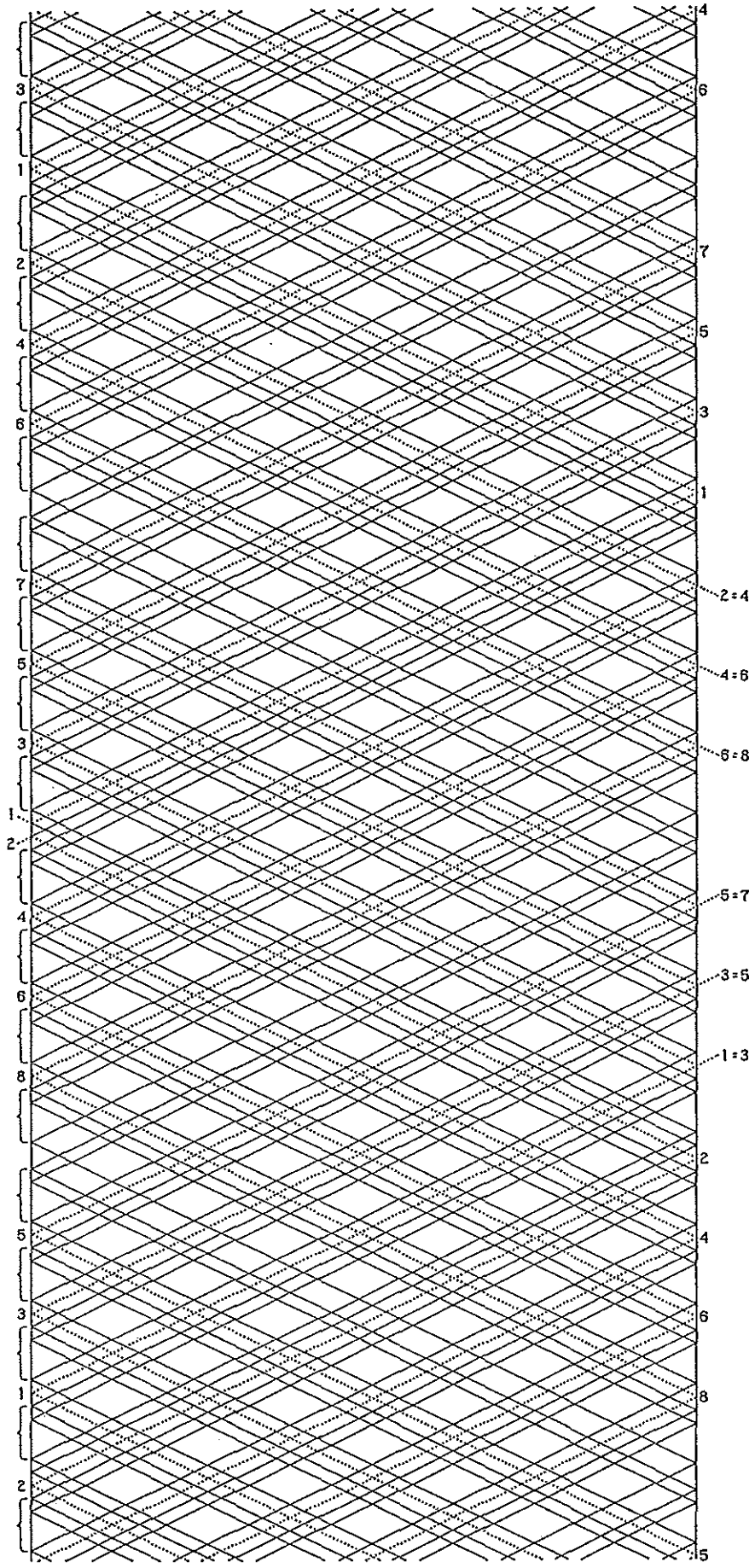


Fig. 225 — A section of a knot as in Fig. 224.

First we lay down the $(2N-1)$ half-cycles marked 1. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down the $(2N-1)$ half-cycles marked 2. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down $2N$ half-cycles marked 3. Next the $2N$ half-cycles marked 4. Then the $2N$ half-cycles marked 5. Then the $2N$ half-cycles marked 6. And so on. Finally we lay down the $2N$ half-cycles marked $2m$ (not shown). Thus we end on the right-hand bight-boundary. For the next evolution from the $(\alpha+1)$-pass to the $(\alpha+2)$-pass we can turn the $(\alpha+1)$-pass knot through 180° in order to get the string-ends on the left-hand bight-boundary again.

If the coding of the first intersection column from the left is \, then the half-cycle algorithms associated with the evolution are:

Half-cycles 1, from **Left** to **Right**:	[repeat $\{\alpha u-\alpha o\}$] $-\alpha u-(\alpha+1)o$.
Half-cycles 1, from **Right** to **Left**:	[repeat $\{\alpha u-\alpha o\}$] $-\alpha u-(\alpha+1)o$.
Half-cycles 2, from **Left** to **Right**:	[repeat $\{\alpha o-\alpha u\}$] $-(\alpha+1)o-(\alpha+1)u$.
Half-cycles 2, from **Right** to **Left**:	[repeat $\{\alpha o-\alpha u\}$] $-(\alpha+1)o-(\alpha+1)u$.
First half-cycle 3, from **Right** to **Left**:	[repeat $\{\alpha u-\alpha o\}$] $-$ $1\times\{(\alpha+1)u-(\alpha+1)o\}$.
Half-cycles 3, from **Right** to **Left**:	[repeat $\{\alpha u-\alpha o\}$] $-\alpha u-(\alpha+1)o-$ $1\times\{(\alpha+1)u-(\alpha+1)o\}$.
Half-cycles 3, from **Left** to **Right**:	[repeat $\{\alpha u-\alpha o\}$] $-\alpha u-(\alpha+1)o-$ $1\times\{(\alpha+1)u-(\alpha+1)o\}$.
First half-cycle 4, from **Right** to **Left**:	[repeat $\{\alpha o-\alpha u\}$] $-\alpha o-(\alpha+1)u-$ $1\times\{(\alpha+1)o-(\alpha+1)u\}$.
Half-cycles 4, from **Right** to **Left**:	[repeat $\{\alpha o-\alpha u\}$] $-$ $2\times\{(\alpha+1)o-(\alpha+1)u\}$.
Half-cycles 4, from **Left** to **Right**:	[repeat $\{\alpha o-\alpha u\}$] $-$ $2\times\{(\alpha+1)o-(\alpha+1)u\}$.
First half-cycle 5, from **Right** to **Left**:	[repeat $\{\alpha o-\alpha u\}$] $-$ $2\times\{(\alpha+1)o-(\alpha+1)u\}$.
Half-cycles 5, from **Right** to **Left**:	[repeat $\{\alpha u-\alpha o\}$] $-\alpha u-(\alpha+1)o-$ $2\times\{(\alpha+1)u-(\alpha+1)o\}$.
Half-cycles 5, from **Left** to **Right**:	[repeat $\{\alpha u-\alpha o\}$] $-\alpha u-(\alpha+1)o-$ $2\times\{(\alpha+1)u-(\alpha+1)o\}$.
First half-cycle 6, from **Right** to **Left**:	[repeat $\{\alpha o-\alpha u\}$] $-\alpha o-(\alpha+1)u-$ $2\times\{(\alpha+1)o-(\alpha+1)u\}$.
Half-cycles 6, from **Right** to **Left**:	[repeat $\{\alpha o-\alpha u\}$] $-$ $3\times\{(\alpha+1)o-(\alpha+1)u\}$.
Half-cycles 6, from **Left** to **Right**:	[repeat $\{\alpha o-\alpha u\}$] $-$ $3\times\{(\alpha+1)o-(\alpha+1)u\}$.

And so on.

If the coding of the first intersection column from the left is /, then replace in the above half-cycle algorithms "o" by "u" and vice versa.

Since the number of bights of a knot is not only a function of its circumferential length, but is also a function of the weaving-pattern, the dimensions and the properties of the braiding material, the most efficient way for the determination of the required number of bights for a band knot is by means of a short length of similar flat braid. Hence for a Gaucho-pattern the number of strings of this similar flat braid is equal to

$[2m(\alpha+1)+1]$. The number of bights required for the band knot must be equal or closely equal to either $[\{2m(\alpha+1)+1\}N+(\alpha+1)]$ or $[\{2m(\alpha+1)+1\}N-(\alpha+1)]$. It will therefore be apparent that especially for higher values of α and/or m it will not always be possible to obtain a value for N which gives a satisfactory solution. For example let $\alpha=2$ and $m=3$, and say we ideally require 87 bights for the band knot envisaged. The nearest solution would be given by $[\{2m(\alpha+1)+1\}N-(\alpha+1)]$ with $N=5$. However, the number of bights will then be equal to 92, a difference which for high quality braidwork is too much. In general there are much better solutions possible, and in *The Braider*, issue No. 13, we shall look into that.

One makes a "Discovery", ... and then .....

We have discussed some aspects of "discoveries" before in Issue No. 8, pg. 162. "Discoveries" are a very personal thing, and hence are of more significance to the "discoverer" than to anyone else. This fact should be realised and appreciated by the "discoverer".

Very few people who make a "discovery" will contemplate the very likely possibility that the "discovery" they made has been made before ... often in fact very many times before. In addition to all this, a "discovery" is very seldom, if ever, the end of a road. Instead, it is generally the beginning of a "new road". Any "discoverer" who does not want to appreciate this gets soon disappointed, and might even accuse other people who continue along the "new road" of piracy. Initially the "discoverer" has the edge on those around him or her, but that edge is soon lost if he or she fails to continue along the "new road". Those "discoverers" who fail to continue should only blame themselves for the missed opportunities which lay in front of them.

The "discovery" of a new knot or braidform is seldom, if ever, an event solely associated with that particular knot or braidform. The "new road" which may have been entered with this "newly discovered" knot or braidform will invaribly lead to other knots or braidforms, hence to generalisations. A "discoverer" should realise that it is not necessarily the "newly discovered" knot or braidform which is important, but that it are rather the generalisations derived from it, since only they lead to further "discoveries".

The existing literature concerned with knots and braids[†] is in fact a collection of individual knots and braids, generally without the slightest reference to relationships and/or generalised construction procedures.

Most braiders don't realise (or don't want to realise) that knots and braids would not have progressed beyond a very primitive form was it not for tools with which we are able to produce various braiding materials. The same applies to generalised construction methods: these can only be developed with the necessary tools. In order to develop generalised construction methods, we must first of course generalise knots and braidforms, as otherwise construction methods remain certain manipulative "tricks" associated with a particular knot or braid or a small set of knots or braids. In the generalisation process of knots and braids we need the help of mathematics as a **tool**.

Unfortunately most people have an erroneous concept of what mathematics is about; they see it either as an art, a science or a form of "black magic" with symbols, depending on their state of mind. Some even worship it like a religion. In the true sense

[†] We are talking here of course only about **real** (physical) knots and braids, not about **fictitious** ones such as with which the mathematical branch of topology is concerned.

mathematics is nothing but a way of writing down relationships in a manner which conveys the intended message unambiguously. It has nothing to do with logic thinking, drawing conclusions and the results thereof. If this was not so then a detective or crime investigator for example would be a mathematician. Obviously with such a statement we might infuriate worshipping mathematicians, but would this not be enough proof by itself that such people are in fact brainwashed and hence cannot think logically? It is only too true that most mathematicians suffer from blind spots in their vision. Thus those people who are not familiar with mathematical writing should not imagine that they cannot learn to read it. After all, everyone who reads this little article has managed to learn the "art" of reading English, and most likely even knows to write it in a much better way: a much more difficult thing than learning to read and write mathematics.

The articles in *The Braider*, and especially the interspersed questions, are intended to encourage the braider to investigate braids, generalise properties and construction methods, and elevate braiding out of the doldrums in which it has resided for centuries. We can only hope that they may have some success.

Symbols used in Mathematical Expressions

In mathematical expressions we make use of various symbols. Besides the letters of the normal alphabet, often the following Greek letters are used:

Lowercase Greek letters.

α = alpha	ι = iota	ϱ = varrho
β = beta	κ = kappa	σ = sigma
γ = gamma	λ = lambda	ς = varsigma
δ = delta	μ = mu	τ = tau
ϵ = epsilon	ν = nu	υ = upsilon
ε = varepsilon	ξ = xi	ϕ = phi
ζ = zeta	o = o	φ = varphi
η = eta	π = pi	χ = chi
θ = theta	ϖ = varpi	ψ = psi
ϑ = vartheta	ρ = rho	ω = omega

Notice that the letter *upsilon* (υ) is a bit wider than v (v); both of them should be distinguished from *nu* (ν). Similarly, *varsigma* (ς) should not be confused with *zeta* (ζ).

Uppercase Greek letters.

Γ = Gamma	Ξ = Xi	Φ = Phi
Δ = Delta	Π = Pi	Ψ = Psi
Θ = Theta	Σ = Sigma	Ω = Omega
Λ = Lambda	Υ = Upsilon	

Note that we often encounter expressions of the form Δb. This expression does not mean $\Delta \cdot b$, hence $\Delta \times b$, but means "*an increment in* b".

THE BRAIDER

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A quarterly publication for the braiding artisan

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Hamilton,
New Zealand.

Solutions to the Questions in Issue No. 11

Question on pg. 229.

We have seen in *The Braider*, Issue No. 6, pp. 114-115 that

$\Delta = \dfrac{n \cdot b + 1}{|p|_b}$ where n is the smallest positive integer such that Δ is an integer.

$\Delta^* = \dfrac{n \cdot b + 1}{|-p|_b}$ where n is the smallest positive integer such that Δ^* is an integer.

Furthermore in *The Braider*, Issue No. 7, pg. 133, we proved that

$\Delta^* = \dfrac{n^* \cdot b - 1}{|p|_b}$ where n^* is the smallest positive integer such that Δ^* is an integer.

From the Diophantine equation $p \cdot \Delta b = b \cdot \Delta p + 1$, which is associated with a Method I Enlargement process, we obtain

$$\Delta_{I,1} b = |\Delta_I|_b = \frac{b \cdot \Delta_{I,1} p + 1}{p},$$

and from the Diophantine equation $p \cdot \Delta b = b \cdot \Delta p - 1$, which is associated with a Method II Enlargement process, we obtain

$$\Delta_{II,1} b = |\Delta_{II}|_b = \frac{b \cdot \Delta_{II,1} p - 1}{p}.$$

Method I Enlargement:

$$\begin{aligned} p \cdot \Delta b &= b \cdot \Delta p + 1, \\ |p \cdot \Delta b|_b &= |b \cdot \Delta p + 1|_b, \\ \big||p|_b \cdot |\Delta b|_b\big|_b &= 1, \\ |p|_b \cdot |\Delta b|_b &= n \cdot b + 1, \\ |\Delta b|_b &= \frac{n \cdot b + 1}{|p|_b}, \\ \text{hence:}\quad |\Delta b|_b &= \Delta_{I,1} b = \Delta. \end{aligned}$$

Method II Enlargement:

$$\begin{aligned} p \cdot \Delta b &= b \cdot \Delta p - 1, \\ |p \cdot \Delta b|_b &= |b \cdot \Delta p - 1|_b, \\ \big||p|_b \cdot |\Delta b|_b\big|_b &= b - 1, \\ |p|_b \cdot |\Delta b|_b &= n^* \cdot b - 1, \\ |\Delta b|_b &= \frac{n^* \cdot b - 1}{|p|_b}, \\ \text{hence:}\quad |\Delta b|_b &= \Delta_{II,1} b = \Delta^*. \end{aligned}$$

Uppermost Question on pg. 232.

The solutions to this question are set out below in Fig. 226.

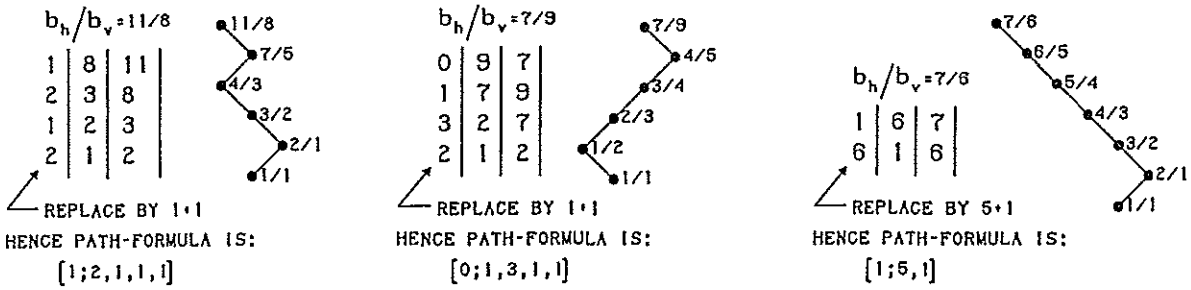


Fig. 226 — The solutions to the uppermost question on pg. 232.

Lowermost Question on pg. 232.

On pp. 228-229, *The Braider*, No. 11, we have seen that the enlargement processes of the same Method, associated with the bight-points x and $(b_v + 1 - x)$, where $1 < x < b_v$, are of the same order. Consequently for upwards and downwards braiding the enlargement processes of the same Method, associated with a bight-point x, where $1 < x < b_v$, are of the same order (turn the string-run diagram through 180° around a horizontal axis).

Say we give a Regular Rectangular Mat an enlargement which, for the Enlargement Method employed, is of the lowest order possible for the bight-point x concerned $(1 < x < b_v)$. Say we continue this process with the resulting enlarged mat, using the same Enlargement Method, again of the lowest possible order.

It is quite easy to find for this further enlargement process the string-run in the vicinity of the Standing End (remember that the sections of string-run due to an enlargement are always "symmetrically paired" with respect to the centre-lines of their associated "lanes"; **these centre-lines must stay free of any string-run**).

Enlargement Method I $(1 < x < b_v)$:

For the string-run in the vicinity of the Standing End we obtain the result as depicted in Fig. 227.

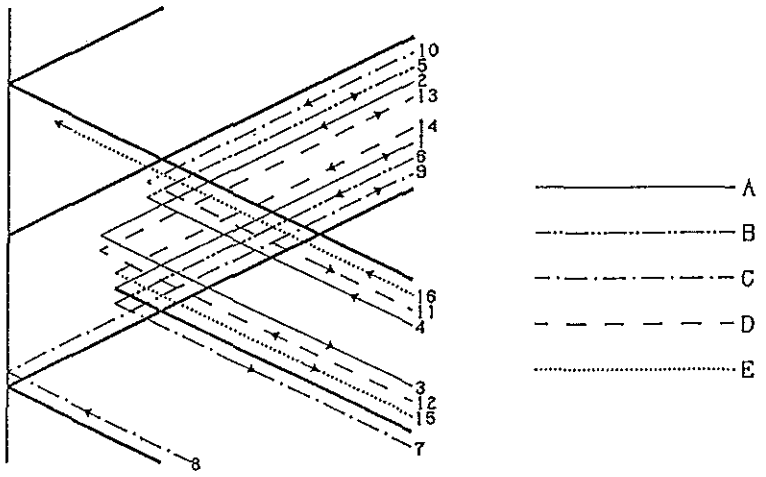


Fig. 227 — The string-run in the vicinity of the Standing End.

The string-run of the original mat is indicated by the thick lines. The first enlargement is indicated by the lines of type 'A', hence the first enlargement ends at the end of line #4. The second enlargement begins at the end of line #4 and ends at the end of

line #16. This second enlargement is by means of the lines 'B','C','D', and 'E' divided into segments.

Segment 'B' "follows" that section of the string-run of the first enlargement which increases the number bights b_v by $\Delta_B b_v = c_I$.

Segment 'C' "follows" the string-run of the original mat, hence it increases the number bights b_v by $\Delta_C b_v = b_v$.

Segment 'D' "follows" the total string-run of the first enlargement, hence it increases the number bights b_v by $\Delta_D b_v = \Delta_I b_v$.

Segment 'E' "follows" that section of the string-run of the first enlargement which increases the number bights b_v by $\Delta_E b_v = c_I^*$.

Thus the total increase in the bights b_v due to this second enlargement is equal to:

$$\Delta_B b_v + \Delta_C b_v + \Delta_D b_v + \Delta_E b_v = c_I + b_v + \Delta_I b_v + c_I^* .$$

Recall from *The Braider*, No. 11, pg. 226, that $c_I + c_I^* = \Delta_I b_v$.

Hence the total increase in the bights b_v due to this second enlargement is equal to:

$$b_v + 2\Delta_I b_v .$$

When $\Delta_I b_v = \Delta_{I,1} b_v$, the total increase in the bights b_v due to this second enlargement is equal to:

$$b_v + 2\Delta_I b_v = b_v + 2\Delta_{I,1} b_v .$$

When $\Delta_I b_v = \Delta_{I,2} b_v$, the total increase in the bights b_v due to this second enlargement is equal to:

$$b_v + 2\Delta_{I,2} b_v = b_v + 2(b_v + \Delta_{I,1} b_v) = 3b_v + 2\Delta_{I,1} b_v .$$

Thus for the bight-points $1 < x < b_v$ the smallest second enlargement of the same Method is always an enlargement of the 2nd-order (see Fig. 228).

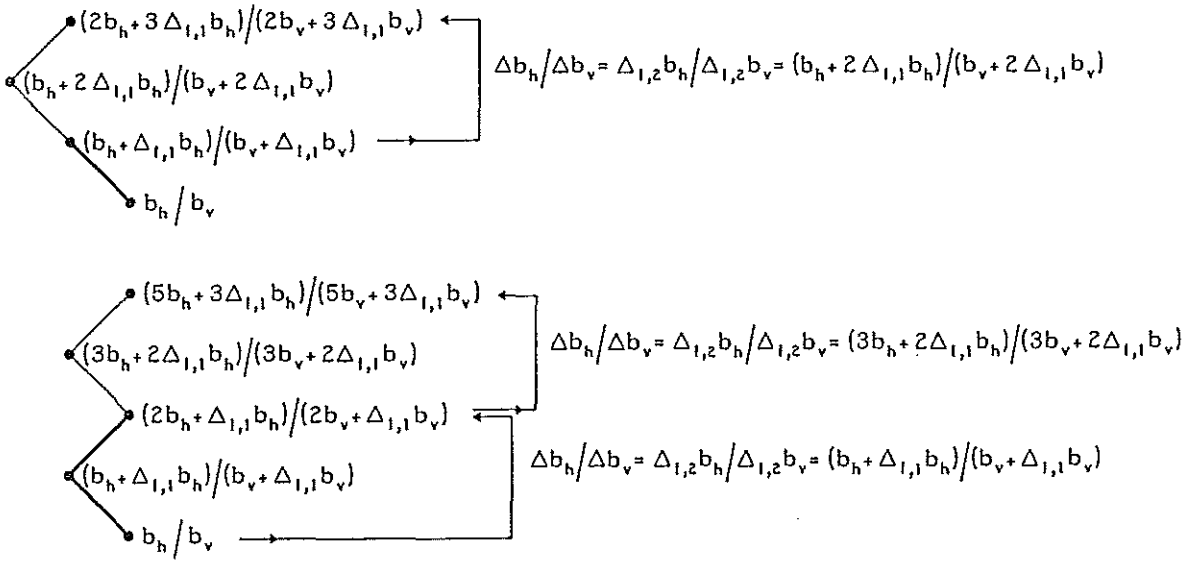


Fig. 228 — The enlargement paths; Enlargement Method I.

Enlargement Method II ($1 < x < b_v$):

For the string-run in the vicinity of the Standing End we obtain the result as depicted in Fig. 229.

The string-run of the original mat is again indicated by the thick lines. The first enlargement is indicated by the lines of type 'A', hence the first enlargement ends at the end of line #4. The second enlargement begins at the end of line #4 and ends at

the end of line #16. This second enlargement is by means of the lines 'B','C','D', and 'E' divided into segments.

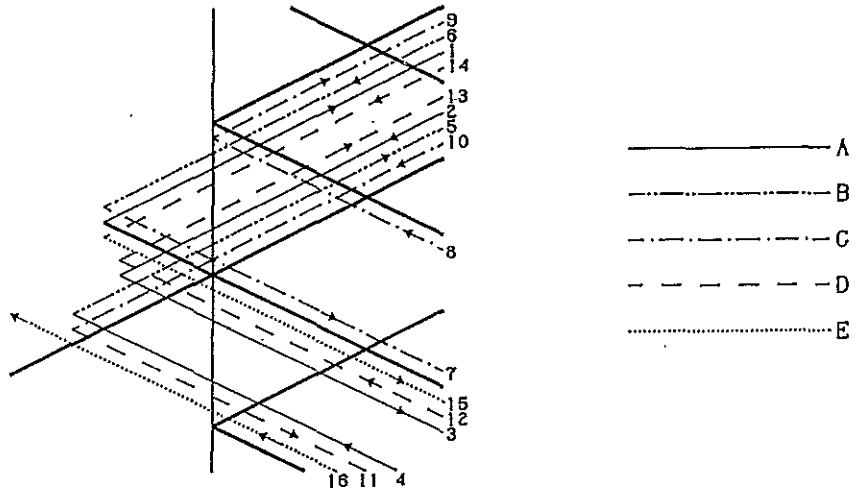


Fig. 229 — The string-run in the vicinity of the Standing End.

Segment 'B' "follows" that section of the string-run of the first enlargement which increases the number bights b_v by $\Delta_B b_v = c_{II}$.

Segment 'C' "follows" the string-run of the original mat, hence it increases the number bights b_v by $\Delta_C b_v = b_v$.

Segment 'D' "follows" the total string-run of the first enlargement, hence it increases the number bights b_v by $\Delta_D b_v = \Delta_{II} b_v$.

Segment 'E' "follows" that section of the string-run of the first enlargement which increases the number bights b_v by $\Delta_E b_v = c^*_{II}$.

Thus the total increase in the bights b_v due to this second enlargement is equal to:

$$\Delta_B b_v + \Delta_C b_v + \Delta_D b_v + \Delta_E b_v = c_{II} + b_v + \Delta_{II} b_v + c^*_{II}.$$

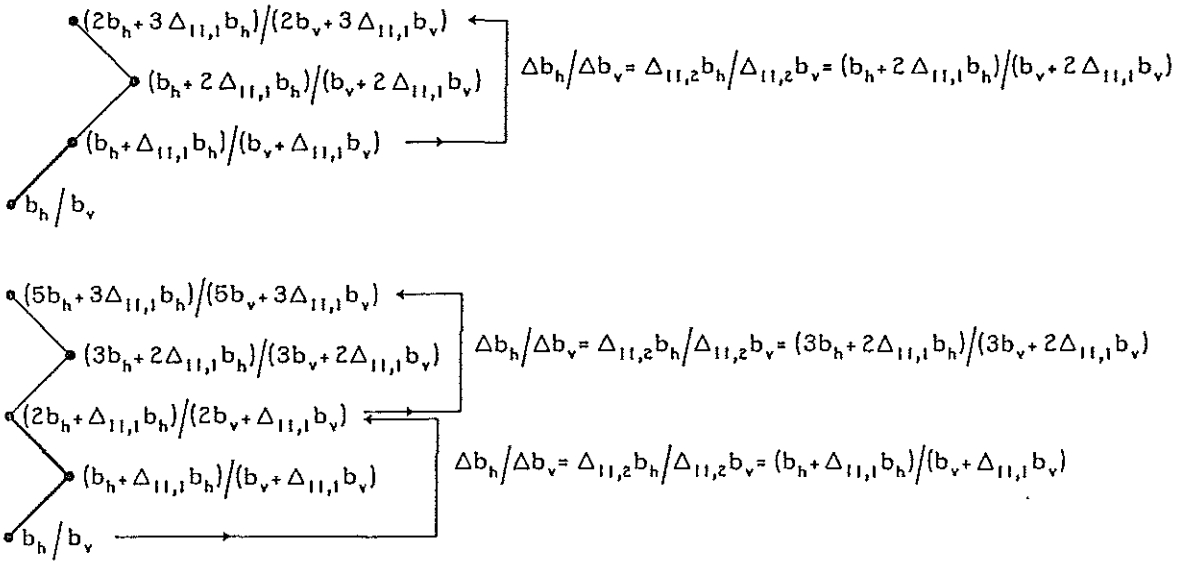


Fig. 230 — The enlargement paths; Enlargement Method II.

Recall from *The Braider*, No. 11, pg. 227, that $c_{II} + c^*_{II} = \Delta_{II} b_v$.

Hence the total increase in the bights b_v due to this second enlargement is equal to:

$$b_v + 2\Delta_{II} b_v.$$

When $\Delta_{II}b_v = \Delta_{II,1}b_v$, the total increase in the bights b_v due to this second enlargement is equal to:

$$b_v + 2\Delta_{II}b_v = b_v + 2\Delta_{II,1}b_v .$$

When $\Delta_{II}b_v = \Delta_{II,2}b_v$, the total increase in the bights b_v due to this second enlargement is equal to:

$$b_v + 2\Delta_{II,2}b_v = b_v + 2(b_v + \Delta_{II,1}b_v) = 3b_v + 2\Delta_{II,1}b_v .$$

Thus for the bight-points $1 < x < b_v$ the smallest second enlargement of the same Method is always an enlargement of the 2nd-order (see Fig. 230).

The bight-points x = 1 and x = b_V:

Method I:

The string-run, in the vicinity of the Standing End, associated with the 1st-order Method I enlargement process for bight-point $x = 1$ is depicted in the left-hand diagram of Fig. 231.

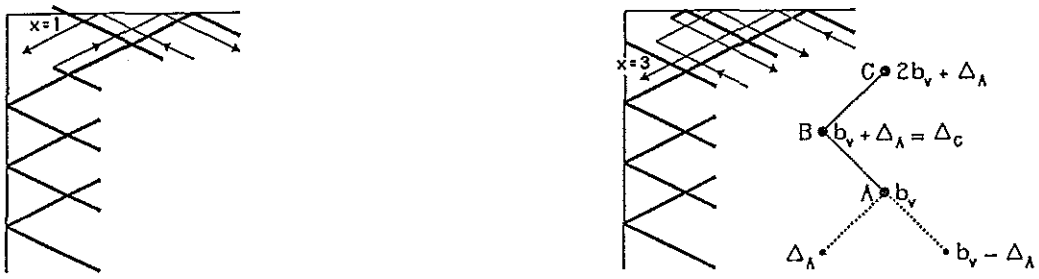


Fig. 231 — Enlargement Methods I, associated with bight-point $x = 1$.

Since the Working End of the string ends in bight-point $x = 1$ of the enlarged mat, this enlarged mat can be given with this Working End either a further 1st-order Method II or a 2nd-order Method I enlargement.

The right-hand diagram of Fig. 231 depicts the string-run, in the vicinity of the Standing End, associated with the 2nd-order Method I enlargement process for bight-point $x = 1$ (note that in the accompanying tree-diagram only the b_v-values are indicated). The Working End of the string ends in bight-point $x = 3$ of the enlarged mat. Since the value for Δ of this enlarged mat is equal to $\Delta_C = b_v + \Delta_A$, and since $\Delta_A < b_v$, we obtain for this bight-point with $x = 3$: $c_I = \left|3\Delta_C\right|_{2b_v+\Delta_A} = \left|3(b_v + \Delta_A)\right|_{2b_v+\Delta_A} = b_v + 2\Delta_A$. Hence $c_I > \Delta_C$, and consequently this enlarged mat can be given with this Working End in bight-point $x = 3$ either a further 1st-order Method II or a 2nd-order Method I enlargement.

It will be needless to say that the above enlargement processes associated with bight-point $x = 1$ can of course be transferred to the other seven extreme bight-points.

Method II:

The string-run, in the vicinity of the Standing End, associated with the 1st-order Method II enlargement process is depicted for bight-point $x = 1$ in the lower two diagrams of Fig. 232. Since the Working End of the string ends in bight-point $x = 1$ of the enlarged mat, this enlarged mat can be given with this Working End either a further 1st-order Method I or a 2nd-order Method II enlargement.

There is a kind of special Method II enlargement for the leftmost upper bight-point (which can of course be transferred to the other seven extreme bight-points of the mat),

which consists of the reversed enlargement string-run associated with the normal 1st-order Method II enlargement process for this bight-point (depicted by the two lower diagrams in Fig. 232). The string-run in the vicinity of the Standing End is depicted in the upper left-hand diagram of Fig. 232 (note again that in the accompanying tree-diagram only the b_v-values are indicated). This special Method II enlargement is of the 1st-order. Let the original mat 'A' be associated with the tree-node 'A', then the enlarged mat 'B' is associated with the tree-node 'B'. If the b_v of the original mat 'A' is equal to b_v, then the b_v of the enlarged mat 'B' is equal to $(2b_v - \Delta_A)$, where $\Delta_A = \Delta_{I,1} b_v$ of mat 'A'. For the enlarged mat 'B' $\Delta_{I,1} b_v = \Delta_B = b_v$. In the enlarged mat 'B' the Working End of the string ends in the bight-point $x = 2$. Hence for this bight-point $c_I = |2 \cdot \Delta_B|_{2b_v - \Delta_A} = \Delta_A$. Since $c_I = \Delta_A < b_v = \Delta_B$, it follows that a further Method I enlargement with the Working End of the string of mat 'B' is of the 1st-order, hence a Method II enlargement would be of the 2nd-order (see *The Braider* No. 11, pg. 227).

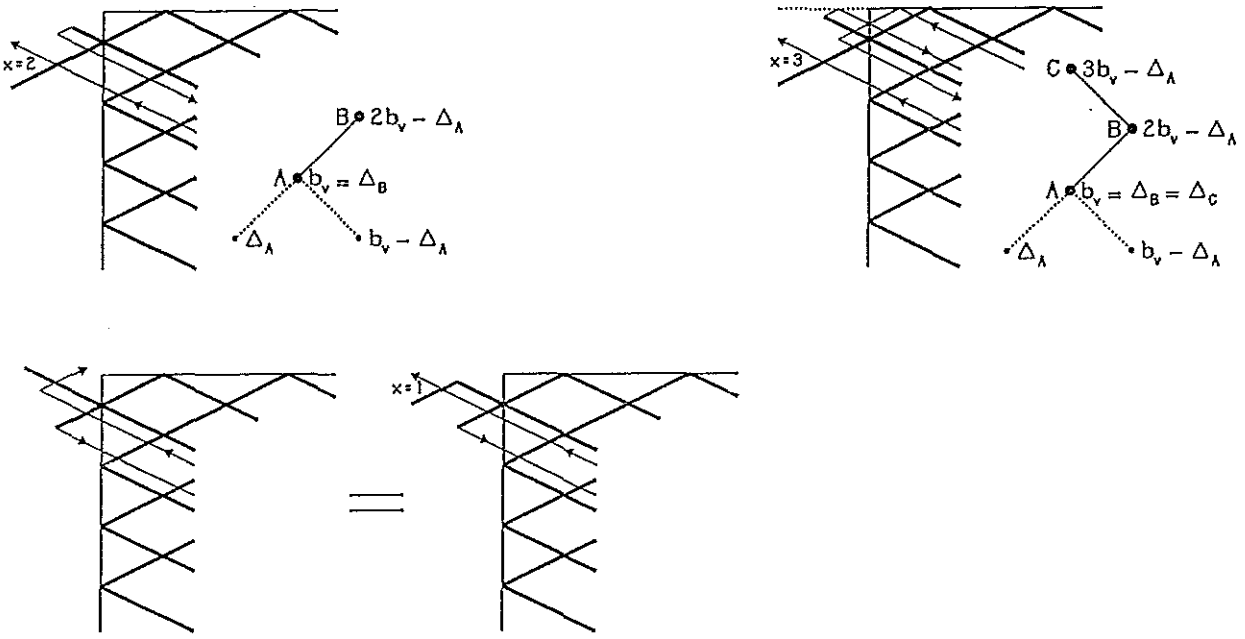


Fig. 232 — Enlargement Methods II, associated with bight-point $x = 1$.

The upper right-hand diagram of Fig. 232 depicts the string-run, in the vicinity of the Standing End, associated with the normal Method II enlargement method for this leftmost upper bight-point of mat 'A', which is thus of the 2nd-order (in the accompanying tree-diagram only the b_v-values are indicated again). For the resulting enlarged mat 'C', $\Delta_{I,1} b_v = \Delta_C = b_v$. In the enlarged mat 'C', the Working End of the string ends in the bight-point $x = 3$. Hence for this bight-point $c_I = |3 \cdot \Delta_C|_{3b_v - \Delta_A} = |3b_v|_{3b_v - \Delta_A} = \Delta_A$. Since $c_I = \Delta_A < b_v = \Delta_C$, it follows that a further Method I enlargement with the Working End of the string of mat 'C' is of the 1st-order, hence a Method II enlargement would be of the 2nd-order.

Thus we found that:

• If in a bight-point x, where $1 \leq x \leq b_v$, the enlargement process for Method I is of the 1st-order, then for the same string-end the enlarged mat may be given a further 1st-order Method II enlargement or a 2nd-order Method I enlargement.

- If in a bight-point x, where $1 \leq x \leq b_v$, the enlargement process for Method I is of the 2nd-order, then for the same string-end the enlarged mat may be given a further 1st-order Method II enlargement or a 2nd-order Method I enlargement.
- If in a bight-point x, where $1 \leq x \leq b_v$, the enlargement process for Method II is of the 1st-order, then for the same string-end the enlarged mat may be given a further 1st-order Method I enlargement or a 2nd-order Method II enlargement.
- If in a bight-point x, where $1 \leq x \leq b_v$, the enlargement process for Method II is of the 2nd-order, then for the same string-end the enlarged mat may be given a further 1st-order Method I enlargement or a 2nd-order Method II enlargement.
- Only when the Standing End and Working End are in one of the eight extreme (corner) bight-points can the mat be enlarged by a 1st-order Method I, a 1st-order Method II, 2nd-order Method I, and a 2nd-order Method II enlargement process. Otherwise the mat can be enlarged by **either** a 1st-order Method I and a 2nd-order Method II, **or** a 2nd-order Method I and a 1st-order Method II enlargement process.

Observe that every bight-point of a Regular Rectangular Mat is associated with a Fibonacci-path in the enlargement tree.† Consequently the *golden ratio* or *devine proportion* is intimately contained in the string-run of these mats.‡

Let's return to the Figs. 227 and 229. We see that the overall enlargement in Fig. 227 consists of the five sections 'A', 'B', 'C', 'D', 'E', where section 'A' ($= \Delta_I b_v =$ section 'D') is the sum of sections 'B' ($= c_I$) and 'E' ($= c_I^*$), while section 'C' is equal to b_v. Similarly, the overall enlargement in Fig. 228 consists of the five sections 'A', 'B', 'C', 'D', 'E', where section 'A' ($= \Delta_{II} b_v =$ section 'D') is the sum of sections 'B' ($= c_{II}$) and 'E' ($= c_{II}^*$), while section 'C' is equal to b_v.

The five sections c_I, c_I^*, c_{II}, c_{II}^* and b_v form the **fundamental enlargement sections** in any enlargement procedure associated with Regular Rectangular Mats. From these sections we can build up various enlargement procedures as we shall show with a few examples below. Note that in the Figs. accompanying these Examples only the b_v-values are indicated in the enlargement path associated with an enlargement process, and that for simplicity in the left-hand path-diagrams $\Delta b_v = \Delta_{I,1} b_v$, while in the right-hand path-diagrams $\Delta b_v = \Delta_{II,1} b_v$.

Example 1:

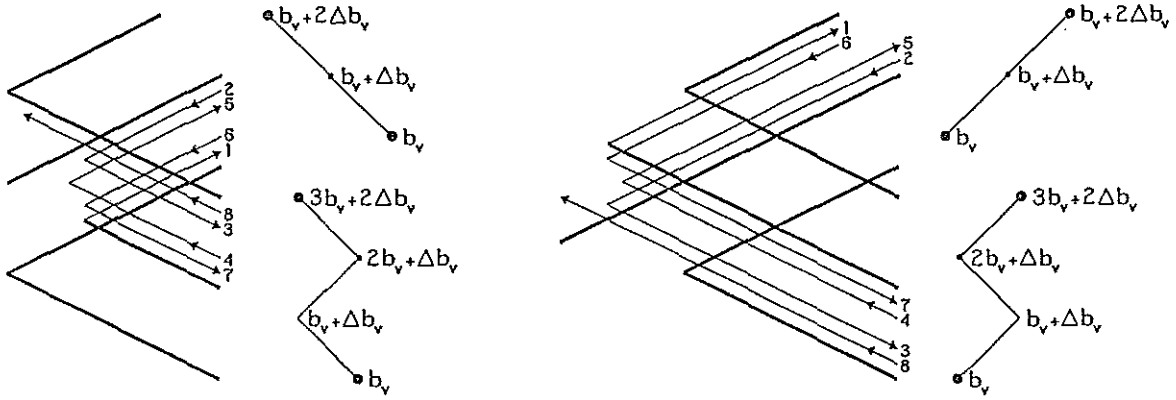


Fig. 233 — Example 1.

† A Fibonacci-path in the enlargement tree consists of alternating 1st-order Method I and Method II enlargement steps.

‡ For the *golden ratio* or *devine proportion* see *The Braider*, Issue No. 8, pg. 171.

For the left-hand enlargement process we obtain:

$$\begin{array}{ll} \text{stringrun } 1\text{-}2: & \longrightarrow \ c_I\,, \\ \text{stringrun } 3\text{-}4: & \longrightarrow \ c_I^*\,, \\ \text{stringrun } 5\text{-}6: & \longrightarrow \ c_I\,, \\ \text{stringrun } 7\text{-}8: & \longrightarrow \ c_I^*\,. \end{array}$$

Hence the overall increase in b_v is equal to $2\Delta_I b_v$. When $\Delta_I b_v = \Delta_{I,1} b_v = \Delta b_v$, the b_v-value of the enlarged mat is then equal to $b_v + 2\Delta b_v$. When $\Delta_I b_v = \Delta_{I,2} b_v = b_v + \Delta_{I,1} b_v = b_v + \Delta b_v$, the b_v-value of the enlarged mat is then equal to $3b_v + 2\Delta b_v$.

For the right-hand enlargement process we obtain:

$$\begin{array}{ll} \text{stringrun } 1\text{-}2: & \longrightarrow \ c_{II}\,, \\ \text{stringrun } 3\text{-}4: & \longrightarrow \ c_{II}^*\,, \\ \text{stringrun } 5\text{-}6: & \longrightarrow \ c_{II}\,, \\ \text{stringrun } 7\text{-}8: & \longrightarrow \ c_{II}^*\,. \end{array}$$

Hence the overall increase in b_v is equal to $2\Delta_{II} b_v$. When $\Delta_{II} b_v = \Delta_{II,1} b_v = \Delta b_v$, the b_v-value of the enlarged mat is then equal to $b_v + 2\Delta b_v$. When $\Delta_{II} b_v = \Delta_{II,2} b_v = b_v + \Delta_{II,1} b_v = b_v + \Delta b_v$, the b_v-value of the enlarged mat is then equal to $3b_v + 2\Delta b_v$.

Example 2:

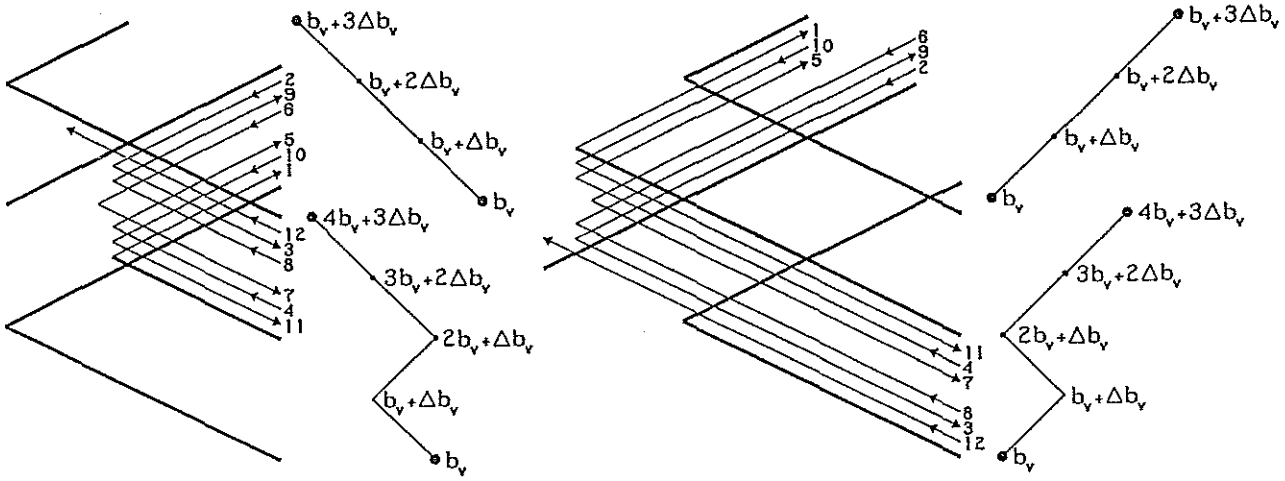


Fig. 234 — Example 2.

For the left-hand enlargement process we obtain:

$$\begin{array}{ll} \text{stringrun } 1\text{-}2: & \longrightarrow \ c_I\,, \\ \text{stringrun } 3\text{-}4: & \longrightarrow \ c_I^*\,, \\ \text{stringrun } 5\text{-}6: & \longrightarrow \ c_I\,, \\ \text{stringrun } 7\text{-}8: & \longrightarrow \ c_I^*\,, \\ \text{stringrun } 9\text{-}10: & \longrightarrow \ c_I\,, \\ \text{stringrun } 11\text{-}12: & \longrightarrow \ c_I^*\,. \end{array}$$

Hence the overall increase in b_v is equal to $3\Delta_I b_v$. When $\Delta_I b_v = \Delta_{I,1} b_v = \Delta b_v$, the b_v-value of the enlarged mat is then equal to $b_v + 3\Delta b_v$. When $\Delta_I b_v = \Delta_{I,2} b_v = b_v + \Delta_{I,1} b_v = b_v + \Delta b_v$, the b_v-value of the enlarged mat is then equal to $4b_v + 3\Delta b_v$.

For the right-hand enlargement process we obtain:

$$\text{stringrun } 1\text{-}2: \quad \longrightarrow \ c_{II}\,,$$

stringrun 3 - 4 : $\longrightarrow$ c^*_{II} ,
stringrun 5 - 6 : $\longrightarrow$ c_{II} ,
stringrun 7 - 8 : $\longrightarrow$ c^*_{II} ,
stringrun 9 - 10 : $\longrightarrow$ c_{II} ,
stringrun 11 - 12 : $\longrightarrow$ c^*_{II} .

Hence the overall increase in b_v is equal to $3\Delta_{II}b_v$. When $\Delta_{II}b_v = \Delta_{II,1}b_v = \Delta b_v$, the b_v-value of the enlarged mat is then equal to $b_v + 3\Delta b_v$. When $\Delta_{II}b_v = \Delta_{II,2}b_v = b_v + \Delta_{II,1}b_v = b_v + \Delta b_v$, the b_v-value of the enlarged mat is then equal to $4b_v + 3\Delta b_v$.

Example 3 :

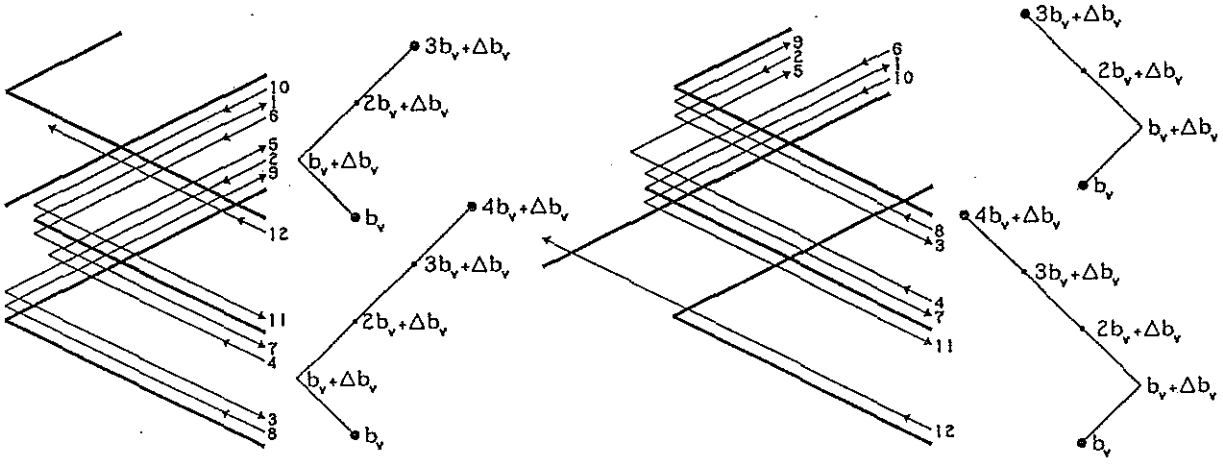


Fig. 235 — Example 3.

For the left-hand enlargement process we obtain :

stringrun 1 - 2 : $\longrightarrow$ c_I ,
stringrun 3 - 6 : $\longrightarrow$ b_v ,
stringrun 7 - 10 : $\longrightarrow$ b_v ,
stringrun 11 - 12 : $\longrightarrow$ c^*_I .

Hence the overall increase in b_v is equal to $2b_v + \Delta_I b_v$. When $\Delta_I b_v = \Delta_{I,1}b_v = \Delta b_v$, the b_v-value of the enlarged mat is then equal to $3b_v + \Delta b_v$. When $\Delta_I b_v = \Delta_{I,2}b_v = b_v + \Delta_{I,1}b_v = b_v + \Delta b_v$, the b_v-value of the enlarged mat is then equal to $4b_v + \Delta b_v$.

For the right-hand enlargement process we obtain :

stringrun 1 - 2 : $\longrightarrow$ c_{II} ,
stringrun 3 - 6 : $\longrightarrow$ b_v ,
stringrun 7 - 10 : $\longrightarrow$ b_v ,
stringrun 11 - 12 : $\longrightarrow$ c^*_{II} .

Hence the overall increase in b_v is equal to $2b_v + \Delta_{II}b_v$. When $\Delta_{II}b_v = \Delta_{II,1}b_v = \Delta b_v$, the b_v-value of the enlarged mat is then equal to $3b_v + \Delta b_v$. When $\Delta_{II}b_v = \Delta_{II,2}b_v = b_v + \Delta_{II,1}b_v = b_v + \Delta b_v$, the b_v-value of the enlarged mat is then equal to $4b_v + \Delta b_v$.

Note that in each of these Examples the enlargement process runs from start to end without any intermediate enlargement stages.

It will no doubt readily be seen that due to the special nature of the eight extreme (corner) bight-points, which allows the use of the 1st-order Method I as well as the 1st-order Method II enlargement method, makes the braiding of Regular Rectangular Mats via the application of these particular enlargement procedures on any one of these bight-points the preferred way. Nevertheless, in most cases even this can be quite cumbersome. Consequently in practice we use another method which in most cases only seems to be quite different, however, in general the mats are still being constructed by consecutive enlargement stages since they can generally only be constructed that way. Before discussing this method, we shall first have a look at how in reality a Regular Rectangular Mat gets constructed via enlargement stages.

$\star\star$ *In each of the following exercises, braiding starts at the left-hand bight-boundary in an upwards direction. Show the stringrun-diagrams for:*

(1). Braiding a $b_h/b_v = 1/5$ *Regular Rectangular Mat by means of enlargement processes, starting with a* $b_h/b_v = 1/1$ *Regular Rectangular Mat.*

(2). Braiding a $b_h/b_v = 1/5$ *Regular Rectangular Mat as normally would be done (there are here five different solutions since there are five different bight-points to start from).*

(3). Braiding a $b_h/b_v = 5/1$ *Regular Rectangular Mat by means of enlargement processes, starting with a* $b_h/b_v = 1/1$ *Regular Rectangular Mat.*

(4). Braiding a $b_h/b_v = 5/1$ *Regular Rectangular Mat as normally would be done.*

(5). Braiding a $b_h/b_v = 2/5$ *Regular Rectangular Mat by means of enlargement processes, starting with a* $b_h/b_v = 1/1$ *Regular Rectangular Mat.*

(6). Braiding a $b_h/b_v = 2/5$ *Regular Rectangular Mat as normally would be done (there are here five different solutions since there are five different bight-points to start from).*

(7). Braiding a $b_h/b_v = 5/2$ *Regular Rectangular Mat by means of enlargement processes, starting with a* $b_h/b_v = 1/1$ *Regular Rectangular Mat.*

(8). Braiding a $b_h/b_v = 5/2$ *Regular Rectangular Mat as normally would be done (there are here two different solutions since there are two different bight-points to start from).*

What do you find by comparing (1) and (2); (3) and (4); (5) and (6); (7) and (8)?

(Regular) Band Knots

In the previous issue we discussed a very select set of Regular Band Knots whose members had either a Spanish ring or Gaucho coding. In this issue we shall discuss a further two very select sets, one whose members have a **Headhunter's** coding, and one whose members have a **Fan** coding. An example of these respective coding forms is depicted in the respective Figs. 236 and 237.

The number of **parts** of a **Headhunter's Knot** is given by the formula: $p = (2m+1)\alpha + 1$, while the number of **parts** of a **Fan Knot** is given by the formula: $p = (2m+2)\alpha - 1$.

The set of Headhunter's Knots we shall discuss have $P = (2m+1)(\alpha+1)+1$ parts and either $B = \{(2m+1)(\alpha+1)+1\}N+(\alpha+1)$ bights or $B = \{(2m+1)(\alpha+1)+1\}N-(\alpha+1)$ bights. Those with $B = \{(2m+1)(\alpha+1)+1\}N+(\alpha+1)$ bights evolve via a Method I enlargement process from Headhunter's Knots with $p = (2m+1)\alpha+1$ parts and

$b = \{(2m+1)\alpha+1\}N+\alpha$ bights, whereas those with $B = \{(2m+1)(\alpha+1)+1\}N-(\alpha+1)$ bights evolve via a Method II enlargement process from Headhunter's Knots with $p = (2m+1)\alpha+1$ parts and $b = \{(2m+1)\alpha+1\}N-\alpha$ bights.

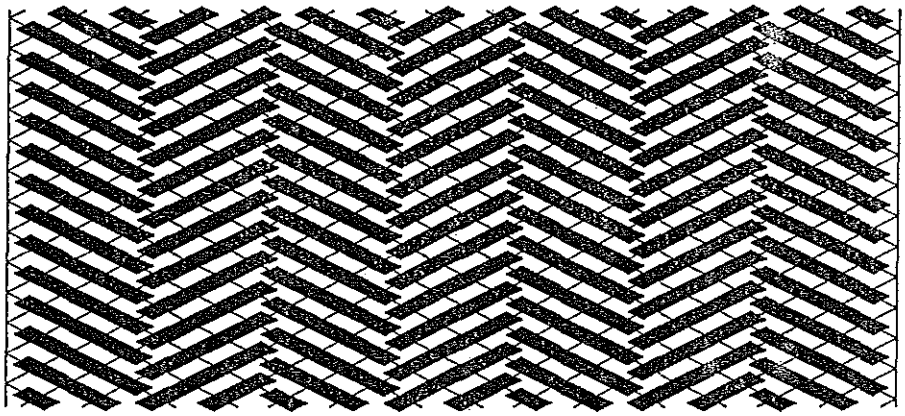


Fig. 236 — A section of a Headhunter's Knot with $\alpha = 4$ and $m = 3$.

The set of Fan Knots we shall discuss have $P = (2m+2)(\alpha+1)-1$ parts and either $B = \{(2m+2)(\alpha+1)-1\}N+(\alpha+1)$ bights or $B = \{(2m+2)(\alpha+1)-1\}N-(\alpha+1)$ bights. Those with $B = \{(2m+2)(\alpha+1)-1\}N-(\alpha+1)$ bights evolve via a Method I enlargement process from Fan Knots with $p = (2m+2)\alpha-1$ parts and $b = \{(2m+2)\alpha-1\}N-\alpha$ bights. Those with $B = \{(2m+2)(\alpha+1)-1\}N+(\alpha+1)$ bights evolve via a Method II enlargement process from Fan Knots with $p = (2m+2)\alpha-1$ parts and $b = \{(2m+2)\alpha-1\}N+\alpha$ bights.

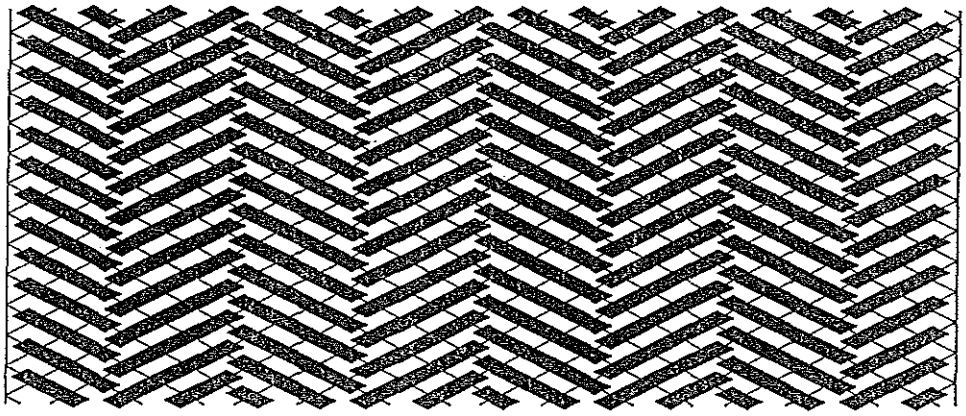
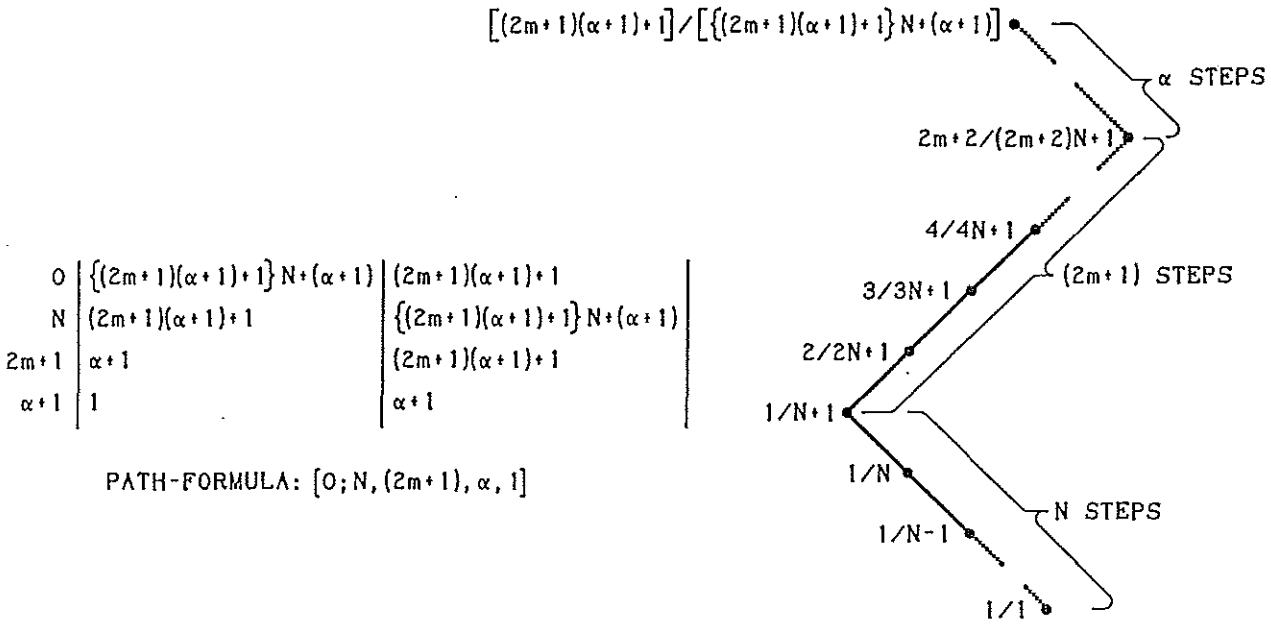


Fig. 237 — A section of a Fan Knot with $\alpha = 4$ and $m = 3$.

The Headhunter's Knots:

Path formulae may be determined with the aid of Euclid's algorithm (see *The Braider*, Issue No. 7, pg. 135). The calculation results for Headhunter's Knots with $P/B = [(2m+1)(\alpha+1)+1]/[\{(2m+1)(\alpha+1)+1\}N+(\alpha+1)]$ are set out in the upper part of Fig. 238. The Headhunter's Knot for $\alpha = 1$ is obtained by evolving the Casa Knot $p/b = [2m+2]/[(2m+2)N+1]$ via enlargement Method I. From the path in the RKT it is evident that this Casa Knot may be obtained by successive 1st-order Method II enlargements from the Column-coded Regular Knot $p/b = 2/[2N+1]$ (or from the Casa Knot $p/b = 4/[4N+1]$).†

† Recall that between any two consecutive Casa Knots we require two consecutive enlargements of the same Method (see *The Braider*, Issue No. 7, pg. 136).



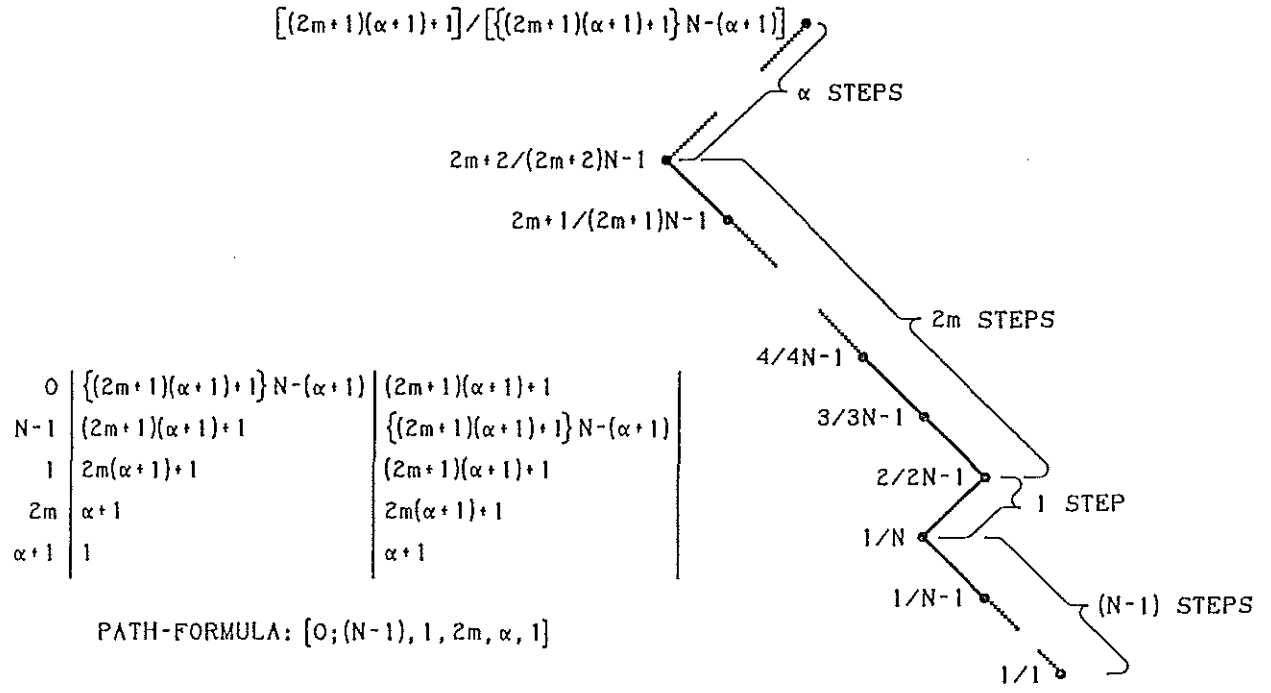


Fig. 238 — Euclid's algorithm and the path formulae.

Similarly, in the lower part of Fig. 238 the calculation results for Headhunter's Knots with $P/B = [(2m+1)(\alpha+1)+1]/[\{(2m+1)(\alpha+1)+1\}N-(\alpha+1)]$ are set out. The Headhunter's Knot for $\alpha = 1$ may be obtained by evolving the Casa Knot $p/b = [2m+2]/[(2m+2)N-1]$ via enlargement Method II. From the path in the RKT it is evident that this Casa Knot may be obtained by successive 1st-order Method I enlargements from the Column-coded Regular Knot $p/b = 2/[2N-1]$ (or from the Casa Knot $p/b = 4/[4N-1]$).

By comparing the formulae of these Headhunter's Knots with the formulae of the Gaucho Knots discussed in the previous issue, a strong similarity will be evident. In fact by replacing the "$2m$" in the Gaucho formulae by "$(2m+1)$" we obtain those of the Headhunter's. Consequently by replacing in the section of string-run depicted in Fig. 221 "$2m$" by "$(2m+1)$", we obtain a section of string-run of an α-pass Headhunter's Knot with $p/b = [(2m+1)\alpha+1]/[\{(2m+1)\alpha+1\}N+\alpha]$ (see Fig. 239).

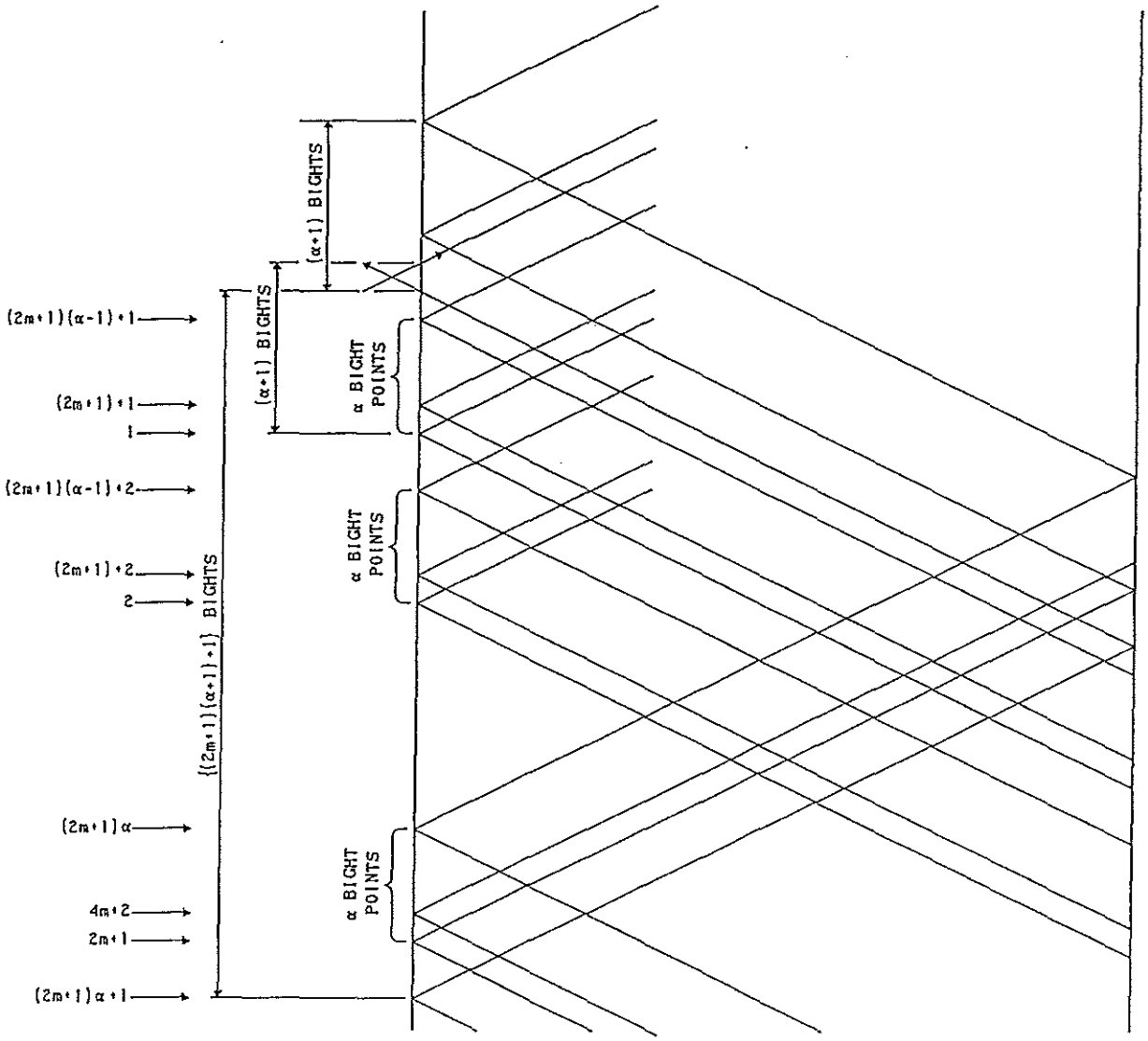


Fig. 239 — A section of the string-run of an α-pass Headhunter's Knot with $p/b = [(2m+1)\alpha+1]/[\{(2m+1)\alpha+1\}N+\alpha]$.

A section of string-run depicted in Fig. 240 shows how the additional $(2m+1)$ revolutions of the string-run associated with the evolution of an α-pass Headhunter's Knot with $p/b = [(2m+1)\alpha+1]/[\{(2m+1)\alpha+1\}N+\alpha]$ to an $(\alpha+1)$-pass Headhunter's Knot with $P/B = [(2m+1)(\alpha+1)+1]/[\{(2m+1)(\alpha+1)+1\}N+(\alpha+1)]$ are being laid down.

Note that in contrast to the Gaucho Knots, we have an **odd** number of additional revolutions, hence each string-end lays down m full revolutions plus a further half revolution. Thus each string-end lays down $\{(2m+1)N+1\}$ half-cycles.

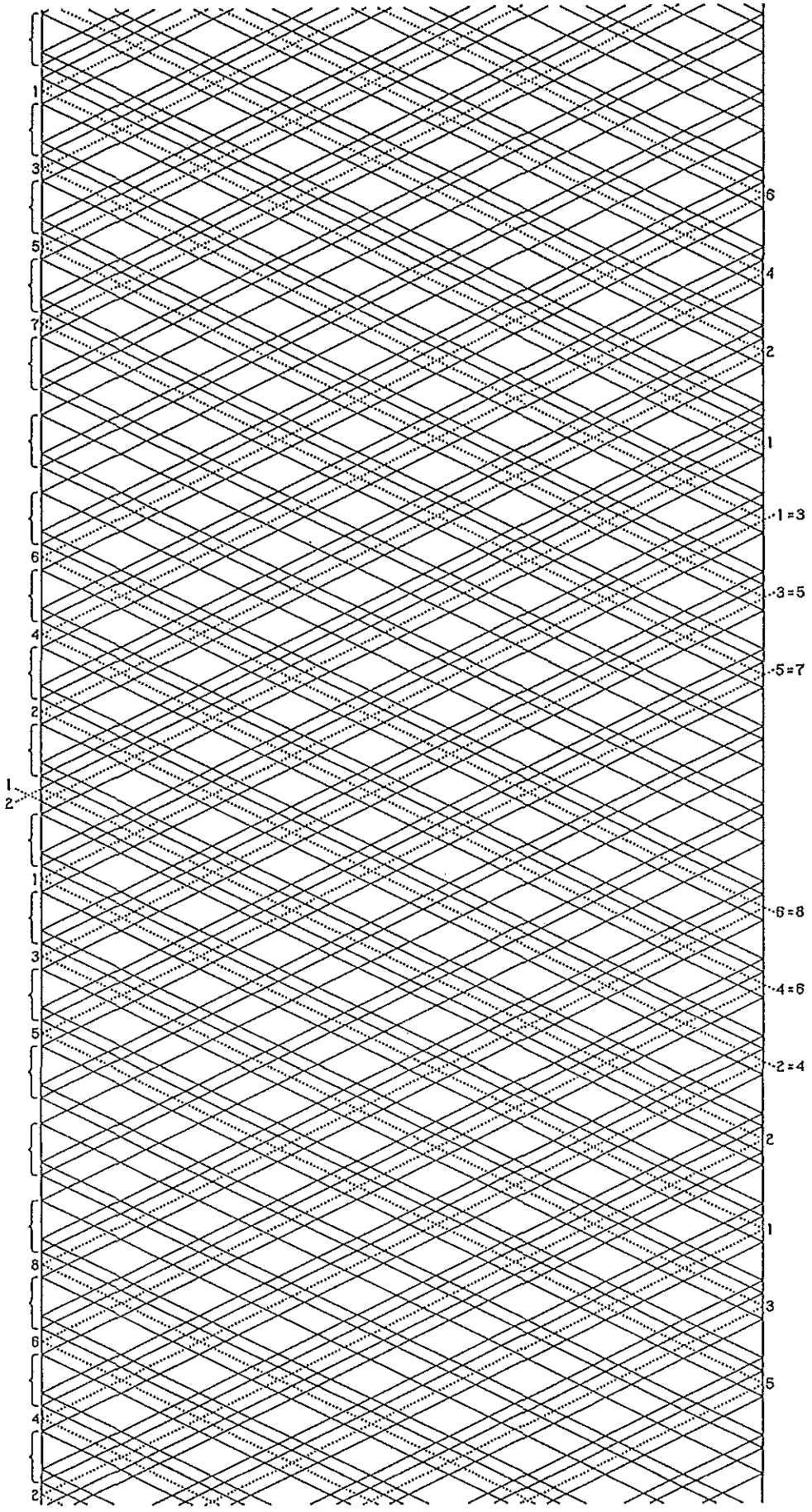


Fig. 240 — A section of a knot as in Fig. 239.

First we lay down the $(2N+1)$ half-cycles marked 1. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down the $(2N+1)$ half-cycles marked 2. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down $2N$ half-cycles marked 3. Next the $2N$ half-cycles marked 4. Then the $2N$ half-cycles marked 5. Then the $2N$ half-cycles marked 6. And so on till we have laid down $(4mN+2)$ half-cycles (the last $2N$ of these half-cycles are marked $2m$). Then each string-end lays down N half-cycles, which means that each string-end makes finally half a revolution. Thus we end on the right-hand bight-boundary on a cylinder generating line which is diametrically opposite to the one from which the evolution process started. For the next evolution from the $(\alpha+1)$-pass to the $(\alpha+2)$-pass we can turn the $(\alpha+1)$-pass knot through 180° in order to get the string-ends on the left-hand bight-boundary again.

If the coding of the first intersection column from the left is \, then the half-cycle algorithms associated with the evolution are:

First half-cycle 1, from **Left** to **Right**:	$\alpha\mathrm{u} - [\text{repeat } \{\alpha\mathrm{o} - \alpha\mathrm{u}\}].$
Half-cycles 1, from **Left** to **Right**:	$(\alpha+1)\mathrm{u} - [\text{repeat } \{\alpha\mathrm{o} - \alpha\mathrm{u}\}].$
Half-cycles 1, from **Right** to **Left**:	$(\alpha+1)\mathrm{o} - [\text{repeat } \{\alpha\mathrm{u} - \alpha\mathrm{o}\}].$
Half-cycles 2, from **Left** to **Right**:	$(\alpha+1)\mathrm{o} - (\alpha+1)\mathrm{u} - \alpha\mathrm{o} -$ [repeat $\{\alpha\mathrm{u} - \alpha\mathrm{o}\}$].
Last half-cycle 2, from **Left** to **Right**:	$(\alpha+1)\mathrm{o} - (\alpha+1)\mathrm{u} - (\alpha+1)\mathrm{o} -$ [repeat $\{\alpha\mathrm{u} - \alpha\mathrm{o}\}$].
Half-cycles 2, from **Right** to **Left**:	$(\alpha+1)\mathrm{u} - (\alpha+1)\mathrm{o} - \alpha\mathrm{u} -$ [repeat $\{\alpha\mathrm{o} - \alpha\mathrm{u}\}$].
Half-cycles 3, from **Right** to **Left**:	$1 \times \{(\alpha+1)\mathrm{o} - (\alpha+1)\mathrm{u}\} - (\alpha+1)\mathrm{o} -$ [repeat $\{\alpha\mathrm{u} - \alpha\mathrm{o}\}$].
Half-cycles 3, from **Left** to **Right**:	$1 \times \{(\alpha+1)\mathrm{u} - (\alpha+1)\mathrm{o}\} - (\alpha+1)\mathrm{u} -$ [repeat $\{\alpha\mathrm{o} - \alpha\mathrm{u}\}$].
Last half-cycle 3, from **Left** to **Right**:	$2 \times \{(\alpha+1)\mathrm{u} - (\alpha+1)\mathrm{o}\} - \alpha\mathrm{u} -$ [repeat $\{\alpha\mathrm{o} - \alpha\mathrm{u}\}$].
Half-cycles 4, from **Right** to **Left**:	$2 \times \{(\alpha+1)\mathrm{u} - (\alpha+1)\mathrm{o}\} - \alpha\mathrm{u} -$ [repeat $\{\alpha\mathrm{o} - \alpha\mathrm{u}\}$].
Half-cycles 4, from **Left** to **Right**:	$2 \times \{(\alpha+1)\mathrm{o} - (\alpha+1)\mathrm{u}\} - \alpha\mathrm{o} -$ [repeat $\{\alpha\mathrm{u} - \alpha\mathrm{o}\}$].
Last half-cycle 4, from **Left** to **Right**:	$2 \times \{(\alpha+1)\mathrm{o} - (\alpha+1)\mathrm{u}\} - (\alpha+1)\mathrm{o} -$ [repeat $\{\alpha\mathrm{u} - \alpha\mathrm{o}\}$].
Half-cycles 5, from **Right** to **Left**:	$2 \times \{(\alpha+1)\mathrm{o} - (\alpha+1)\mathrm{u}\} - (\alpha+1)\mathrm{o} -$ [repeat $\{\alpha\mathrm{u} - \alpha\mathrm{o}\}$].
Half-cycles 5, from **Left** to **Right**:	$2 \times \{(\alpha+1)\mathrm{u} - (\alpha+1)\mathrm{o}\} - (\alpha+1)\mathrm{u} -$ [repeat $\{\alpha\mathrm{o} - \alpha\mathrm{u}\}$].
Last half-cycle 5, from **Left** to **Right**:	$3 \times \{(\alpha+1)\mathrm{u} - (\alpha+1)\mathrm{o}\} - \alpha\mathrm{u} -$ [repeat $\{\alpha\mathrm{o} - \alpha\mathrm{u}\}$].
Half-cycles 6, from **Right** to **Left**:	$3 \times \{(\alpha+1)\mathrm{u} - (\alpha+1)\mathrm{o}\} - \alpha\mathrm{u} -$ [repeat $\{\alpha\mathrm{o} - \alpha\mathrm{u}\}$].
Half-cycles 6, from **Left** to **Right**:	$3 \times \{(\alpha+1)\mathrm{o} - (\alpha+1)\mathrm{u}\} - \alpha\mathrm{o} -$ [repeat $\{\alpha\mathrm{u} - \alpha\mathrm{o}\}$].
Last half-cycle 6, from **Left** to **Right**:	$3 \times \{(\alpha+1)\mathrm{o} - (\alpha+1)\mathrm{u}\} - (\alpha+1)\mathrm{o} -$ [repeat $\{\alpha\mathrm{u} - \alpha\mathrm{o}\}$].

And so on.

Note that the numbers associated with the half-cycles indicate, as in the previous Issue of *The Braider*, the revolution to which the half-cycles belong; they do not indicate the half-cycle number!

If the coding of the first intersection column from the left is /, then replace in the above half-cycle algorithms "o" by "u" and vice versa.

When we replace "$2m$" by "$(2m+1)$" in the section of string-run depicted in Fig. 224, we obtain a section of string-run of an α-pass Headhunter's Knot with $p/b = [(2m+1)\alpha+1]/[\{(2m+1)\alpha+1\}N-\alpha]$ (see Fig. 241).

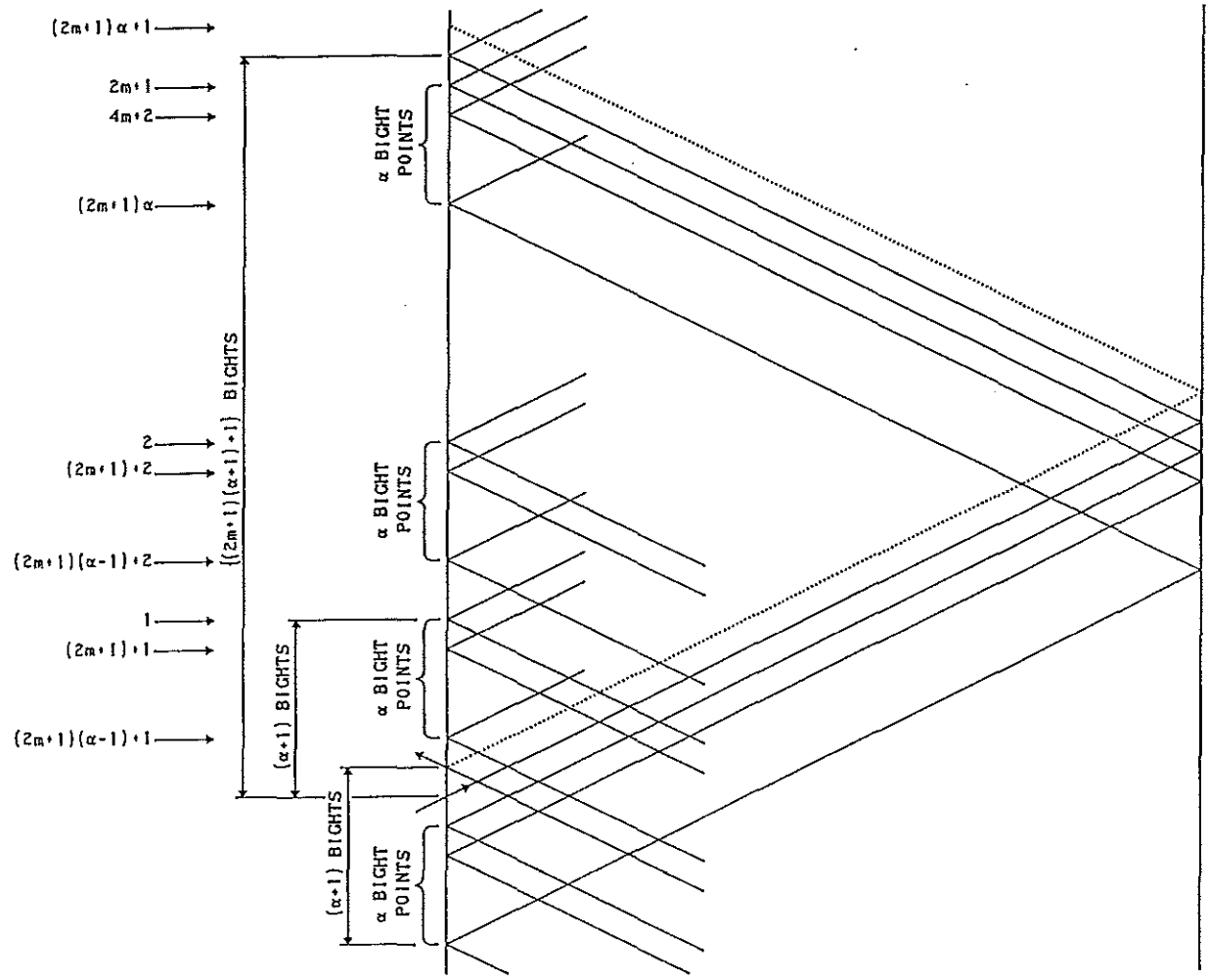


Fig. 241 — A section of the string-run of an α-pass Headhunter's Knot with $p/b = [(2m+1)\alpha+1]/[\{(2m+1)\alpha+1\}N-\alpha]$.

A section of string-run depicted in Fig. 242 shows how the additional $(2m+1)$ revolutions of the string-run associated with the evolution of an α-pass Headhunter's Knot with $p/b = [(2m+1)\alpha+1]/[\{(2m+1)\alpha+1\}N-\alpha]$ to an $(\alpha+1)$-pass Headhunter's Knot with $P/B = [(2m+1)(\alpha+1)+1]/[\{(2m+1)(\alpha+1)+1\}N-(\alpha+1)]$ are being laid down.

Note again that in contrast to the Gaucho Knots, we have an **odd** number of additional revolutions, hence each string-end lays down m full revolutions plus a further half revolution. Thus each string-end lays down $\{(2m+1)N-1\}$ half-cycles.

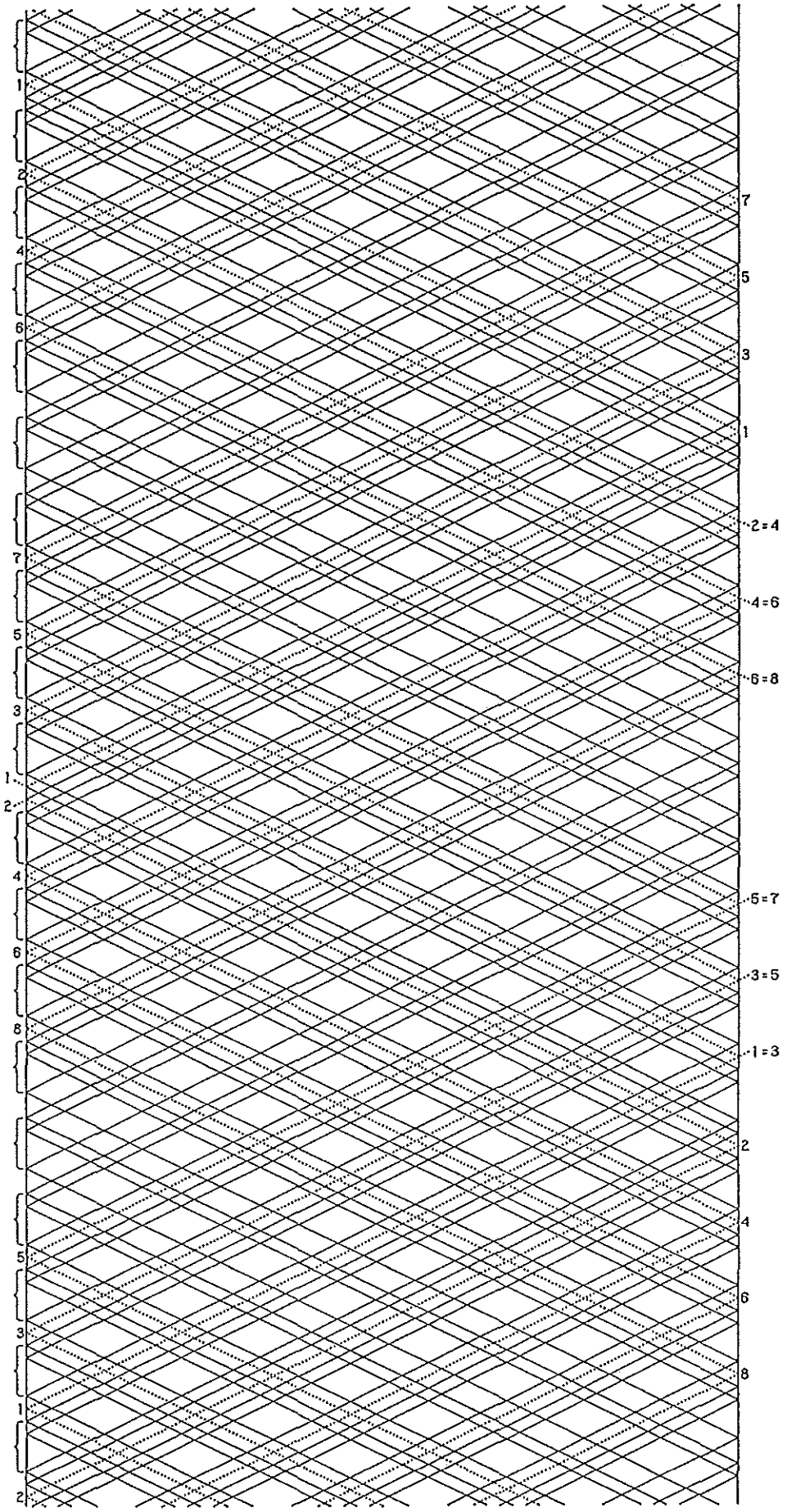


Fig. 242 — A section of a knot as in Fig. 241.

First we lay down the $(2N-1)$ half-cycles marked 1. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down the $(2N-1)$ half-cycles marked 2. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down $2N$ half-cycles marked 3. Next the $2N$ half-cycles marked 4. Then the $2N$ half-cycles marked 5. Then the $2N$ half-cycles marked 6. And so on till we have laid down $(4mN-2)$ half-cycles (the last $2N$ of these half-cycles are marked $2m$). Then each string-end lays down N half-cycles, which means that each string-end makes finally half a revolution. Thus we end on the right-hand bight-boundary on a cylinder generating line which is diametrically opposite to the one from which the evolution process started. For the next evolution from the $(\alpha+1)$-pass to the $(\alpha+2)$-pass we can turn the $(\alpha+1)$-pass knot through 180° in order to get the string-ends on the left-hand bight-boundary again.

If the coding of the first intersection column from the left is \, then the half-cycle algorithms associated with the evolution are:

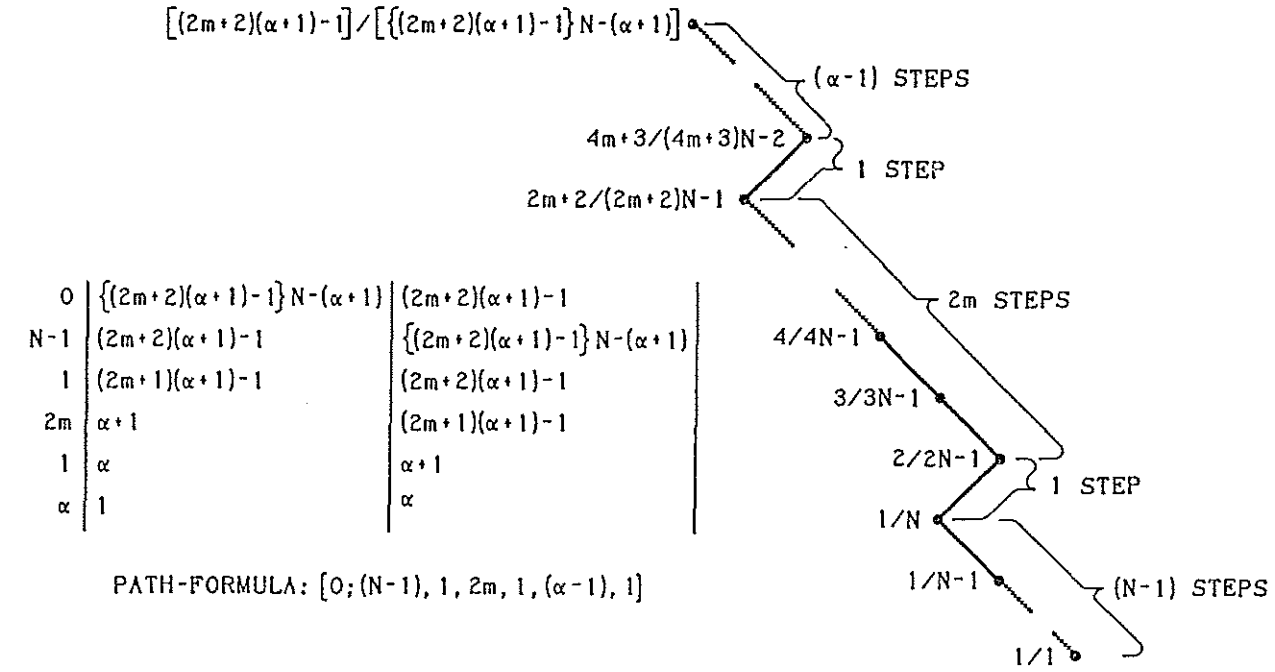
Half-cycles **1**, from **Left** to **Right**:	$[\text{repeat } \{\alpha\text{u} - \alpha\text{o}\}] - (\alpha+1)\text{u}.$
Half-cycles **1**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{o} - \alpha\text{u}\}] - (\alpha+1)\text{o}.$
Half-cycles **2**, from **Left** to **Right**:	$[\text{repeat } \{\alpha\text{o} - \alpha\text{u}\}] - \alpha\text{o} - (\alpha+1)\text{u} - (\alpha+1)\text{o}.$
Half-cycles **2**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{u} - \alpha\text{o}\}] - \alpha\text{u} - (\alpha+1)\text{o} - (\alpha+1)\text{u}.$
First half-cycle **3**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{o} - \alpha\text{u}\}] - \alpha\text{o} - 1 \times \{(\alpha+1)\text{u} - (\alpha+1)\text{o}\}.$
Half-cycles **3**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{o} - \alpha\text{u}\}] - (\alpha+1)\text{o} - 1 \times \{(\alpha+1)\text{u} - (\alpha+1)\text{o}\}.$
Half-cycles **3**, from **Left** to **Right**:	$[\text{repeat } \{\alpha\text{u} - \alpha\text{o}\}] - (\alpha+1)\text{u} - 1 \times \{(\alpha+1)\text{o} - (\alpha+1)\text{u}\}.$
First half-cycle **4**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{u} - \alpha\text{o}\}] - (\alpha+1)\text{u} - 1 \times \{(\alpha+1)\text{o} - (\alpha+1)\text{u}\}.$
Half-cycles **4**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{u} - \alpha\text{o}\}] - \alpha\text{u} - 2 \times \{(\alpha+1)\text{o} - (\alpha+1)\text{u}\}.$
Half-cycles **4**, from **Left** to **Right**:	$[\text{repeat } \{\alpha\text{o} - \alpha\text{u}\}] - \alpha\text{o} - 2 \times \{(\alpha+1)\text{u} - (\alpha+1)\text{o}\}.$
First half-cycle **5**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{u} - \alpha\text{o}\}] - \alpha\text{u} - 2 \times \{(\alpha+1)\text{o} - (\alpha+1)\text{u}\}.$
Half-cycles **5**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{o} - \alpha\text{u}\}] - (\alpha+1)\text{o} - 2 \times \{(\alpha+1)\text{u} - (\alpha+1)\text{o}\}.$
Half-cycles **5**, from **Left** to **Right**:	$[\text{repeat } \{\alpha\text{u} - \alpha\text{o}\}] - (\alpha+1)\text{u} - 2 \times \{(\alpha+1)\text{o} - (\alpha+1)\text{u}\}.$
First half-cycle **6**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{u} - \alpha\text{o}\}] - (\alpha+1)\text{u} - 2 \times \{(\alpha+1)\text{o} - (\alpha+1)\text{u}\}.$
Half-cycles **6**, from **Right** to **Left**:	$[\text{repeat } \{\alpha\text{u} - \alpha\text{o}\}] - \alpha\text{u} - 3 \times \{(\alpha+1)\text{o} - (\alpha+1)\text{u}\}.$
Half-cycles **6**, from **Left** to **Right**:	$[\text{repeat } \{\alpha\text{o} - \alpha\text{u}\}] - \alpha\text{o} - 3 \times \{(\alpha+1)\text{u} - (\alpha+1)\text{o}\}.$

And so on.

If the coding of the first intersection column from the left is /, then replace in the above half-cycle algorithms "o" by "u" and vice versa.

The Fan Knots:

We shall again calculate the path formulae by means of Euclid's algorithm; for Fan Knots with $P/B = [(2m+2)(\alpha+1)-1]/[\{(2m+2)(\alpha+1)-1\}N-(\alpha+1)]$, the calculation results are set out in the upper part of Fig. 243.



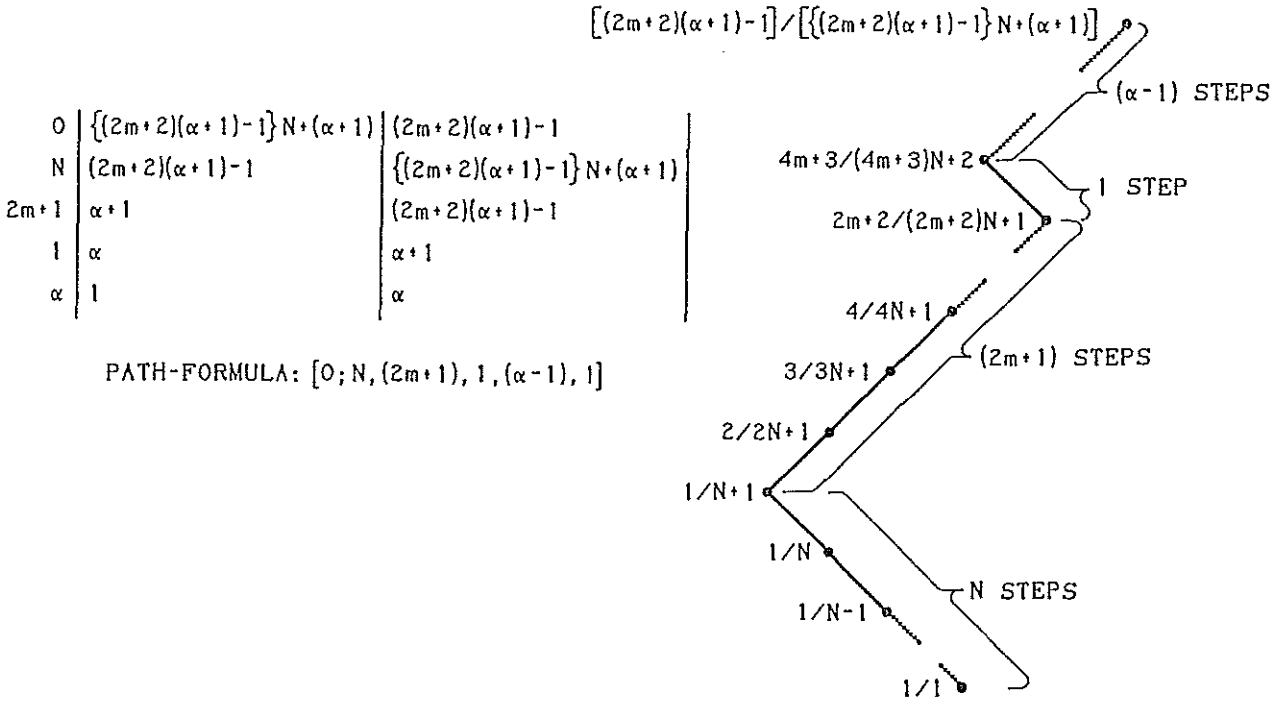


Fig. 243 — Euclid's algorithm and the path formulae.

The Fan Knot for $\alpha = 1$ is obtained by evolving, via a 2nd-order Method I enlarge-

ment, the Casa Knot $p/b = [2m+1]/[(2m+1)N-1]$. From the path in the RKT it is evident that this Casa Knot may be obtained by successive 1st-order Method I enlargements from the Casa Knot $p/b = 3/[3N-1]$.

Similarly, in the lower part of Fig. 243 the calculation results for Fan Knots with $P/B = [(2m+2)(\alpha+1)-1]/[\{(2m+2)(\alpha+1)-1\}N+(\alpha+1)]$ are set out. The Fan Knot for $\alpha = 1$ may be obtained by evolving the Casa Knot $p/b = [2m+1]/[(2m+1)N+1]$ via a 2nd-order Method II enlargement. From the path in the RKT it is evident that this Casa Knot may be obtained by successive 1st-order Method II enlargements from the Casa Knot $p/b = 3/[3N+1]$.

In Fig. 244 is depicted a section of the string-run which shows how for the Fan Knots with $p/b = [(2m+2)\alpha-1]/[\{(2m+2)\alpha-1\}N-\alpha]$ the successive revolutions relate to each other.

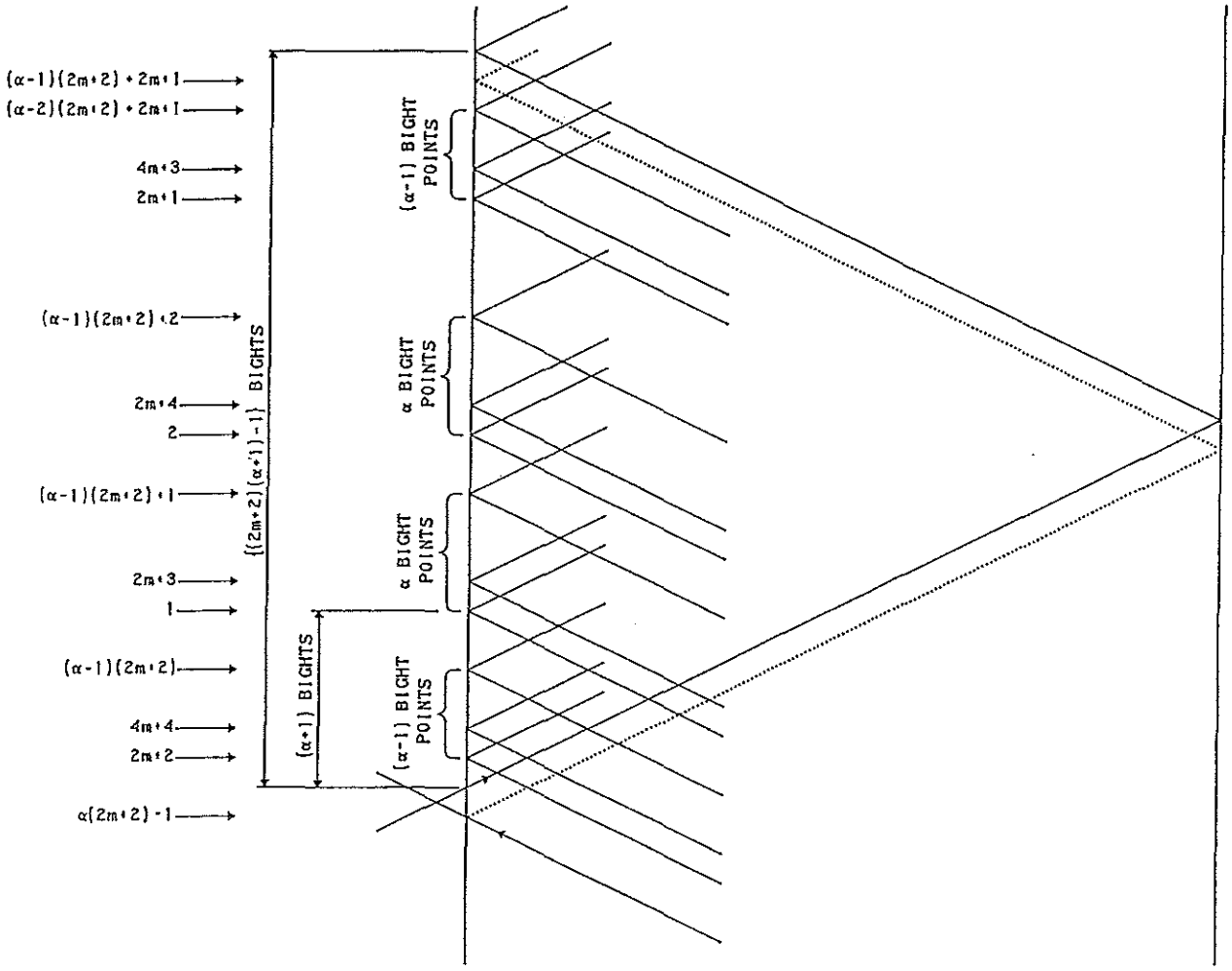


Fig. 244 — A section of the string-run of an α-pass Fan Knot with $p/b = [(2m+2)\alpha-1]/[\{(2m+2)\alpha-1\}N-\alpha]$.

A section of string-run depicted in Fig. 245 shows how the additional $(2m+2)$ revolutions of the string-run associated with the evolution of an α-pass Fan Knot with $p/b = [(2m+2)\alpha-1]/[\{(2m+2)\alpha-1\}N-\alpha]$ to an $(\alpha+1)$-pass Fan Knot with $P/B = [(2m+2)(\alpha+1)-1]/[\{(2m+2)(\alpha+1)-1\}N-(\alpha+1)]$ are being laid down.

Note that we have an **even** number of additional revolutions, hence each string-end lays down $(m+1)$ full revolutions $(= (2m+2)N-1$ half-cycles).

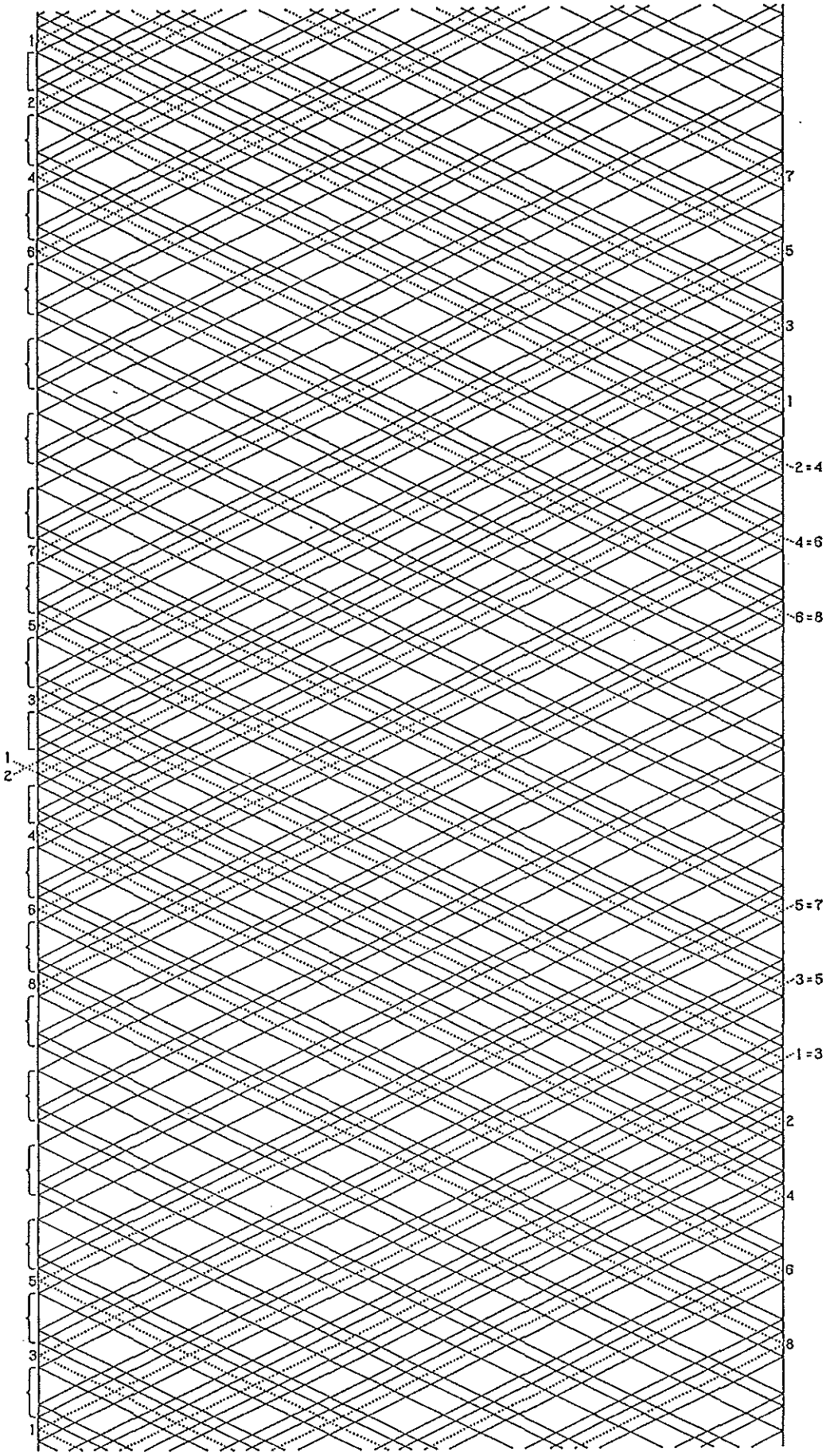


Fig. 245 — A section of a knot as in Fig. 244.

First we lay down the $(2N-1)$ half-cycles marked 1. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down the $(2N-1)$ half-cycles marked 2. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down $2N$ half-cycles marked 3. Next the $2N$ half-cycles marked 4. Then the $2N$ half-cycles marked 5. Then the $2N$ half-cycles marked 6. And so on till we have laid down $(4m+4)N-2)$ half-cycles (the last $2N$ of these half-cycles are marked $(2m+2)$). Thus we end on the right-hand bight-boundary on the same cylinder generating line from which the evolution process started. For the next evolution from the $(\alpha+1)$-pass to the $(\alpha+2)$-pass we can turn the $(\alpha+1)$-pass knot through 180° in order to get the string-ends on the left-hand bight-boundary again.

If the coding of the first intersection column from the left is \, then the half-cycle algorithms associated with the evolution are:

First Half-cycle **1**, from **Left** to **Right**:	$(\alpha-1)\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]-(\alpha-1)\mathrm{o}.$
Half-cycles **1**, from **Left** to **Right**:	$\alpha\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]-(\alpha-1)\mathrm{o}.$
Half-cycles **1**, from **Right** to **Left**:	$\alpha\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]-(\alpha-1)\mathrm{o}.$
Half-cycles **2**, from **Left** to **Right**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}].$
Half-cycles **2**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}].$
First half-cycle **3**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}].$
Half-cycles **3**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]-(\alpha+1)\mathrm{u}-\alpha\mathrm{o}.$
Half-cycles **3**, from **Left** to **Right**:	$[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]-(\alpha+1)\mathrm{u}-\alpha\mathrm{o}.$
First half-cycle **4**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]-(\alpha+1)\mathrm{o}-\alpha\mathrm{u}.$
Half-cycles **4**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]-\alpha\mathrm{o}-1\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-\alpha\mathrm{u}.$
Half-cycles **4**, from **Left** to **Right**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]-\alpha\mathrm{o}-1\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-\alpha\mathrm{u}.$
First half-cycle **5**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]-\alpha\mathrm{u}-1\times\{(\alpha+1)\mathrm{o}-(\alpha+1)\mathrm{u}\}-\alpha\mathrm{o}.$
Half-cycles **5**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]-(\alpha+1)\mathrm{u}-1\times\{(\alpha+1)\mathrm{o}-(\alpha+1)\mathrm{u}\}-\alpha\mathrm{o}.$
Half-cycles **5**, from **Left** to **Right**:	$[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]-(\alpha+1)\mathrm{u}-1\times\{(\alpha+1)\mathrm{o}-(\alpha+1)\mathrm{u}\}-\alpha\mathrm{o}.$
First half-cycle **6**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]-(\alpha+1)\mathrm{o}-1\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-\alpha\mathrm{u}.$
Half-cycles **6**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]-\alpha\mathrm{o}-2\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-\alpha\mathrm{u}.$
Half-cycles **6**, from **Left** to **Right**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]-\alpha\mathrm{o}-2\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-\alpha\mathrm{u}.$

And so on.

If the coding of the first intersection column from the left is /, then replace in the above half-cycle algorithms "o" by "u" and vice versa.

For the Fan Knot with $p/b=[(2m+2)\alpha-1]/[\{(2m+2)\alpha-1\}N+\alpha]$ a section of its string-run is shown in Fig. 246. Again, the end points of the successive revolutions are indicated. Fig. 247 shows how, in the evolution process from an α-pass Fan Knot with $p/b=[(2m+2)\alpha-1]/[\{(2m+2)\alpha-1\}N+\alpha]$ to an $(\alpha+1)$-pass Fan Knot with $P/B=[(2m+2)(\alpha+1)-1]/[\{(2m+2)(\alpha+1)-1\}N+(\alpha+1)]$, the additional $(2m+2)$ revolutions of the string-run are being laid down.

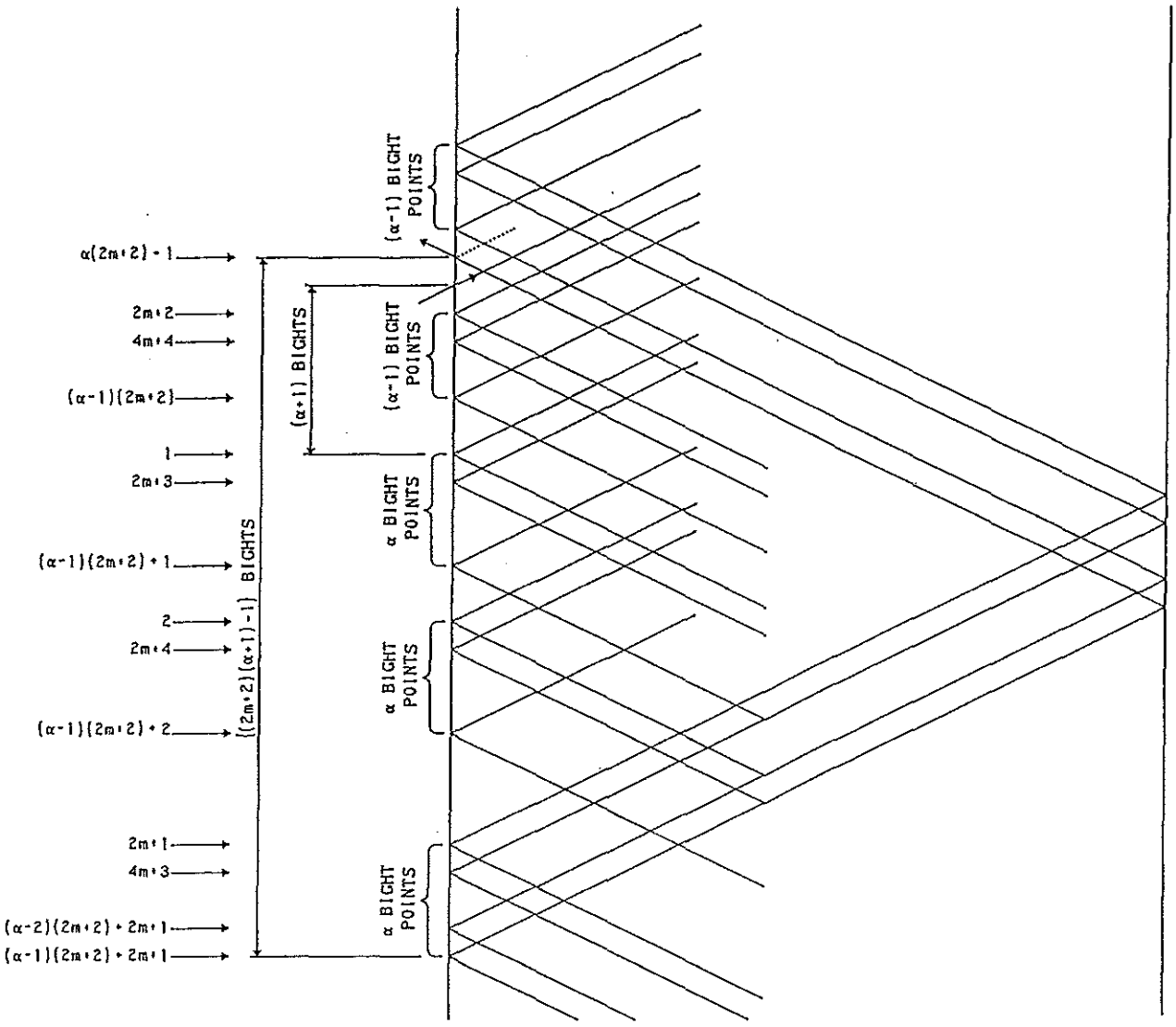


Fig. 246 — A section of the string-run of an α-pass Fan Knot with $p/b = [(2m+2)\alpha - 1]/[\{(2m+2)\alpha - 1\}N + \alpha]$.

Note that we have an **even** number of additional revolutions, hence each string-end lays down $(m+1)$ full revolutions $(= (2m+2)N + 1$ half-cycles).

First we lay down the $(2N+1)$ half-cycles marked 1. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down the $(2N+1)$ half-cycles marked 2. Starting at the left-hand bight-boundary, we finish at the right-hand bight-boundary as indicated. Next we lay down $2N$ half-cycles marked 3. Next the $2N$ half-cycles marked 4. Then the $2N$ half-cycles marked 5. Then the $2N$ half-cycles marked 6. And so on till we have laid down $(4m+4)N + 2)$ half-cycles (the last $2N$ of these half-cycles are marked $(2m+2)$). Thus we end on the right-hand bight-boundary on the same cylinder generating line from which the evolution process started. For the next evolution from the $(\alpha+1)$-pass to the $(\alpha+2)$-pass we can turn the $(\alpha+1)$-pass knot through 180° in order to get the string-ends on the left-hand bight-boundary again.

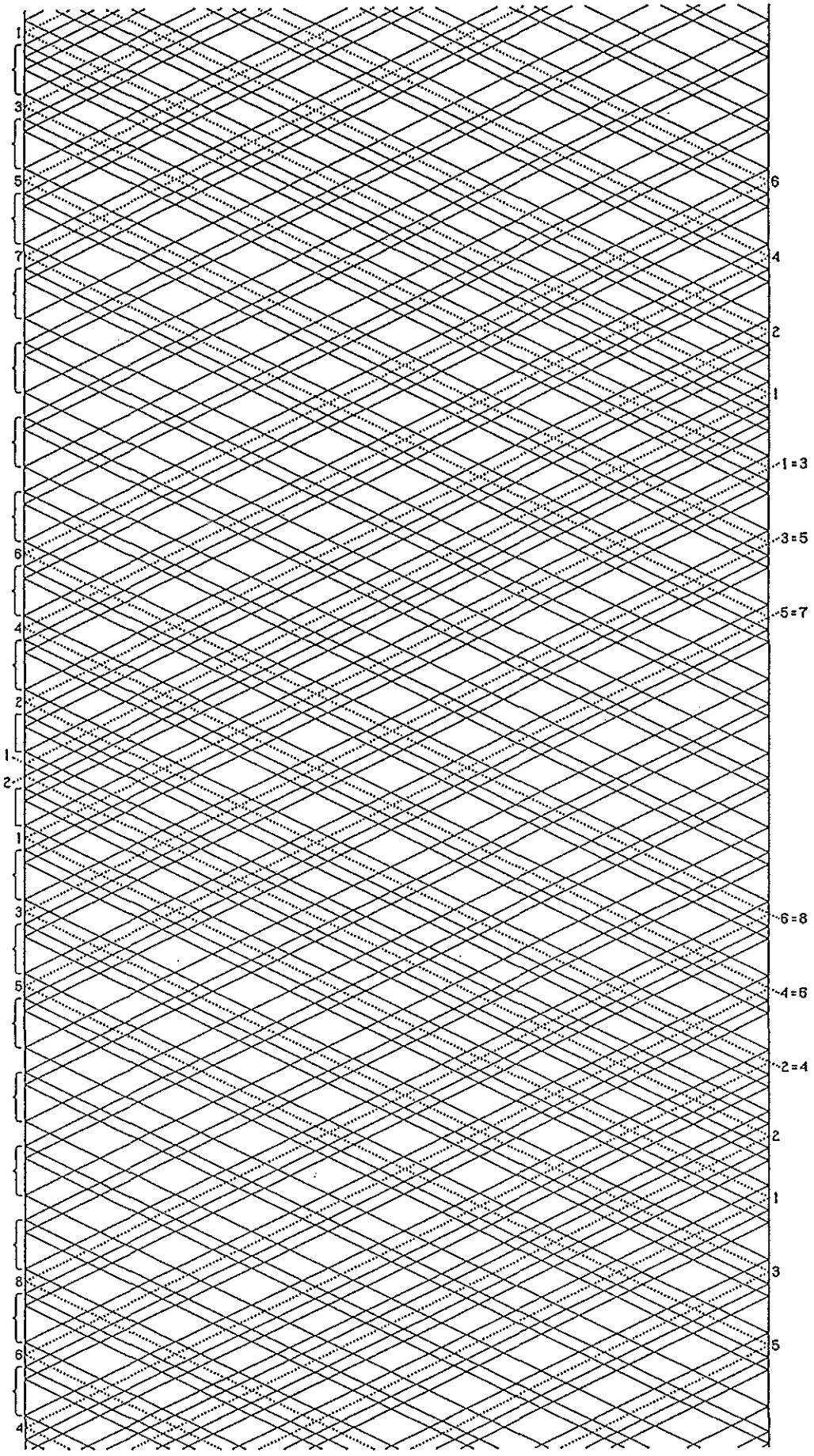


Fig. 247 — A section of a knot as in Fig. 246.

If the coding of the first intersection column from the left is \, then the half-cycle algorithms associated with the evolution are:

Half-cycles **1**, from **Left** to **Right**:	$(\alpha-1)\mathrm{u}-\alpha\mathrm{o}-[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]$.
Half-cycles **1**, from **Right** to **Left**:	$(\alpha-1)\mathrm{u}-\alpha\mathrm{o}-[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]$.
Half-cycles **2**, from **Left** to **Right**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]$.
Last half-cycle **2**, from **Left** to **Right**:	$\alpha\mathrm{o}-(\alpha+1)\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]$.
Half-cycles **2**, from **Right** to **Left**:	$[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]$.
Half-cycles **3**, from **Right** to **Left**:	$\alpha\mathrm{u}-(\alpha+1)\mathrm{o}-[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]$.
Half-cycles **3**, from **Left** to **Right**:	$\alpha\mathrm{u}-(\alpha+1)\mathrm{o}-[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]$.
Last half-cycle **3**, from **Left** to **Right**:	$\alpha\mathrm{u}-1\times\{(\alpha+1)\mathrm{o}-(\alpha+1)\mathrm{u}\}-\alpha\mathrm{o}-[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]$.
Half-cycles **4**, from **Right** to **Left**:	$\alpha\mathrm{o}-1\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-\alpha\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]$.
Half-cycles **4**, from **Left** to **Right**:	$\alpha\mathrm{o}-1\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-\alpha\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]$.
Last half-cycle **4**, from **Left** to **Right**:	$\alpha\mathrm{o}-1\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-(\alpha+1)\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]$.
Half-cycles **5**, from **Right** to **Left**:	$\alpha\mathrm{u}-1\times\{(\alpha+1)\mathrm{o}-(\alpha+1)\mathrm{u}\}-(\alpha+1)\mathrm{o}-[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]$.
Half-cycles **5**, from **Left** to **Right**:	$\alpha\mathrm{u}-1\times\{(\alpha+1)\mathrm{o}-(\alpha+1)\mathrm{u}\}-(\alpha+1)\mathrm{o}-[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]$.
Last half-cycle **5**, from **Left** to **Right**:	$\alpha\mathrm{u}-2\times\{(\alpha+1)\mathrm{o}-(\alpha+1)\mathrm{u}\}-\alpha\mathrm{o}-[\text{repeat }\{\alpha\mathrm{u}-\alpha\mathrm{o}\}]$.
Half-cycles **6**, from **Right** to **Left**:	$\alpha\mathrm{o}-2\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-\alpha\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]$.
Half-cycles **6**, from **Left** to **Right**:	$\alpha\mathrm{o}-2\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-\alpha\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]$.
Last half-cycle **6**, from **Left** to **Right**:	$\alpha\mathrm{o}-2\times\{(\alpha+1)\mathrm{u}-(\alpha+1)\mathrm{o}\}-(\alpha+1)\mathrm{u}-[\text{repeat }\{\alpha\mathrm{o}-\alpha\mathrm{u}\}]$.

And so on.

If the coding of the first intersection column from the left is /, then replace in the above half-cycle algorithms "o" by "u" and vice versa.

There are many other general coding patterns for which we can derive in a similar way the general half-cycle braiding algorithms associated with the successive revolutions. However, as already mentioned on pg.245, there is for high-class work often too much discrepancy between the number of bights required and the possible number of bights. Furthermore, general half-cycle algorithms are not particularly user-friendly. Although some readers might not immediately have realised that we encountered here in fact a '*braiding to pattern*' method (the '*braiding to pattern*' is a commonly encountered braiding method which is responsible for the very restrictive braiding-repertoire of many braiders; most knots described in the braiding literature are intimately associated with this method), but by a careful study of the various diagrams presented, this should become quite apparent. Since '*braiding to pattern*' methods are always very restrictive and not particularly user-friendly, it should therefore be obvious that our braiding methods so far discussed for Regular Band Knots are similarly handicapped. In the next issue No. 13 of *The Braider* we shall discuss a general method which does not suffer from these maladies.

A Pineapple Knot

The following Pineapple Knot is an interweave of three Casa Knots, each of an **even** number of parts.

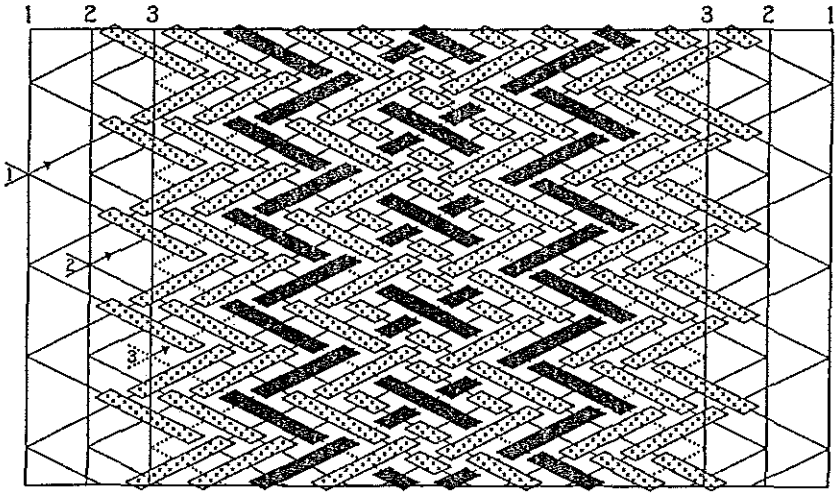


Fig. 248 — $A = 3$, $B^* = 5$, $P = 22$, $B = 15$, $P_{c_1} = P_{c_2} = 8$, $P_{c_3} = 6$.

1st component; 1st colour; between bight-boundaries 1 (left) and 2 (right).

1. $L \longrightarrow R$: Free Run.
2. $R \longrightarrow L$: o.
3. $L \longrightarrow R$: u.
4. $R \longrightarrow L$: $u - 2o$.
5. $L \longrightarrow R$: $o - 2u$.
6. $R \longrightarrow L$: $2u - 2o$.
7. $L \longrightarrow R$: $2o - 2u$.
8. $R \longrightarrow L$: $o - 2u - o - u - o$.
9. $L \longrightarrow R$: $u - 2o - u - o - u$.
10. $R \longrightarrow L$: $o - u - o - u - o - u - o$.

2nd component; 1st colour; between bight-boundaries 2 (left) and 1 (right).

1. $L \longrightarrow R$: $u - o - 2u - o - u - o$.
2. $R \longrightarrow L$: $o - u - 3o - u - o - u$.
3. $L \longrightarrow R$: $u - o - 2u - o - 2u - o$.
4. $R \longrightarrow L$: $o - 2u - 3o - u - 2o - u$.
5. $L \longrightarrow R$: $u - 2o - 2u - o - 2u - o - u$.
6. $R \longrightarrow L$: $o - 2u - o - u - 2o - u - 2o - u$.
7. $L \longrightarrow R$: $u - 2o - 2u - 2o - 2u - o - u$.
8. $R \longrightarrow L$: $2o - 2u - o - u - 2o - 2u - 2o - u$.
9. $L \longrightarrow R$: $2u - 2o - 2u - 2o - 2u - 2o - u$.
10. $R \longrightarrow L$: $2o - 2u - 2o - u - 2o - 2u - 2o - u$.

3rd component; 2nd colour; between bight-boundaries 3 (left) and 3 (right).

1. $L \longrightarrow R$: $2u - 2o - u - 2o - u - 2o - 2u$.
2. $R \longrightarrow L$: $2u - 2o - u - 2o - u - 3o - 2u$.
3. $L \longrightarrow R$: $2u - 2o - u - 2o - u - 2o - 3u$.
4. $R \longrightarrow L$: $2u - 2o - u - 2o - 2u - 3o - 2u$.
5. $L \longrightarrow R$: $2u - 2o - u - 2o - u - 3o - 3u$.
6. $R \longrightarrow L$: $2u - 2o - u - 3o - 2u - 3o - 2u$.
7. $L \longrightarrow R$: $2u - 2o - u - o - u - o - u - 3o - 3u$.
8. $R \longrightarrow L$: $2u - 2o - 2u - 3o - 2u - 3o - 2u$.
9. $L \longrightarrow R$: $2u - 3o - u - o - u - o - u - 3o - 3u$.
10. $R \longrightarrow L$: $2u - 3o - 2u - 3o - 2u - 3o - 2u$.

THE BRAIDER

APPENDIX 1997

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A quarterly publication
for
the braiding artisan

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Some comments we received

Besides some positive comments as regards the contents of *The Braider*, we received a negative comment from Tom Hall with reference to Issue No. 10, pp. 209 – 215 as follows: *'The piece on the 3 ply 3 part/4 bight knots was good. It might help some people with the dropping of the end. I know I do not always do it in the best way. I did think you were a little hard on all of the whipmakers. Even though what you said was true it made the article sound tacky. It does not make you or "The Braider" look very professional. You could of got the same point across without all the name calling.'*

We also received some comments from Pieter Van de Griend. He also was critical about some remarks relating to some references in this article, but unfortunately he seems to have missed the gist of the discussion. Furthermore he seems to be quite allergic to the comments about the value or purpose of mathematics as well as to the adverse comments about some fashionable, but nevertheless fallacious trends in mathematics (he is a mathematician by the way). No doubt, he is of course not the only mathematician who very much dislikes the comments about their 'holy' subject. It is interesting though that to date none of their fraternity has submitted a solution to any of the questions posed. Since their 'holy' subject plays an important role in nearly all these questions, one can only speculate as to why not.

It is to be expected that some may dislike certain references in *The Braider* to the relevant published material under discussion and their author(s) as well as to fashionable acceptances and practices. Although we limit our references to published cases which need a critical evaluation or have a direct connection with the subject matter under discussion, hence often a detailed or specific reference is not only fair but is also essential to the reader in order to obtain a full understanding of the matter, this may not be appreciated by those who worship fashionable acceptances and/or believe in political toadyism. For these people it seems that detailed referencing constitutes name-calling; they obviously don't like to correct misleading or incorrect information.

Braiding has suffered and still suffers from accepted fashionable nonsense and secrecy. It is therefore high time to expose myths and fallacies, and hence it is one of the prime purposes of *The Braider* to do so. Since *The Braider* is committed to a high technical standard, it will therefore not hesitate to give critical comments in areas where such comments are not only more than justified, but are in fact much overdue. Those who can't hack the course should get off the horse.

We like to thank Doug Van Tassel for a most justified comment concerning the lack of specific subject matter indication in the listings under the *Contents* on the front-cover of each issue. We should of course have addressed this item much earlier, and explained the reason for the form of this list and its purpose. The **form** of the *Contents* list is to enable us to keep each entry on one line as much as possible. The **purpose** of the *Contents* list is to indicate only the *number* of articles in the issue concerned, hence not to indicate any specific subject matter dealt with. Consequently the *Contents* list is of little or no real reference value and hence the front-cover carries no page-numbers.

In order to find the relevant pages concerning a specific subject, one needs at least a reasonably comprehensive **index**. However, *The Braider* is a continuing series of quarterly issues, and hence an up-to-date index cannot be provided. The best solution therefore is for those who would like to possess a more detailed contents-list to compile their own index and keep it up-to-date.

Ed's Algorithm

The English philosopher, and essayist Francis Bacon said in his *Essayes*, 1597 : *"Some bookes are to bee tasted, others to bee swallowed, and some few to bee chewed and digested"*. The same may be said for subjects, and braiding is one which certainly can do with a fair amount of chewing and digesting. Even seemingly 'simple' braidforms such as Regular Knots require a lot of chewing and digesting before one can say that he or she got it all under his or her belt. It appears that only a very few people do appreciate this (certainly none in the academic world) and, in stark contrast to most braiders, realise that there is much more to those seemingly 'simple' Regular Knots than meets the eye. One of those rare 'birds' who does realise that there is much more to those 'simple' Regular Knots is Ed Pass from Florence, Arizona, U.S.A., who derived an alternative algorithm diagram to the one we have used so far. In recognition to his realisation and effort, we shall call his algorithm diagram: **Ed's Algorithm**. Although Ed's Algorithm may be obtained in different ways, we shall here derive it from our conventional algorithm diagram. The conventional algorithm diagram can readily be written down with the aid of the value for Δ^*, where Δ^* is the increase in bights when the Regular Knot p/b gets enlarged by a fundamental enlargement step in accordance with Method II (this is clearly indicated by the formula for Δ^* in the question on pg. 115 of *The Braider*, Issue No. 6). In the same Issue on pg. 117 it was mentioned that the sum of any two vertically aligned i-values is equal to $b-1$. By making use of this fact and of Δ^*, we obtain the uppermost algorithm diagram in Fig. 1. Then in *The Braider*, Issue No. 6, pg. 115, the formula for Δ indicates that its value is equal to the increase in bights when the Regular Knot p/b gets enlarged by a fundamental enlargement step in accordance with Method I, and on pg. 133 of *The Braider*, Issue No. 7 we noted that $\Delta + \Delta^* = b$. With these relationships, the uppermost algorithm diagram in Fig. 1 readily transforms via the second algorithm diagram from the top into the third algorithm diagram from the top. This third algorithm diagram from the top can be written as the fourth algorithm diagram from the top. When we now add 1 to every i-value in this fourth algorithm diagram from the top, we obtain the lowermost algorithm, which is **Ed's Algorithm**.

In our conventional algorithm diagram we have the relationships:

$$\text{for an } \mathbf{even} \text{ numbered half-cycle } h_e: \qquad i = \frac{h_e - 2}{2}.$$

$$\text{for an } \mathbf{odd} \text{ numbered half-cycle } h_o: \qquad i = \frac{h_o - 3}{2}.$$

Lets call the values associated with the intersection columns in Ed's Algorithm the ι-values (pronounced: iota-values) in order to distinguish them from the i-values in the conventional algorithm diagram. Since $\iota = i + 1$ we thus obtain:†

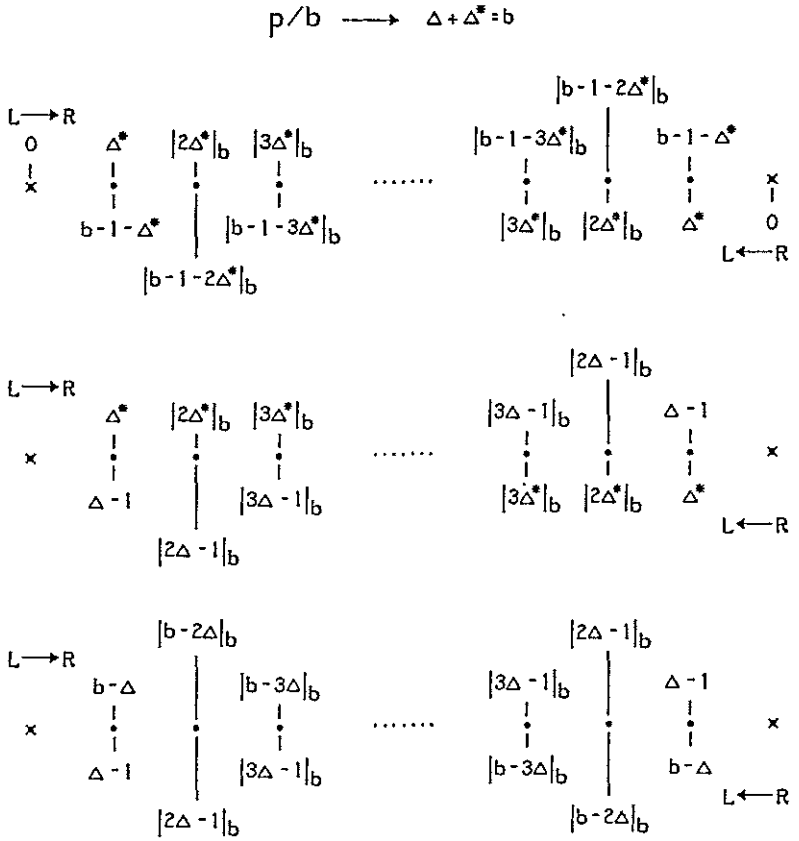
$$\text{for an } \mathbf{even} \text{ numbered half-cycle } h_e: \qquad \iota = \frac{h_e}{2} = \left\lfloor \frac{h}{2} \right\rfloor.$$

$$\text{for an } \mathbf{odd} \text{ numbered half-cycle } h_o: \qquad \iota = \frac{h_o - 1}{2} = \left\lfloor \frac{h}{2} \right\rfloor.$$

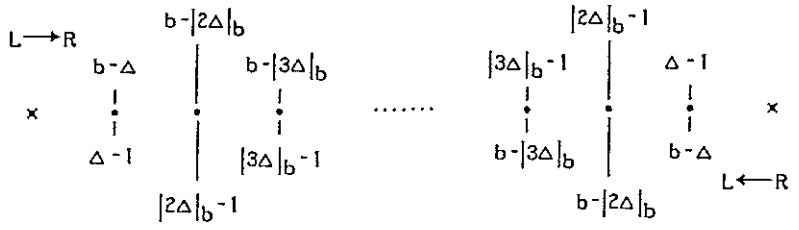
Thus in Ed's Algorithm we have to read for half-cycle h the ι-values which are:

$$\leq \left\lfloor \frac{h}{2} \right\rfloor.$$

† $\lfloor x \rfloor$ denotes the greatest whole number equal to or smaller than x.



ONLY WHEN $\triangle = 1$ ARE THERE n-VALUES FOR WHICH $|n\triangle|_b = 0$, WHERE n IS A NATURAL NUMBER. WHEN WE THEN TAKE INSTEAD OF 0 THE VALUE b, WE OBTAIN



WHEN WE ADD 1 TO EACH i-VALUE WE OBTAIN ED'S ALGORITHM SHOWN BELOW

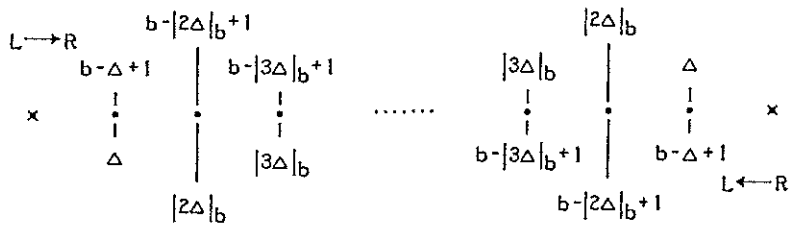


Fig. 1 — The derivation of Ed's Algorithm from the algorithm diagram.

Of the examples Ed Pass sent us, three are shown in Fig. 2, and one in Fig. 3.

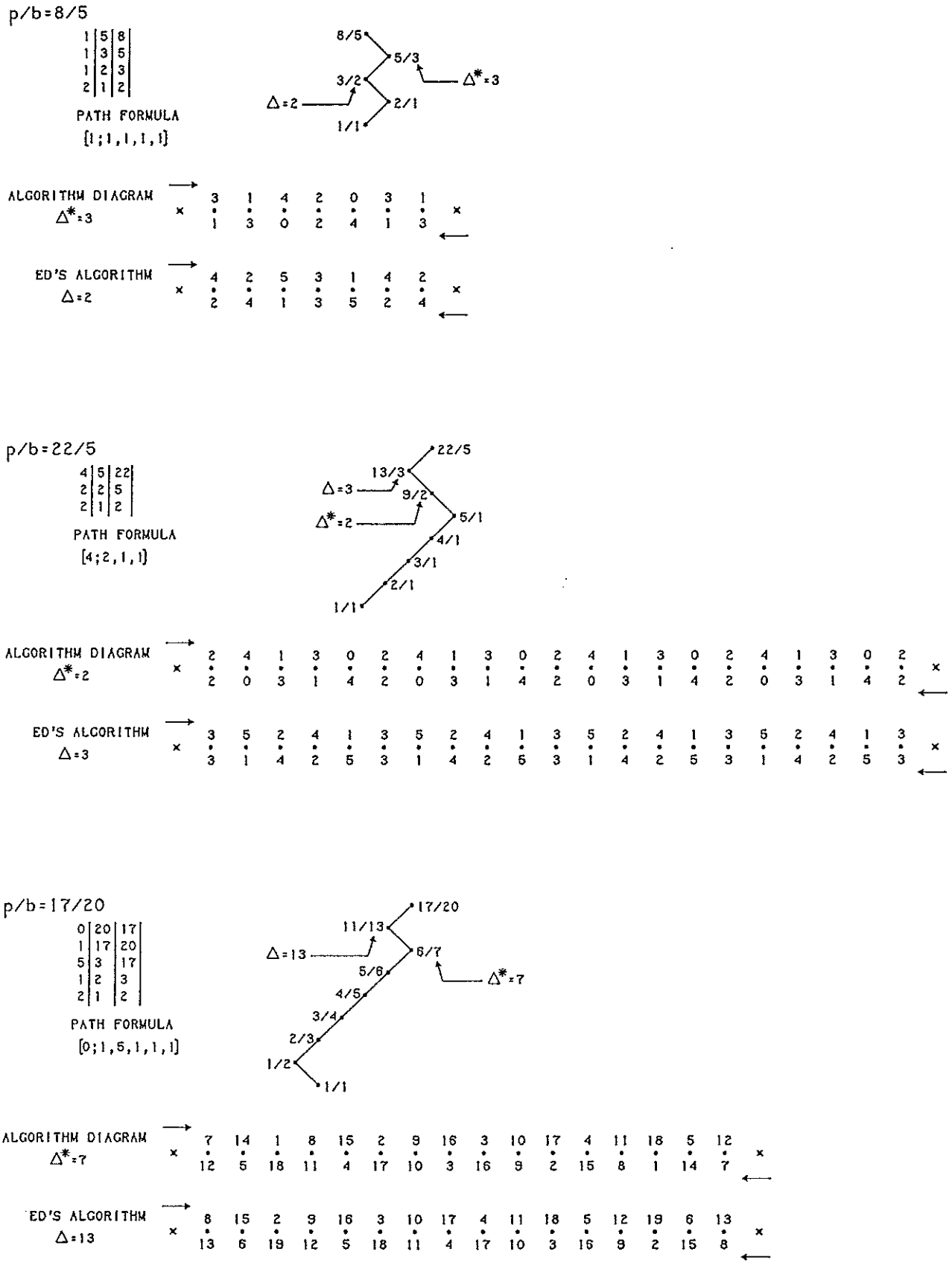


Fig. 2 — Ed's Algorithm and the algorithm diagram (codings have been omitted).

With the aid of Euclid's algorithm (refer to *The Braider*, Issue No. 7, pg. 135) we determine the path-formula of the Regular knot, which gives us its path in the RKT. From this path we obtain its Δ-value and Δ^*-value. With the Δ^*-value we construct the algorithm diagram, and with the Δ-value we construct Ed's algorithm.

Fig. 3 shows an example of a Regular Knot which is neither column nor row coded.

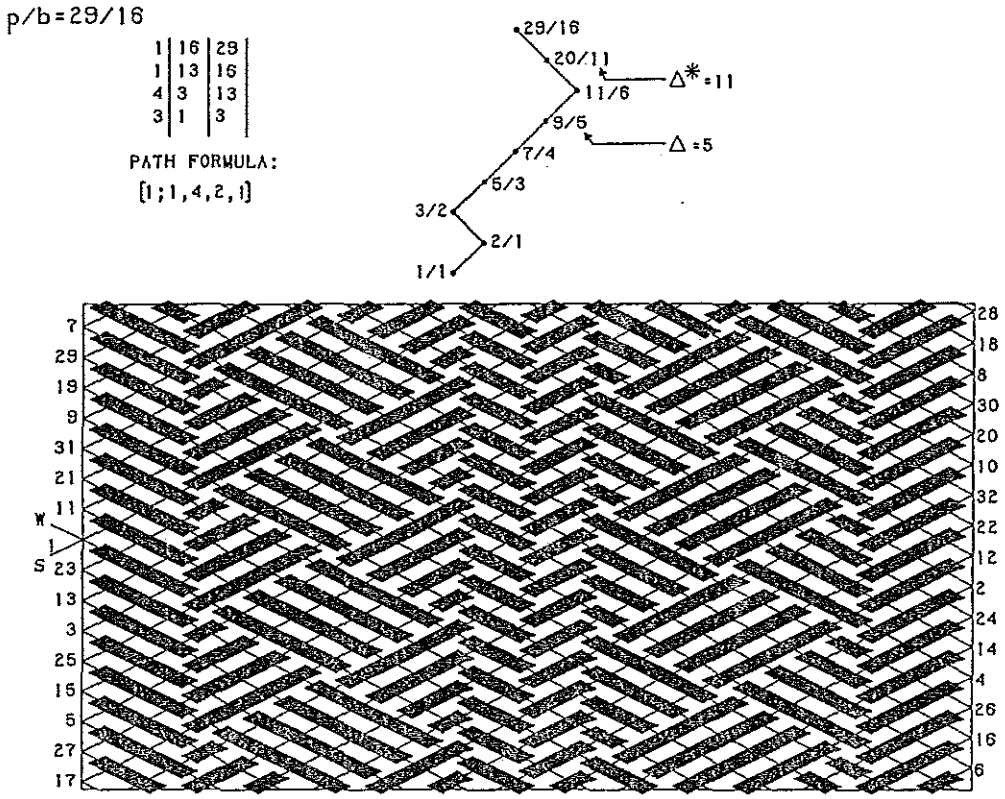


Fig. 3 — A Pampas-style knot with $p/b = 29/16$.

Since there are four different coded half-cycles from lower-left to upper-right we need an algorithm diagram for each type, and since there are four different coded half-cycles from lower-right to upper-left, none of which can be coupled with any of the four types from lower-left to upper-right, we also need an algorithm diagram for each of them. Hence we require a total of eight algorithm diagrams.

L ⟶ R

11	6	1	12	7	2	13	8	3	14	9	4	15	10	5	0	11	6	1	12	7	2	13	8	3	14	9	4	HALF-CYCLE			
u	u	u	o	o	o	o	u	u	u	u	o	u	u	o	o	u	o	o	o	o	u	u	u	u	o	o	o	1	9	17	25
u	u	u	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	o	u	o	o	o	3	11	19	27
u	u	u	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	o	o	o	5	13	21	29
u	u	u	o	u	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	o	o	o	7	15	23	31

R ⟶ L

11	6	1	12	7	2	13	8	3	14	9	4	15	10	5	0	11	6	1	12	7	2	13	8	3	14	9	4	HALF-CYCLE			
u	u	u	o	u	u	u	u	o	o	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	o	o	o	2	10	18	26
u	u	u	o	o	o	o	u	u	u	u	o	u	u	o	o	u	u	o	o	o	o	u	u	u	o	o	o	4	12	20	28
u	u	u	o	o	o	u	u	u	u	o	o	u	u	o	o	u	o	o	o	o	u	u	u	u	o	o	o	6	14	22	30
u	u	u	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	u	u	o	o	o	o	u	o	o	o	8	16	24	32

For convenience sake we have arranged the algorithm diagrams for the lower-right to

upper-left half-cycles so that we can read them from left to right. The conventional algorithm diagrams with their i-values are shown above on pg. v; the half-cycles associated with each one are indicated to the right of the thick vertical line.

Similarly the eight diagrams for Ed's algorithms with their ι-values are shown below.

L ⟶ R

12	7	2	13	8	3	14	9	4	15	10	5	16	11	6	1	12	7	2	13	8	3	14	9	4	15	10	5	HALF-CYCLE			
u	u	u	o	o	o	o	u	u	u	u	o	u	u	o	o	u	o	o	o	o	u	u	u	u	o	o	o	1	9	17	25
u	u	u	o	o	o	u	u	u	u	o	o	u	u	o	o	u	u	u	u	o	o	o	o	u	o	o	o	3	11	19	27
u	u	u	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	o	o	o	5	13	21	29
u	u	u	o	u	u	u	u	o	o	o	o	u	u	o	o	u	u	o	o	o	o	u	u	u	o	o	o	7	15	23	31

R ⟶ L

12	7	2	13	8	3	14	9	4	15	10	5	16	11	6	1	12	7	2	13	8	3	14	9	4	15	10	5	HALF-CYCLE			
u	u	u	o	u	u	u	u	o	o	o	o	u	u	o	o	u	u	u	o	o	o	o	u	u	o	o	o	2	10	18	26
u	u	u	o	o	o	o	u	u	u	u	o	u	u	o	o	u	u	o	o	o	o	u	u	u	o	o	o	4	12	20	28
u	u	u	o	o	o	u	u	u	u	o	o	u	u	o	o	u	o	o	o	o	u	u	u	u	o	o	o	6	14	22	30
u	u	u	o	o	u	u	u	u	o	o	o	u	u	o	o	u	u	u	u	o	o	o	o	u	o	o	o	8	16	24	32

The half-cycle algorithms are as follows:

1. Free Run.
2. o.
3. o.
4. $u - 2o$.
5. $u - o - u$.
6. $u - 3o - u$.
7. $2u - 3o$.
8. $3u - o - u - o - u$.
9. $u - o - u - 2o - 2u$.
10. $2u - 3o - u - o - u - o$.
11. $u - o - u - 2o - u - o - u - o$.
12. $u - o - u - 5o - u - o$.
13. $3u - 3o - u - o - u - o$.
14. $2u - o - u - 5o - 2u - o$.
15. $3u - 4o - u - 2o - u - o$.
16. $2u - o - 2u - 3o - 2u - 2o - u - o$.
17. $2u - 2o - u - 6o - 2u - o$.
18. $5u - 4o - 2u - 2o - 2u - o$.
19. $2u - 2o - 2u - 3o - 2u - 3o - u - o$.
20. $2u - 2o - 3u - 3o - u - 3o - 2u - 2o$.
21. $2u - o - 3u - 4o - 2u - 2o - 2u - 2o$.
22. $2u - 2o - 2u - 2o - u - 5o - 3u - 2o$.
23. $5u - 3o - u - 2o - u - 3o - 2u - 2o$.
24. $3u - o - 3u - 2o - u - 2o - 3u - 3o - u - 2o$.
25. $3u - 2o - 3u - o - u - 2o - u - 3o - 3u - 2o$.
26. $3u - o - 3u - 3o - u - 2o - 3u - 3o - 2u - 2o$.
27. $3u - 3o - 2u - 2o - u - 2o - 4u - 3o - u - 2o$.
28. $3u - 4o - 3u - o - u - 2o - 2u - 4o - 3u - 2o$.

29. $3u - 2o - 4u - 2o - u - 2o - 3u - 4o - 2u - 2o$.
30. $3u - 3o - 4u - 2o - u - 2o - u - 4o - 4u - 3o$.
31. $3u - o - 4u - 4o - u - 2o - 2u - 4o - 3u - 3o$.
32. $3u - 2o - 4u - 3o - 2u - 2o - 4u - 4o - u - 3o$.

It is important to check whether or not the half-cycle algorithms for a Regular Knot are possibly correct. A simple check is offered by the number of crossings in a half-cycle.

(1). The first half-cycle is always a free run, hence no crossings.

(2). The half-cycles $2n$ and $2n+1$ have each $\left\lfloor \frac{np}{b} \right\rfloor$ crossings, where $1 \leq n \leq b-1$.

(3). The half-cycle $2b$ has $p-1$ crossings.

(4). The sum of the crossings in the half-cycles m and $2b+1-m$ is equal to $p-1$, where $1 \leq m \leq 2b$.

Then there is a further check for Regular Knots with $p < b$. When $p < b$ the overall number of half-cycles of the string-run consist of p sets of half-cycles, where all the half-cycles of a set have each the same number of crossings. The number of half-cycles in the first set (the free-run half-cycles, the half-cycles each of which intersects zero other half-cycles) is equal to:

$$2\left\lfloor \frac{b}{p} \right\rfloor + 1 .$$

The number of half-cycles in the subsequent consecutive sets greater equal 2 and less than p, hence excepting the last set (the pth set), is equal to:

$$2\left(\left\lfloor \frac{(n+1)b}{p} \right\rfloor - \left\lfloor \frac{nb}{p} \right\rfloor\right), \qquad \text{where} \quad n = 1, 2, 3, \cdots, (p-2).$$

The number of half-cycles in the last set (the pth set) is equal to:

$$2\left(b - \left\lfloor \frac{(p-1)b}{p} \right\rfloor\right) - 1 = 2\left\lfloor \frac{b}{p} \right\rfloor + 1 .$$

With these formulae it is easy to prove that the sequence of the number of half-cycles in the sequential sets is palindromic†. Once again, **Ed Pass** has to be **congratulated** with discovering by himself these relationships, the more so since only recently in the Dutch publication "Het Knoopeknauwertje" (The Unraveller of Knot-mysteries), Issue No.7, its writer/editor (who is a mathematician) is obviously unaware of these half-cycle properties as otherwise he would not have written the following half-cycle algorithms for the well-known $p/b = 5/6$ Spanish Ring Knot:

1. $L \longrightarrow R$: Free Run.
2. $R \longrightarrow L$: Free Run.
3. $L \longrightarrow R$: o.
4. $R \longrightarrow L$: o.
5. $L \longrightarrow R$: $2o$.
6. $R \longrightarrow L$: $2o$.
7. $L \longrightarrow R$: $2o$.
8. $R \longrightarrow L$: $2o - u$.
9. $L \longrightarrow R$: $2o - u$.
10. $R \longrightarrow L$: $2o - 2u$.
11. $L \longrightarrow R$: $2o - 2u$.
12. $R \longrightarrow L$: $2o - 2u$.

It will immediately be seen that the sequence of the number of half-cycles in the sequential sets is $2, 2, 3, 2, 3$, which is not palindromic and hence the table of half-cycle algorithms must be incorrect. The sum of the crossings in the half-cycles 3 and 10 should be $p - 1 = 4$ and not 5. Furthermore, the sum of the crossings in the half-cycles

† A palindromic sequence of numbers is a sequence of numbers which, when taken in reverse order, gives the same sequence of numbers. For example: $5, 4, 6, 4, 6, 4, 5$.

5 and 8 should be $p-1=4$ and not 5. Apart from all this, it should be well known that the half-cycles $2n$ and $2n+1$, where $1 \leq n \leq b-1$, each have the same number of crossings always, and hence in the above case there is an error in the half-cycles 2 and/or 3, and in the half-cycles 4 and/or 5. It is obvious that we are here not dealing with a typing error, but rather with a case which shows that the nature of Regular Knots is not understood at all.

We hope that other braiders will follow the example set by Ed Pass, and hence will discover by themselves relationships which rule the world of braids. They will then obtain the satisfaction which undoubtedly Ed Pass obtains from braiding as a craft.

Knotters, Nutters and Conferences

Conferences are the ideal venues for people with limited capabilities who seek the limelight. This statement should only be seen as a general one of course since there are always innocent nonlimelight seeking participants who, with respect to conferences, don't know the ropes yet. Those novices believe that at these venues they will hear the experts in the field and learn a great deal from them, not realising yet that the participating experts are in general ex-spurts (ex = have been; spurts = drips under pressure). The conference organisers are, consciously or subconsciously, well aware of this and hence normally give each speaker roughly half an hour in which to perform his act. This ensures that a speaker doesn't get the time to either present anything properly (hence of real value to the audience) or make a fool of himself. Nevertheless, conferences are great venues at which to underline one's importance and to rub shoulders, even if it is only for acquiring the smell of a well established ex-spurt.

Was the IGKT-North American Branch Conference in New Bedford, Massachusetts, commemorating the 50th aniversary of Ashley's death, such an event???

Going by the various reports, it certainly looks very much like it as far as the subject knot or braid is concerned. Of course, newspaper reports can be very misleading, but the reports in the *Boston Sunday Globe* of 3/8/1997, *The Standard-Times* of 7/8/1997, the *New Bedford Standard Times* of 10/8/1997 are in agreement with the Conference Report by the offical-unofficial chronicler of the event, Robert M. Wolfe, M.D.

The reader should not get the impression that the 50th aniversary of Ashley's death should not have been commemorated, to the contrary it should have been commemorated in a way which, in a technical sense, did not degrade the achievements of Ashley. Then what was so degrading???

It is the so-called **Ashley bend** #1452 which the ex-spurts like to see on a commemorative postage stamp. Those who read *The Braider* properly, and can even faintly remember the general gist of some discussions, will recall that we did discuss the "Ashley bend" in some previous issue (Issue No. 5) and showed that this bend was nothing more and nothing less than the two string 3-part/2-bight over-under coded Regular Cylindrical Braid, often referred to as the two string 3-part/2-bight over-under coded lanyard knot, the two string 3-part/2-bight Turk's Head lanyard knot, the two string walled crown knot, or the Footrope knot #783. It is a two string **crown knot** with a **wall knot** under it, the working ends of which are fed through the eye of the **crown knot**. It may be used as a bend when the string-ends through the eye of the crown are kept free of any load. In order to obtain then the maximum strength it is important to

tie and tighten this bend as a lanyard knot before using it as a bend as otherwise the lay of the strings is normally not as good as it can and should be. Since Ashley and apparently all "experts" missed the real nature of this bend it was a shame that two readers of *The Braider*, one being a speaker at this Conference and editor of the knotting journal *Het Knoopeknauwertje* in which was published (Issue No. 9) a letter of the other concerning this bend in connection with, among other, the Ashley commemorative postage stamp, either could not remember the general gist of our discussion concerning the "Ashley bend" or preferred to keep the other ex-spurts in the dark. Whatever the case may be, Ashley surely deserved to be commemorated for the book he wrote or by one of the knots he did discover, rather than by an already existing and ancient one he apparently didn't recognise the nature of. It is of course understandable that mistakes will be made and that oversights will take place, but there is no excuse for propagating them by those who should know better.

There was in the past, and still is at the present, a passionate discussion going on in the IGKT about **new knots** (see *Knotting Matters*, Issue No. 57, pp. 57–61). They seem to be the self-appointed franchise holders for approving the quality "**new**". This not only shows their inflated ego, but also their ignorance as far as knots are concerned. Not only is it rather irrelevant whether a knot is "new" or not, but we shall never know whether or not this is the case, because for the rather irrelevant determination of "new" or "not new" we need to be able to speak all the present languages and dialects in the world, be able to read all the written languages and dialects of the present and the past in the world, and have access to the knotting information associated with them. Seeing that the ex-spurts in the IGKT apparently don't know and seem to be unable to "discover" that **Ashley's bend** #1452 and the **Footrope knot** #783 are the same (both written down in Ashley's book in the English language), one can only wonder about their self-conferred expertise in knotting matters.

We have mentioned in the past that knots and braids are heavily infested with a good deal of nonsense from a practical as well as from a theoretical angle. There was with respect to this an interesting statement in the report (prepared by Robert Wolfe, M.D.) of the Conference as follows:
Vaughan Jones, professor of mathematics at the University of California, Berkeley, and Hon. Vice President of the IGKT [International Guild of Knot Tyers], gave a surprisingly understandable introduction to knots from the mathematician's perspective. He explained that, if the standing end and free end aren't fused together, you have mathematically, no-knot at all, and that only what we call closed loops (with their various twists, crossings, and other permutations) are considered knots.

Hence from the mathematician's perspective the fusing of the ends must be considered an essential part of knotting — no fusing, no knot. But also when one starts with a string of which the ends are fused, whatever is produced is from a mathematicians perspective no-knot at all. In real, hence physical, knots, however, the ends are very rarely fused and if they are fused this fusing has no effect on the physical knot since it does not form a part of the knotting process. Thus mathematical knots are not physical knots and hence whatever the mathematician's theory states about their knots cannot be considered to be of value to physical knots; if there is at times a correlation, then that is obviously purely coincidental. The mathematician's knots are not real, hence physical, knots, but are instead hypothetical fantasy knots, and consequently so is their associated theory with respect to the real physical world around us. We have stressed this several times before and it is prudent to appreciate the inevitable consequences —

the introduction of a great deal of hairy-fairy nonsense derived from hairy-fairy hypothetical theories.

A braider or knotter should be aware that Conferences as well as Publications can be, and often are, the propagation beds for nonsense and confusion, originating from both the practical and theoretical angle. Hence it is very important to evaluate the presented material in a critical way and to ask questions (the more the better) where obscurity lingers.

It is of some interest to note that roughly over the last decade or so there is a conspicuous infiltration of topological knot theorists (they belong to the fraternity of the theoretical mathematicians) into the realms of the practical knotters and braiders. It is fair to say that these mathematicians have no genuine interest in real, hence physical, knots and braids, which is of course understandable. One therefore wonders why they like to be associated with people whose field of interest is really quite different, and vice versa one wonders why practical knotting and braiding organisations are apparently more than delighted to have them as members or office holders. Wouldn't the most likely reason be the acquisition of the aromatic odour of each other, which from the mathematician's point of view would give them some practical credibility, and which from the knotter's and braider's point of view would give them some academic status. It certainly is all a world of pretence.

Errata — *The Braider*

Some typing errors were noted in at least some copies of the following Issues of *The Braider*. Our grateful thanks again to Doug Van Tassel who detected them. The corrections required are indicated below in bold:

No. 9

none

No. 10

pg. 213, line 2 — **choose**

No. 11

pg. 232, line 17 — **general**
pg. 245, line 8 — **general**

No. 12

pg. 257, line 7 from bottom — delete **thinspace**

THE BRAIDER

No.13 FEBRUARY 1998.

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A quarterly publication for the braiding artisan

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Solution to the Question in Issue No. 12

Question on pg. 256.

First we shall present the solutions to the sub-questions 1-8, then we shall discuss in more detail the even-numbered sub-questions of these. In each of the Figs. 249 - 252 the top-row of diagrams depicts the string-run diagrams associated with the sequential enlargement steps in the braiding process by means of enlargements, while each string-run diagram in the bottom row of diagrams depicts one of the normally used braiding methods. Where appropriate, under each diagram in the bottom row are indicated the sequential enlargement steps.

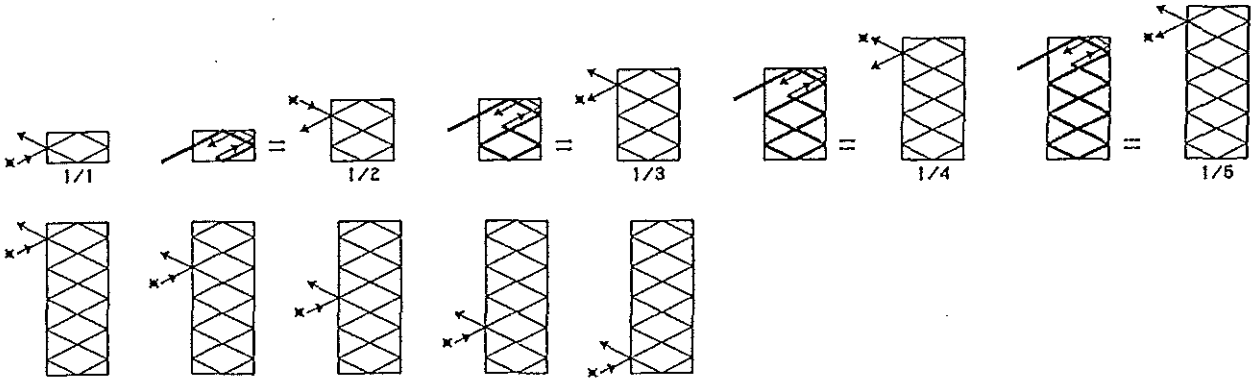


Fig. 249 — Solutions to the sub-questions (1) and (2).

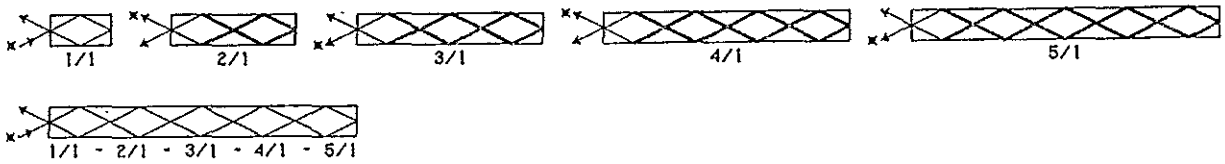


Fig. 250 — Solutions to the sub-questions (3) and (4).

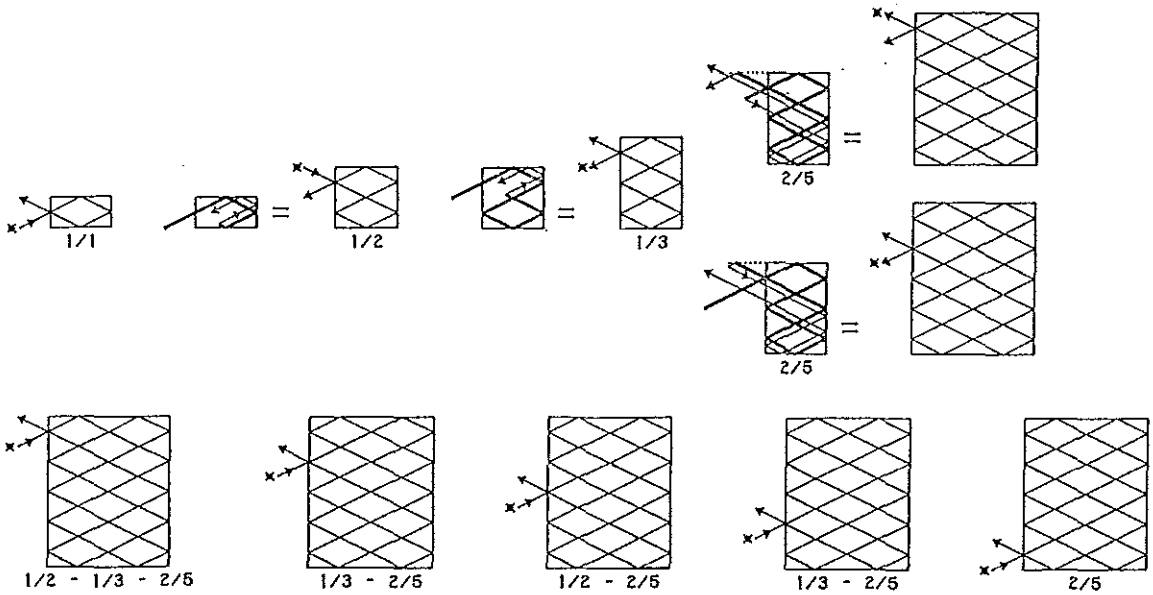


Fig. 251 — Solutions to the sub-questions (5) and (6).

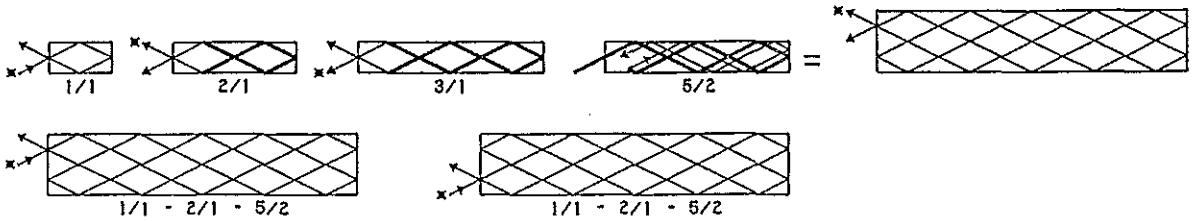


Fig. 252 — Solutions to the sub-questions (7) and (8).

In the question it was assumed that the braiding process started at the **left-hand** bight-boundary in an **upwards** direction. The bight-index of our starting-point is 0. The first bight-point on the left-hand bight-boundary which the string-run reaches after leaving the starting-point (bight-index 0) has the bight-index 1, the second bight-point on the left-hand bight-boundary which the string-run reaches after leaving the starting-point has the bight-index 2, the third bight-point on the left-hand bight-boundary which the string-run reaches after leaving the starting-point has the bight-index 3, and so on. When the string-run reaches bight-index 'x', a Regular Rectangular Mat is obtained only when x fulfils one of the conditions Ⓐ, Ⓑ, Ⓒ, or Ⓓ, depicted in Fig. 253, where the black dot becomes a left-hand bight-point.

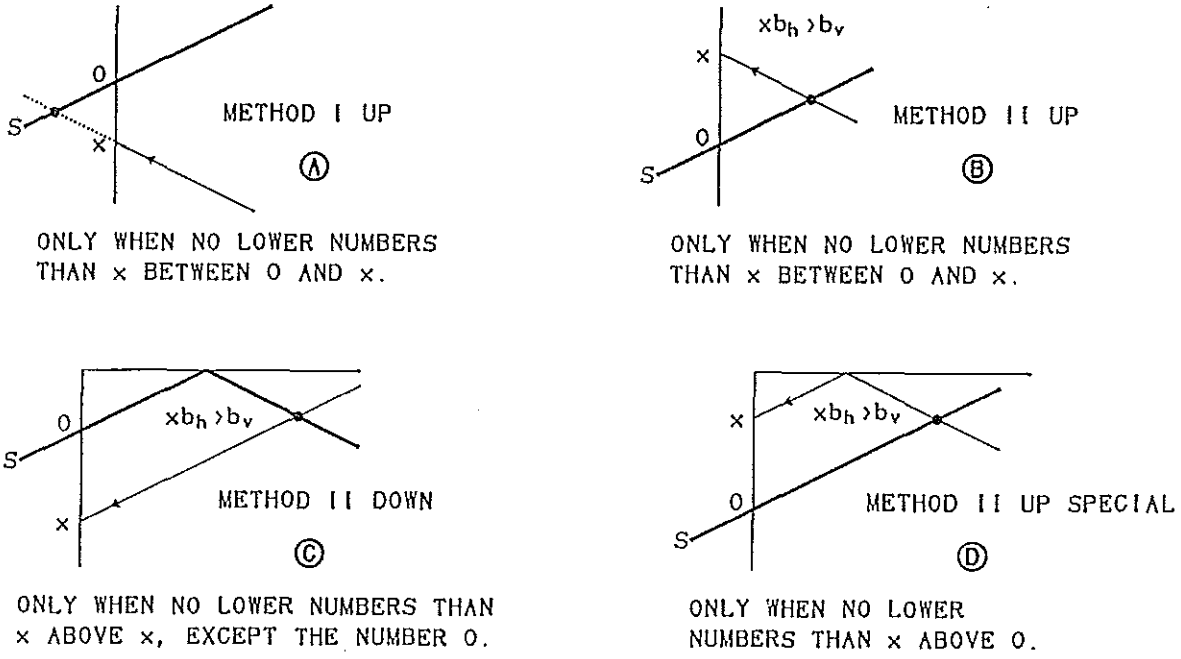


Fig. 253 — Conditions for an intermediate size Regular Rectangular Mat.

Note that in the cases Ⓑ, Ⓒ, and Ⓓ, the condition $x \cdot b_h > b_v$ must apply.

When $b_h > b_v$, we can write $b_h = nb_v + |b_h|_{b_v}$ where n is a positive integer, consequently for $x = 1$ we obtain the intermediate Regular Rectangular Mats with $b_h/b_v = 1/1, \ldots, n/1$; and when Ⓐ applies for $x = 1$ then also $\overline{b_h/b_v} = (n+1)/1$.

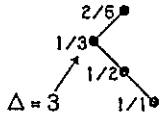
Hence in none of the five cases of sub-question (2) (see Fig. 249, bottom row of diagrams) do we encounter intermediate Regular rectangular Mats.

In the case of sub-question (4) (see Fig. 250, diagram in bottom row) we encounter the intermediate Regular Rectangular Mats $b_h/b_v = 1/1$, $2/1$, $3/1$ and $4/1$.

In order to examine the solutions to sub-question (6) we make use of Fig. 254 in which the leftmost column with dots, crosses and bight-indices is associated with the

leftmost string-run diagram in the bottom row of Fig. 251. The second column from the left with dots, crosses and bight-indices in Fig. 254 is associated with the second string-run diagram from the left in the bottom row of Fig. 251. And so on.

0-•	3 ×	3 •	1 ×	1 •
3 ×	0-•	1 ×	3 •	4 ×
2 •	1 ×	0-•	4 ×	3 •
1 ×	2 •	4 ×	0-•	2 ×
4 •	4 ×	2 •	2 ×	0-•



0	5	2
2	2	5
2	1	2

PATH FORMULA:
[0; 2, 1, 1]

Fig. 254 — Bight-indices associated with the solutions to sub-question (6).

The bight-point where the string-run starts (the Standing End) receives the bight-index 0, then we move **upwards** two bight-points each time, whereby each such second bight-point receives a dot. Continue **past** the uppermost bight-point, then move **downwards**. Each second bight-point, while our movement is **downwards**, receives a cross. Continue **past** the lowermost bight-point, then move **upwards** again, whereby each second bight-point, while our movement is **upwards**, receives a dot. Continue till we arrive in the bight-point of the Standing End (bight-index 0).

Thus, when the uppermost bight-point received a dot, then the next bight-point, which we now have to give a cross, is the second from the top; and when the second bight-point from the top received a dot, then the next bight-point, which we now have to give a cross, is the uppermost bight-point (we come to this bight-point again on our **downwards** movement). Similarly, when the lowermost bight-point received a cross, then the next bight-point, which we now have to give a dot, is the second from the bottom; and when the second bight-point from the bottom received a cross, then the next bight-point, which we now have to give a dot, is the lowermost bight-point (we come to this bight-point again on our **upwards** movement). Note that we **enter** a bight-point with a dot from the SE direction (↖), and that we **enter** a bight-point with a cross from the NE direction (↙).

Next we have to give each bight-point a bight-index. We have already given the bight-point with the Standing End the bight-index 0. The easiest way in assigning bight-indices to the bight-points is to give each bight-point a bight-index at the time we give it a dot or a cross. To do this, we add the value of Δ to the previous bight-index and determine the mod b_v value[†] of this sum, which is then the bight-index of our bight-point which has to be marked with a dot or a cross depending on our direction of movement.[‡] The value of Δ can readily be obtained from the path in the RKT, which in turn may be obtained with Euclid's algorithm via the path formula.

With the aid of the conditions outlined in Fig. 253 we can now readily determine from Fig. 254 the intermediate Regular Rectangular Mats we encounter during each of the five different construction processes associated with sub-question (6). These successive intermediate Regular Rectangular Mats are indicated in Fig. 251 (the bottom row of diagrams) for each process under its relevant string-run diagram.

Both solutions to sub-question (8) contain the same intermediate Regular Rectangular Mats, as is indicated in Fig. 252 (the bottom row of diagrams) for each process under its relevant string-run diagram. The specifications of these intermediate Regular Rectangular Mats follow immediately from what has been said on pg. 274.

[†] See *The Braider* No. 4, pg. 77.

[‡] Note that Δ is used here in a different context than in *The Braider* No. 11, pp. 230 - 231.

By comparing the sub-questions (1) and (2); (3) and (4); (5) and (6); (7) and (8), we see that only in the case of sub-question (4) we pass successively through every Regular Rectangular Mat on the path to the final one. We also see that sometimes we don't pass through any intermediate one. Hence the vast difference between Regular Knots and Regular Rectangular Mats is that in the construction of Regular Knots we **always** pass through **all** the intermediate ones on the path to the final one, whereas in the construction of Regular Rectangular Mats this is in general **not** the case.

Since the examples in our question were associated with very small Regular Rectangular Mats, it will be instructive to look at two larger Regular Rectangular Mats.

Example 1:

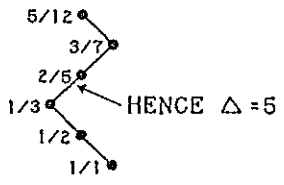
A Regular Rectangular Mat with $b_h = 5$ and $b_v = 12$.

S=1	S=2	S=3	S=4	S=5	S=6	S=7	S=8	S=9	S=10	S=11	S=12
0-•	5 ×	5 •	10 ×	10 •	3 ×	3 •	8 ×	8 •	1 ×	1 •	6 ×
5 ×	0-•	10 ×	5 •	3 ×	10 •	8 ×	3 •	1 ×	8 •	6 ×	1 •
7 •	10 ×	0-•	3 ×	5 •	8 ×	10 •	1 ×	3 •	6 ×	8 •	11 ×
10 ×	7 •	3 ×	0-•	8 ×	5 •	1 ×	10 •	6 ×	3 •	11 ×	8 •
2 •	3 ×	7 •	8 ×	0-•	1 ×	5 •	6 ×	10 •	11 ×	3 •	4 ×
3 ×	2 •	8 ×	7 •	1 ×	0-•	6 ×	5 •	11 ×	10 •	4 ×	3 •
9 •	8 ×	2 •	1 ×	7 •	6 ×	0-•	11 ×	5 •	4 ×	10 •	9 ×
8 ×	9 •	1 ×	2 •	6 ×	7 •	11 ×	0-•	4 ×	5 •	9 ×	10 •
4 •	1 ×	9 •	6 ×	2 •	11 ×	7 •	4 ×	0-•	9 ×	5 •	2 ×
1 ×	4 •	6 ×	9 •	11 ×	2 •	4 ×	7 •	9 ×	0-•	2 ×	5 •
11 •	6 ×	4 •	11 ×	9 •	4 ×	2 •	9 ×	7 •	2 ×	0-•	7 ×
6 ×	11 •	11 ×	4 •	4 ×	9 •	9 ×	2 •	2 ×	7 •	7 ×	0-•

S=1	S=2	S=3	S=4	S=5
1 → NO	1 → NO	1 → NO	1 → NO	1 → NO
2 → YES (A)	2 → YES (A)	2 → YES (A)	2 → NO	2 → NO
3 → NO	3 → YES (C)	3 → YES (C)	3 → YES (D)	3 → YES (D)
5 → YES (C)	5 → YES (D)	5 → YES (B)	5 → NO	5 → YES (B)
7 → NO	7 → YES (A)	7 → NO	7 → YES (A)	7 → NO
1/2 - 2/5 - 5/12	1/2 - 1/3 - 2/5 - 3/7 - 5/12	1/2 - 1/3 - 2/5 - 5/12	1/3 - 3/7 - 5/12	1/3 - 2/5 - 5/12

S=6	S=7	S=8	S=9	S=10	S=11
1 → NO	1 → NO	1 → NO	1 → NO	1 → NO	1 → NO
2 → YES (A)	2 → YES (A)	2 → YES (A)	2 → NO	2 → NO	2 → NO
3 → NO	3 → NO	3 → NO	3 → YES (B)	3 → YES (B)	3 → NO
5 → NO	5 → YES (B)	5 → YES (B)	5 → NO	5 → YES (B)	5 → NO
7 → NO	7 → YES (A)	7 → NO	7 → YES (A)	7 → NO	7 → NO
1/2 - 5/12	1/2 - 2/5 - 3/7 - 5/12	1/2 - 2/5 - 5/12	1/3 - 3/7 - 5/12	1/3 - 2/5 - 5/12	5/12

S=12
1 → NO
2 → NO
3 → NO
5 → YES (B)
7 → NO
2/5 - 5/12



0	12	5
2	5	12
2	2	5
2	1	2

PATH FORMULA:

[0; 2, 2, 1, 1]

The Standing End of the string (bight-index 0) can be positioned in any of the 12 bight-points on the left-hand bight-boundary. The 12 uppermost columns in the table above give the bight-indices (obtained with the aid of Δ for the upwards braiding

direction) of these bight-points for each such a Standing-End position. Since the b_v-values of the nodes on the path from 1/1 to 5/12 are $1, 2, 3, 5, 7$ and 12 it is sufficient to investigate the bight-indices $1, 2, 3, 5$ and 7 with respect to the conditions depicted in Fig. 253. This has been done in the table above, and under each such an investigation are indicated the consecutive Regular Rectangular Mats through which we pass. Note that the Working End in each consecutive Regular Rectangular Mat must finish at the left bight-boundary. Observe that we cannot start with the 1/1 Regular Rectangular Mat, and that in general not every node on the path represents a consecutive Regular Rectangular Mat in the braiding process under consideration.

In order to form a better understanding of the successive enlargement stages through which each construction process proceeds, we have depicted these stages, for each Starting End position under consideration, in Fig. 255.

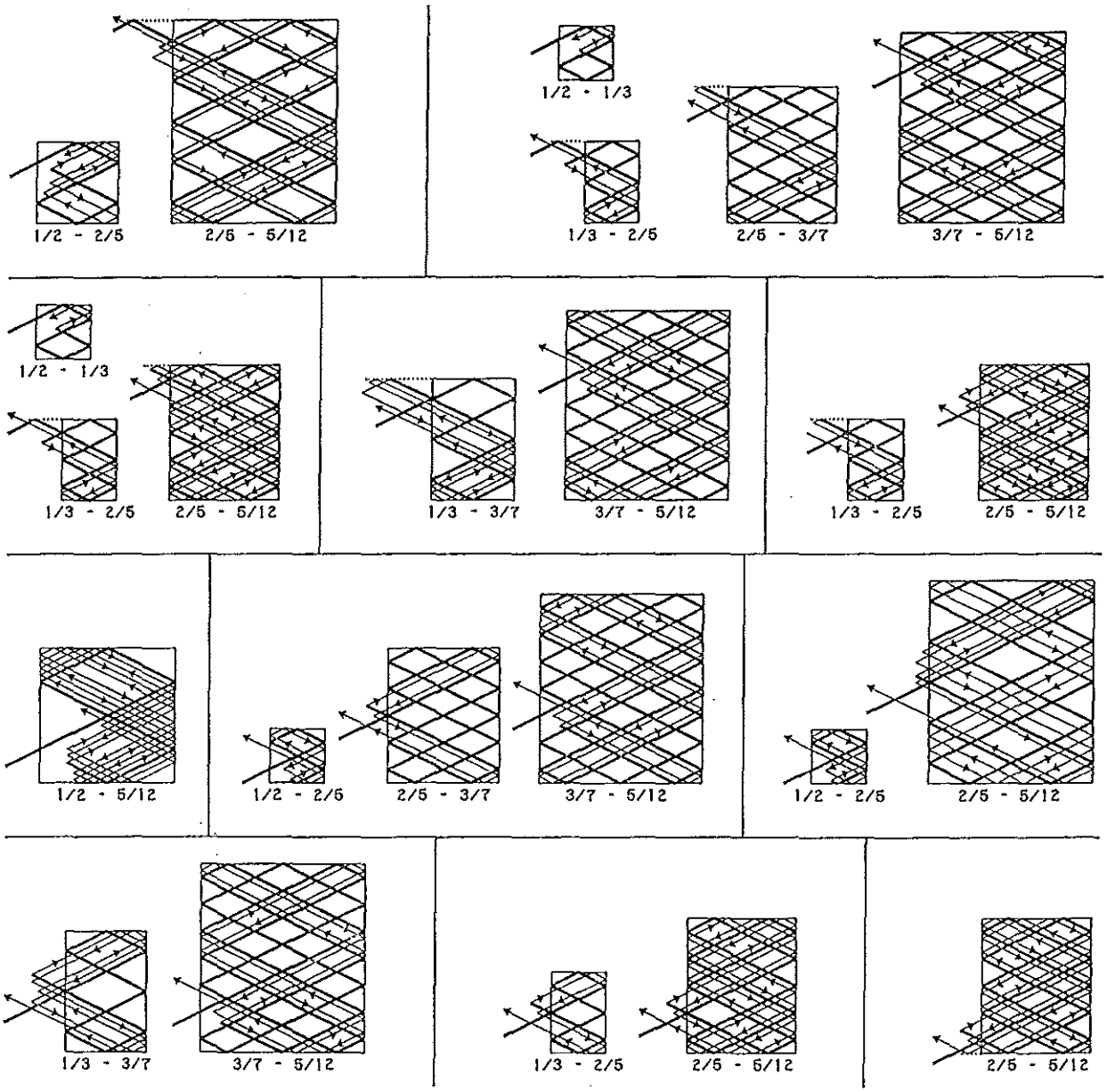


Fig. 255 — The consecutive enlargements associated with the construction procedures in Example 1.

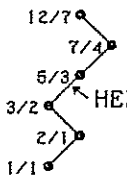
Example 2:

A Regular Rectangular Mat with $b_h = 12$ and $b_v = 7$.

S=1	S=2	S=3	S=4	S=5	S=6	S=7
0-•	3 ×	3 •	6 ×	6 •	2 ×	2 •
3 ×	0-•	6 ×	3 •	2 ×	6 •	5 ×
4 •	6 ×	0-•	2 ×	3 •	5 ×	6 •
6 ×	4 •	2 ×	0-•	5 ×	3 •	1 ×
1 •	2 ×	4 •	5 ×	0-•	1 ×	3 •
2 ×	1 •	5 ×	4 •	1 ×	0-•	4 ×
5 •	5 ×	1 •	1 ×	4 •	$_4$×	0-•

S=1	S=2	S=3
1 → YES (C);(A)	1 → YES (C);(A)	1 → YES (C);(A)
2 → NO	2 → YES (C)	2 → YES (C)
3 → YES (C)	3 → YES (D)	3 → YES (B)
4 → NO	4 → YES (A)	4 → NO
1/1 - 2/1 - 5/3 - 12/7	1/1 - 2/1 - 3/2 - 5/3 - 7/4 - 12/7	1/1 - 2/1 - 3/2 - 5/3 - 12/7

S=4	S=5	S=6	S=7
1 → YES (C)	1 → YES (C)	1 → YES (D)	1 → YES (D)
2 → YES (D)	2 → YES (D)	2 → NO	2 → NO
3 → NO	3 → YES (B)	3 → NO	3 → YES (B)
4 → YES (A)	4 → NO	4 → NO	4 → NO
1/1 - 3/2 - 7/4 - 12/7	1/1 - 3/2 - 5/3 - 12/7	1/1 - 12/7	1/1 - 5/3 - 12/7



HENCE $\Delta = 3$

1	7	12
1	5	7
2	2	5
2	1	2

PATH FORMULA:

[1;1,2,1,1]

The various data obtained for this Example are displayed in the table above. The successive enlargement stages through which each construction process proceeds is depicted in Fig. 256.

It will be evident that the braiding of Regular Rectangular Mats by means of enlargement methods is in general not a practical proposition. Even the use of the consistent 1st-order Method I and Method II enlargement processes† associated with any of the eight extreme corner bight-points has its practical limitations. Even for most Casacoded Regular Rectangular Mats is this the case. In fact, the braiding of nearly any braidform by means of enlargement methods has serious drawbacks when used as a general construction method. In due course we shall discuss the reasons why. It is nevertheless very important to fully understand, and hence be able to visualise these enlargement processes. After all, irrespective which construction procedure we employ in practice, we will generally always go through enlargement stages. Furthermore, other quite significant relationships are displayed by enlargement procedures.

† See *The Braider*, Issue No. 10, pg. 205.

In the next issue of *The Braider* we shall discuss the practical method by which to braid a Regular Rectangular Mat. Since this method is a practical one, it is not restricted to specific coding forms.

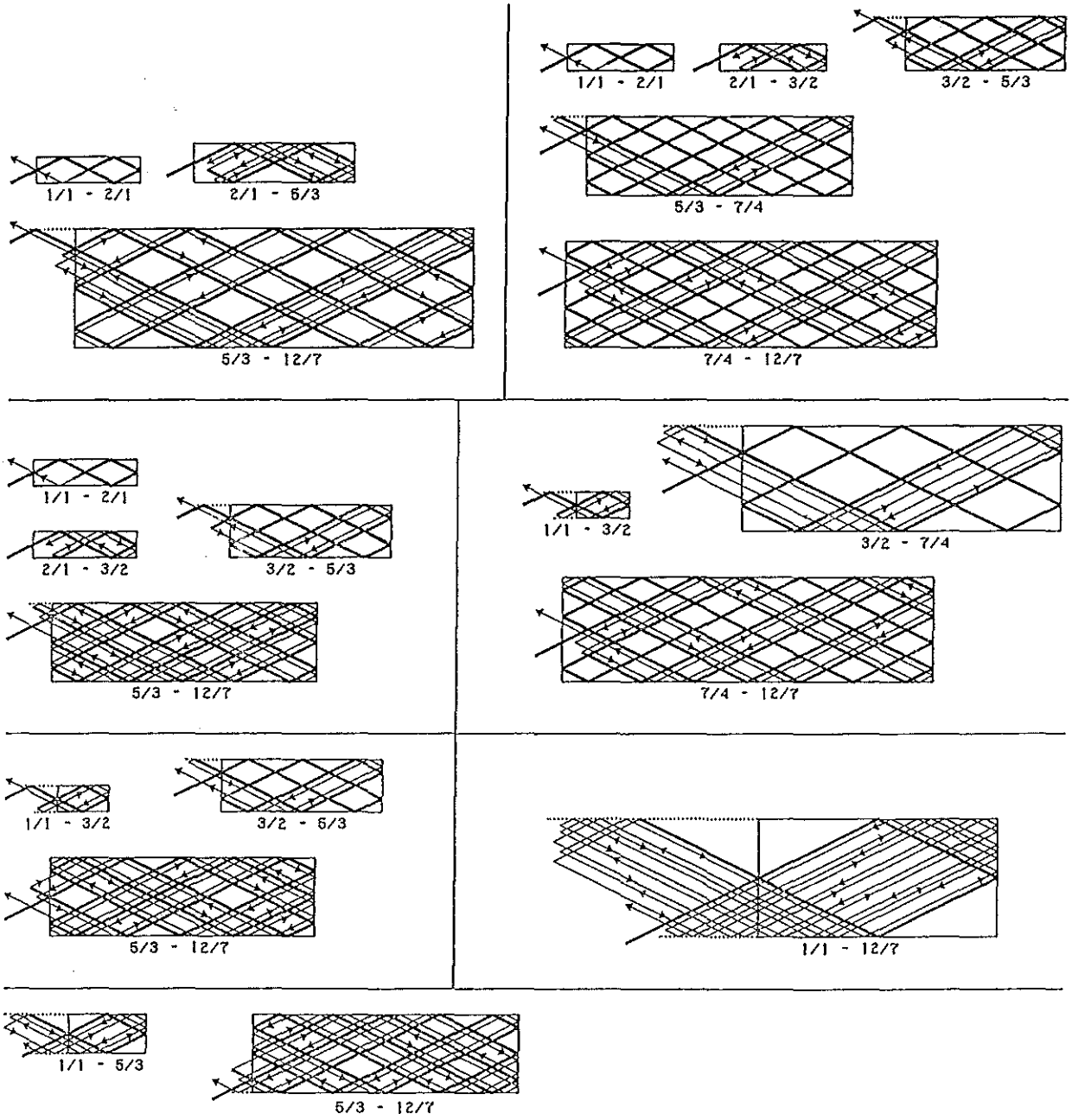


Fig. 256 — The consecutive enlargements associated with the construction procedures in Example 2.

** *In The Braider, Issue No. 11, we have seen the important role* c_{I_x} *and* c_{II_x} *play in the enlargement processes associated with Regular Rectangular Mats. Any particular* c_{I_x} *sequence is* **not** *specifically associated with only one* b_h/b_v *combination, but belongs instead to very many (in fact an infinite number) of* b_h/b_v *combinations. The same applies to any particular* c_{II_x} *sequence.*

What is the relationship between b_h *and* b_v *for Regular Rectangular Mats which have identical* c_{I_x} *sequences, and what is the relationship between* b_h *and* b_v *for*

Regular Rectangular Mats which have identical c_{II_x} *sequences?*

In The Braider, Issue No. 11, we have also discussed some **complementary** *relationships. Say the* $c_{I_{x_1}}$ *sequence belongs to a certain set of Regular Rectangular Mats. In relation to this set of mats, to which set of Regular Rectangular Mats belongs the* **complementary** $c_{I_{x_2}}$ *sequence, where* $c_{I_{x_2}} = b_v - c_{I_{x_1}}$ *? Similarly, to which set of Regular Rectangular Mats belongs the* **complementary** $c_{II_{x_2}}$ *sequence, where* $c_{II_{x_2}} = b_v - c_{II_{x_1}}$ *?*

Recall from The Braider, Issue No. 8, the Fig. 149 on pg. 169. With regards the Regular Rectangular Mats, what can we say about the sequences $F_n/F_{(n+1)}$ *and* $F_n/F_{(n+2)}$*, and what can we say about the sequences* $F_{(n+1)}/F_n$ *and* $F_{(n+2)}/F_n$ *?*

(Regular) Band Knots

Not only are the Regular Band Knots so far discussed very special ones within the families which have a Gaucho-coding, a Headhunter's-coding or a Fan-coding, but in addition most braiders will not be able to fully comprehend and visualise their discussed construction processes readily. Only the general construction method, which we shall discuss in this Issue by means of a few Examples, will enable us to do so.♣ Then, in *The Braider*, Issue No. 14, we will revisit the Regular Band Knots discussed in the previous two Issues, in order to see how and where they fit into this general method and so obtain a full understanding of the way in which they may be constructed.†

The general construction method makes use of the **algorithm diagram**‡. Since in Regular Band Knots the number of *bights* (B) always greatly exceeds the number of *parts* (P), only the initial section of the **complementary cyclic bight-number scheme**†† takes part in the **algorithm diagram**. Hence instead of first working out this *complementary cyclic bight-number scheme*, we set off $(P + 1)$ points along a horizontal line and first assign to these points the complementary cyclic bight-numbers by using the Δ^*-value which readily may be obtained from the appropriate node on the Regular Band Knot's path in the RKT. This path follows from **the path-formula** which is obtained by means of **Euclid's algorithm.**‡‡

The possible correctness or otherwise of the assigned bight-numbers to the points of the algorithm diagram can readily be checked with the condition given in the question at the bottom of pg. 117 of *The Braider*, Issue No. 6 (see also *The Braider*, Issue No. 7, pg. 133).

Next, the inner $(P - 1)$ points, which represent the intersection-columns of the Regular Band Knot, have to receive a coding arrangement. An essential requirement

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♣ We shall limit our discussion here to Column-coded Regular Band Knots.

† It should be needless to say that a thorough understanding of what really goes on during a braiding process is of the utmost importance with respect to progress in the braiding arena. It also enables the braider to select the most appropriate construction process for an envisaged braid.

‡ See *The Braider*, Issue No. 6, pp. 114-117.

†† See *The Braider*, Issue No. 5, pg. 93.

‡‡ See *The Braider*, Issue No. 7, pg. 135, and Issue No. 12, pg. 247.

to which this coding arrangement must fulfil is that during the construction process we pass either through a 2-parts column-coded Regular Knot or through a 3-parts Casa-coded Regular Knot (see *The Braider*, Issue No. 11, pg. 232). Hence for the 2-parts column-coded Regular Knot case, two points receive an appropriate coding, while for the 3-parts Casa-coded Regular Knot case, four points receive an appropriate coding. The coding of the remaining points may then be chosen so as to give a pleasing pattern.

Example 1 :

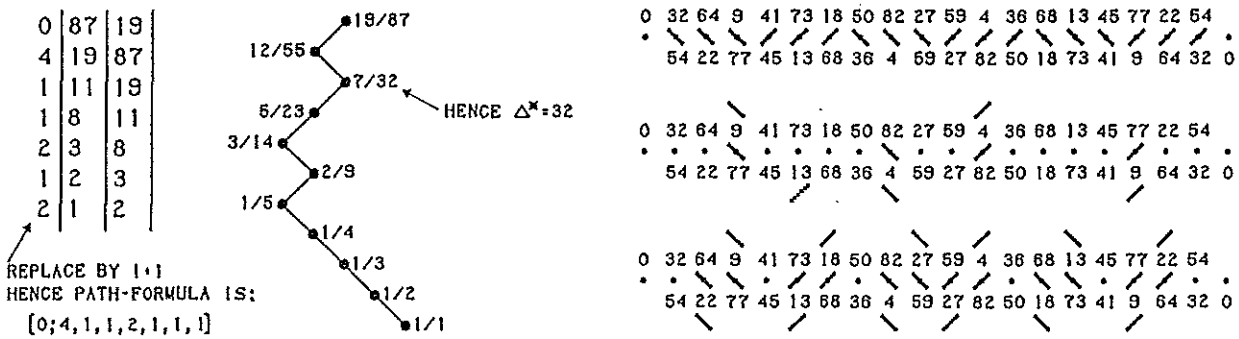


Fig. 257 — $P/B = 19/87$; Euclid's algorithm; path in the RKT; algorithm diagram.

BIGHT-NUMBER		HALF-CYCLE	NUMBER OF HALF-CYCLES		HALF-CYCLE		BIGHT-NUMBER
31	←	64			64	→	31
31	←	65	10	10	65	→	31
32	→	66			66	→	32
36	→	74			74	→	36
36	←	75	10	10	75	→	36
41	→	84			84	→	41
41	←	85	8	8	85	→	41
45	→	92			92	→	45
45	←	93	10	8	93	→	45
					100	→	49
				10	101	→	49
50	→	102			102	→	50
50	←	103	8		103	→	50
54	→	110			110	→	54
54	←	111	10	10	111	→	54
59	→	120			120	→	59
59	←	121	10	10	121	→	59
64	→	130			130	→	64
64	←	131	8	10	131	→	64
68	→	138			138	→	68
68	←	139	10		139	→	68
				8	140	→	69
73	→	148			141	→	69
73	←	149	8	8	148	→	73
77	→	156			149	→	73
77	←	157	10	9	156	→	77
					157	→	77
82	→	166			165	→	81
82	←	167	8	9	166	→	82
86	←	174			167	→	82
					174	→	86

Table 1 — Example 1.

The Δ^*-value is equal to the increase in the number of bights which $P/B = 19/87$ receives when subjected to a 1st-order Method II enlargement; hence $\Delta^* = 32$. Say that during the construction process we want to pass through the 3-parts Casa-coded Regular Knot, hence through $p/b = 3/14$ as a Casa-coded Regular Knot. The last half-cycle of this knot is half-cycle #28 which has the bight-number $i = \frac{28-2}{2} = 13$.† Hence in the algorithm diagram we must give the bight-numbers $i \leq 13$ a coding which is compatible with a Casa-coded Regular Knot. However, since the continued enlargement of the $p/b = 3/14$ Regular Knot is according to Enlargement Method II, the bight-number $i = 13$ associated with half-cycle 28 is not associated with a crossing, but with the point where the Working End meets the Standing End and so completes the string-run of the $p/b = 3/14$ Regular Knot. Consequently, only the points of the algorithm diagram associated with the bight-numbers $i < 13$ should be given a coding which is compatible with a Casa-coded Regular Knot.

The bight-numbers above the points of the algorithm diagram are associated with the half-cycles which run from the left-hand bight-boundary to the right-hand bight-boundary and the bight-numbers below the points of the algorithm diagram are associated with the half-cycles which run from the right-hand bight-boundary to the left-hand bight-boundary.

Let the first half-cycle run from left to right, then the *even* numbered half-cycles run from right to left and are associated with the bight-numbers below the points of the algorithm diagram.

At the right-hand side of Fig. 257 are depicted three algorithm diagrams, the central one of which depicts a Casa-coding for the $p/b = 3/14$ Regular Knot. The remaining non-coded inner $P - 1 - 4 = 19 - 5 = 14$ points may be given any coding we desire.

From the lower algorithm diagram we observe that also the Regular Knot $p/b = 7/32$ can be given a Casa-coding. After doing this, we are left with $P - 1 - 2(7 - 1) = 19 - 1 - 12 = 6$ non-coded inner points which may be given any coding we desire.

We can code these remaining six inner points in such a way that we obtain the 3-pass Gaucho-coding depicted in the upper algorithm diagram.

Since the total required string-length for a Band Knot is very long, we braid the knot by using each string-end alternatively. First we braid the $p/b = 3/14$ Casa-coded Regular Knot (see *The Braider*, Issue No. 11, pg. 233), which we enlarge to the $p/b = 7/32$ Casa-coded Regular Knot (one string-end does the 'track-laying' followed by the other string-end's 'track-splitting' operation)‡.

Thus in general:

We braid, by using the string-ends alternatively, the 2-parts Column-coded Regular Knot or the 3-parts Casa-coded Regular Knot first. Then, if applicable, we enlarge it, by using the string-ends alternatively, to the maximum size Casa-coded Regular Knot on the path to the final Regular Band Knot. We continue the braiding process by using each string-end alternatively, whereby each string-end braids a number of consecutive half-cycles. Thus the further braiding procedure of the Regular Band Knot consists of a number of braiding sequences, whereby each braiding sequence consists of two sub-sequences: one sub-sequence performed by one string-end and the other sub-sequence performed by the other string-end. These continued braiding procedures may be divided into two types:

† See *The Braider*, Issue No. 5, pp. 89-95.

‡ See *The Braider*, Issue No. 7, pg. 143.

(1) For each braiding sequence, the two sub-sequences consist both of the same number of half-cycles (note that the braiding sequences do **not** have to consist of an equal number of half-cycles).
(2) Not in all braiding sequences do the sub-sequences fulfil the condition under **(1)**.

Since the number of half-cycles is large, even in our Example from $p/b = 7/32$ to $P/B = 19/87$ (110 half-cycles), it is most inconvenient to keep track of the half-cycle numbers during the further construction procedure. Fortunately, the algorithm diagram provides us with an easy orientation method, the essence of which we met in *The Braider*, Issue No. 6, pp. 115-116, namely the half-cycle which intersects the Standing End half-cycle. We reach this orientation half-cycle once in every revolution of the knot.

From the algorithm diagram of the $P/B = 19/87$ Regular 3-pass Gaucho-coded Regular Band Knot (upper right-hand side of Fig. 257) we compile the left-hand side of Table 1 (pg. 281). This table is associated with the braiding of this Band Knot from $p/b = 7/32$ to $P/B = 19/87$. In this table the consecutive braiding sequences are separated by thick horizontal line segments, while the two sub-sequences of each braiding sequence are separated by a shorter thin line segment. On the left-hand side of the table, every sub-sequence ends with a half-cycle which intersects the Standing End half-cycle.

On the right-hand side of the table a new set of braiding sequences and sub-sequences has been displayed which comply with braiding procedure type 1. Except for the last braiding sequence, note how every first sub-sequence in a braiding sequence still ends with a half-cycle which intersects the Standing End half-cycle. However, the last half-cycle of the second sub-sequence of a braiding sequence either ends with a half-cycle which intersects the Standing End half-cycle or ends with a half-cycle which can easily be referenced to such a half-cycle. This last case also applies to the last half-cycle of the first sub-sequence of the last braiding sequence. Hence braiding the $P/B = 19/87$ 3-pass Gaucho-coded Regular Band Knot from the $p/b = 7/32$ Casa-coded Regular Knot in accordance with the scheme depicted on the right-hand side of Table 1 (pg. 281) is relatively simple since the reference half-cycles (the ones which intersect the Standing End half-cycles) are easy to locate and since half-cycle braiding algorithms only change when their associated bight-number (i) attains the value $|n \cdot \Delta^*|_B$, where $0 < n < P$. Note that this braiding procedure allows us to start braiding from the centre of the string-length.

In the half-cycle algorithms associated with the half-cycles which intersect the Standing End half-cycle, we shall indicate this crossing position in the following manner:

An entry $\ldots - (1, s, 3)5o - \ldots$ indicates that in this set of 5 *overs*, the 2nd *over* crosses the Standing End half-cycle.

An entry $\ldots - (s)u - \ldots$ indicates that this single *under* crosses the Standing End half-cycle.

The reader is furthermore referred to Pamphlet No. 7, *The Braiding of Column-coded Regular Knots*, pp. 29-31.

For the braiding process from the $p/b = 7/32$ Casa-coded Regular Knot to the $P/B = 19/87$ 3-pass Gaucho-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of Table 1 (pg. 281) we obtain from the algorithm diagram (upper right-hand side of Fig. 257) the following half-cycle algorithms:

$(i = 31)$ 65 ↗ : $u - o - u - o - u - o.$
$(i = \mathbf{32})$ **66** ↖ : $(s, 1)2u - o - u - o - u - o.$
$(i = 32)$ 67 ↗ : $2u - o - u - o - u - o.$
$(i = 33)$ 68 ↖ : $2u - o - u - o - u - o.$
↗ half-cycles as half-cycle #67.
↖ half-cycles as half-cycle #68.
$(i = \mathbf{36})$ **74** ↖ : $2u - o - u - (1, s)2o - u - o.$

$(i = 36)$ 75 ↘ : $2o - u - o - 2u - o - u.$
$(i = 37)$ 76 ↙ : $2o - u - o - 2u - o - u.$
↘ half-cycles as half-cycle #75.
↙ half-cycles as half-cycle #76.
$(i = \mathbf{41})$ **84** ↙ : $2o - (s, 1)2u - o - 2u - o - u.$

$(i = 41)$ 85 ↗ : $2u - 2o - u - 2o - u - o.$
$(i = 42)$ 86 ↖ : $2u - 2o - u - 2o - u - o.$
↗ half-cycles as half-cycle #85.
↖ half-cycles as half-cycle #86.
$(i = \mathbf{45})$ **92** ↖ : $2u - 2o - u - 2o - (1, s)2u - o.$

$(i = 45)$ 93 ↘ : $2o - 2u - o - 2u - 2o - u.$
$(i = 46)$ 94 ↙ : $2o - 2u - o - 2u - 2o - u.$
↘ half-cycles as half-cycle #93.
↙ half-cycles as half-cycle #94.
$(i = 49)$ 100 ↙ : $2o - 2u - o - 2u - 2o - u.$

$(i = 49)$ 101 ↗ : $2u - 2o - u - 2o - 2u - o.$
$(i = \mathbf{50})$ **102** ↖ : $2u - 2o - (s, 1)2u - 2o - 2u - o.$
$(i = 50)$ 103 ↗ : $2u - 2o - 2u - 2o - 2u - o.$
$(i = 51)$ 104 ↖ : $2u - 2o - 2u - 2o - 2u - o.$
↗ half-cycles as half-cycle #103.
↖ half-cycles as half-cycle #104.
$(i = \mathbf{54})$ **110** ↖ : $2u - 2o - 2u - 2o - 2u - (1, s)2o.$

$(i = 54)$ 111 ↘ : $2o - 2u - 2o - 2u - 2o - 2u.$
$(i = 55)$ 112 ↙ : $2o - 2u - 2o - 2u - 2o - 2u.$
↘ half-cycles as half-cycle #111.
↙ half-cycles as half-cycle #112.
$(i = \mathbf{59})$ **120** ↙ : $2o - 2u - 2o - (s, 2)3u - 2o - 2u.$

$(i = 59)$ 121 ↗ : $2u - 2o - 2u - 3o - 2u - 2o.$
$(i = 60)$ 122 ↖ : $2u - 2o - 2u - 3o - 2u - 2o.$
↗ half-cycles as half-cycle #121.
↖ half-cycles as half-cycle #122.
$(i = \mathbf{64})$ **130** ↖ : $(1, s, 1)3u - 2o - 2u - 3o - 2u - 2o.$

$(i = 64)$ 131 ↘ : $3o - 2u - 2o - 3u - 2o - 2u.$
$(i = 65)$ 132 ↙ : $3o - 2u - 2o - 3u - 2o - 2u.$
↘ half-cycles as half-cycle #131.

			↙ half-cycles as half-cycle #132.
$(i = \mathbf{68})$	**138** ↙	:	$3o - 2u - 2o - 3u - (s,2)3o - 2u.$
$(i = 68)$	139 ↘	:	$3o - 2u - 2o - 3u - 3o - 2u.$
$(i = 69)$	140 ↙	:	$3o - 2u - 2o - 3u - 3o - 2u.$
$(i = 69)$	141 ↗	:	$3u - 2o - 2u - 3o - 3u - 2o.$
$(i = 70)$	142 ↖	:	$3u - 2o - 2u - 3o - 3u - 2o.$
			↗ half-cycles as half-cycle #141.
			↖ half-cycles as half-cycle #142.
$(i = \mathbf{73})$	**148** ↖	:	$3u - (1,s,1)3o - 2u - 3o - 3u - 2o.$
$(i = 73)$	149 ↘	:	$3o - 3u - 2o - 3u - 3o - 2u.$
$(i = 74)$	150 ↙	:	$3o - 3u - 2o - 3u - 3o - 2u.$
			↘ half-cycles as half-cycle #149.
			↙ half-cycles as half-cycle #150.
$(i = \mathbf{77})$	**156** ↙	:	$3o - 3u - 2o - 3u - 3o - (s,2)3u.$
$(i = 77)$	157 ↗	:	$3u - 3o - 2u - 3o - 3u - 3o.$
$(i = 78)$	158 ↖	:	$3u - 3o - 2u - 3o - 3u - 3o.$
			↗ half-cycles as half-cycle #157.
			↖ half-cycles as half-cycle #158.
$(i = 81)$	165 ↗	:	$3u - 3o - 2u - 3o - 3u - 3o.$
$(i = \mathbf{82})$	**166** ↘	:	$3o - 3u - (1,s,1)3o - 3u - 3o - 3u.$
$(i = 82)$	167 ↙	:	$3o - 3u - 3o - 3u - 3o - 3u.$
$(i = 83)$	168 ↘	:	$3o - 3u - 3o - 3u - 3o - 3u.$
			↙ half-cycles as half-cycle #167.
			↘ half-cycles as half-cycle #168.
$(i = 86)$	174 ↘	:	$3o - 3u - 2o - 3u - 3o - 3u.$

The numbers in bold are associated with the half-cycles which intersect the Standing End half-cycle. Note that the Standing End half-cycle of a sub-sequence is the final half-cycle of the immediately preceeding sub-sequence, hence each sub-sequence has a different Standing End half-cycle. Also note that this braiding process ends at the bight-boundary opposite to the one from which we started, but on the same cylinder generating line.

Although in this braiding process it is easy enough to determine when the last half-cycle in each sub-sequence has been laid down, a much easier process is offered by the arrangement of the braiding sequences and sub-sequences shown on the right-hand side of Table 2 (pg. 286). Except for the last braiding sequence, the last half-cycle of a sub-sequence intersect the Standing End half-cycle, which makes orientation easier. Note that the total number of half-cycles in the sub-sequences marked 'A' is equal to the total number of half-cycles in the sub-sequences marked 'B'. Hence by dividing the last braiding sequence (of 18 half-cycles) into two sub-sequences, each with an equal number of half-cycles, we can start the overall braiding process from the mid-length position of the string. Note that this braiding process starts and ends in exactly the same way as the previous one did.

EIGHT-NUMBER 1	⇄	HALF-CYCLE		NUMBER OF HALF-CYCLES		HALF-CYCLE	→	EIGHT-NUMBER 1
31	←	64				64	→	31
31	←	65	A	10	10	65	→	31
32	→	66				66	→	32
36	→	74				74	→	36
36	←	75	B	10	10	75	→	36
41	→	84				84	→	41
41	←	85	A	8	8	85	→	41
45	→	92				92	→	45
45	←	93	B	10	10	93	→	45
50	→	102				102	→	50
50	←	103	A	8	8	103	→	50
54	→	110				110	→	54
54	←	111	B	10	10	111	→	54
59	→	120				120	→	59
59	←	121	A	10	10	121	→	59
64	→	130				130	→	64
64	←	131	B	8	8	131	→	64
68	→	138				138	→	68
68	←	139	A	10	10	139	→	68
73	→	148				148	→	73
73	←	149	B	8	8	149	→	73
77	→	156				156	→	77
77	←	157		10	9	157	→	77
						165	→	81
82	→	166				166	→	82
82	←	167		8	9	167	→	82
86	←	174				174	→	86

Table 2 — Example 1.

For the braiding process from the $p/b = 7/32$ Casa-coded Regular Knot to the $P/B = 19/87$ 3-pass Gaucho-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of Table 2 (pg. 286) we obtain from the algorithm diagram (upper right-hand side of Fig. 257) the following half-cycle algorithms:

$(i = 31)$ $65\ \nearrow$: $u - o - u - o - u - o.$
$(i = \mathbf{32})$ $\mathbf{66}\ \nwarrow$: $(s,1)2u - o - u - o - u - o.$
$(i = 32)$ $67\ \nearrow$: $2u - o - u - o - u - o.$
$(i = 33)$ $68\ \nwarrow$: $2u - o - u - o - u - o.$
$\nearrow$ half-cycles as half-cycle #67.
$\nwarrow$ half-cycles as half-cycle #68.
$(i = \mathbf{36})$ $74\ \nwarrow$: $2u - o - u - (1,s)2o - u - o.$

$(i = 36)$ $75\ \searrow$: $2o - u - o - 2u - o - u.$
$(i = 37)$ $76\ \swarrow$: $2o - u - o - 2u - o - u.$
$\searrow$ half-cycles as half-cycle #75.
$\swarrow$ half-cycles as half-cycle #76.
$(i = 41)$ $84\ \swarrow$: $2o - (s,1)2u - o - 2u - o - u.$

$(i = 41)$ $85\ \nearrow$: $2u - 2o - u - 2o - u - o.$
$(i = 42)$ $86\ \nwarrow$: $2u - 2o - u - 2o - u - o.$
$\nearrow$ half-cycles as half-cycle #85.

$\nwarrow$ half-cycles as half-cycle #86.

$(i = 45)$ **92** $\nwarrow$: $2u - 2o - u - 2o - (1, s)2u - o.$

$(i = 45)$ 93 $\searrow$: $2o - 2u - o - 2u - 2o - u.$

$(i = 46)$ 94 $\swarrow$: $2o - 2u - o - 2u - 2o - u.$

$\searrow$ half-cycles as half-cycle #93.

$\swarrow$ half-cycles as half-cycle #94.

$(i = 50)$ **102** $\swarrow$: $2o - 2u - (s, 1)2o - 2u - 2o - u.$

$(i = 50)$ 103 $\nearrow$: $2u - 2o - 2u - 2o - 2u - o.$

$(i = 51)$ 104 $\nwarrow$: $2u - 2o - 2u - 2o - 2u - o.$

$\nearrow$ half-cycles as half-cycle #103.

$\nwarrow$ half-cycles as half-cycle #104.

$(i = 54)$ **110** $\nwarrow$: $2u - 2o - 2u - 2o - 2u - (1, s)2o.$

$(i = 54)$ 111 $\searrow$: $2o - 2u - 2o - 2u - 2o - 2u.$

$(i = 55)$ 112 $\swarrow$: $2o - 2u - 2o - 2u - 2o - 2u.$

$\searrow$ half-cycles as half-cycle #111.

$\swarrow$ half-cycles as half-cycle #112.

$(i = 59)$ **120** $\swarrow$: $2o - 2u - 2o - (s, 2)3u - 2o - 2u.$

$(i = 59)$ 121 $\nearrow$: $2u - 2o - 2u - 3o - 2u - 2o.$

$(i = 60)$ 122 $\nwarrow$: $2u - 2o - 2u - 3o - 2u - 2o.$

$\nearrow$ half-cycles as half-cycle #121.

$\nwarrow$ half-cycles as half-cycle #122.

$(i = 64)$ **130** $\nwarrow$: $(1, s, 1)3u - 2o - 2u - 3o - 2u - 2o.$

$(i = 64)$ 131 $\searrow$: $3o - 2u - 2o - 3u - 2o - 2u.$

$(i = 65)$ 132 $\swarrow$: $3o - 2u - 2o - 3u - 2o - 2u.$

$\searrow$ half-cycles as half-cycle #131.

$\swarrow$ half-cycles as half-cycle #132.

$(i = 68)$ **138** $\swarrow$: $3o - 2u - 2o - 3u - (s, 2)3o - 2u.$

$(i = 68)$ 139 $\nearrow$: $3u - 2o - 2u - 3o - 3u - 2o.$

$(i = 69)$ 140 $\nwarrow$: $3u - 2o - 2u - 3o - 3u - 2o.$

$\nearrow$ half-cycles as half-cycle #139.

$\nwarrow$ half-cycles as half-cycle #140.

$(i = 73)$ **148** $\nwarrow$: $3u - (1, s, 1)3o - 2u - 3o - 3u - 2o.$

$(i = 73)$ 149 $\searrow$: $3o - 3u - 2o - 3u - 3o - 2u.$

$(i = 74)$ 150 $\swarrow$: $3o - 3u - 2o - 3u - 3o - 2u.$

$\searrow$ half-cycles as half-cycle #149.

$\swarrow$ half-cycles as half-cycle #150.

$(i = 77)$ **156** $\swarrow$: $3o - 3u - 2o - 3u - 3o - (s, 2)3u.$

$(i = 77)$ 157 $\nearrow$: $3u - 3o - 2u - 3o - 3u - 3o.$

$(i = 78)$ 158 $\nwarrow$: $3u - 3o - 2u - 3o - 3u - 3o.$

$\nearrow$ half-cycles as half-cycle #157.

			$\nwarrow$ half-cycles as half-cycle #158.
$(i = 81)$	165 $\nearrow$	:	$3u - 3o - 2u - 3o - 3u - 3o.$
$(i = 82)$	**166** $\searrow$	:	$3o - 3u - (1, s, 1)3o - 3u - 3o - 3u.$
$(i = 82)$	167 $\swarrow$	:	$3o - 3u - 3o - 3u - 3o - 3u.$
$(i = 83)$	168 $\searrow$	:	$3o - 3u - 3o - 3u - 3o - 3u.$
			$\swarrow$ half-cycles as half-cycle #167.
			$\searrow$ half-cycles as half-cycle #168.
$(i = 86)$	174 $\searrow$	:	$3o - 3u - 2o - 3u - 3o - 3u.$

Example 2:

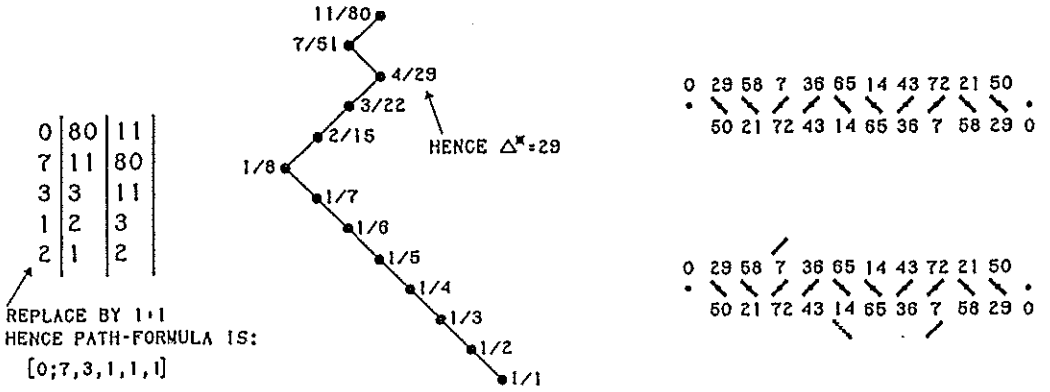


Fig. 258 — $P/B = 11/80$; Euclid's algorithm; path in the RKT; algorithm diagram.

BIGHT-NUMBER ⟶ HALF-CYCLE				NUMBER OF HALF-CYCLES		HALF-CYCLE ⟶ BIGHT-NUMBER		
14	←	30				30	⟶	14
14	←	31	A	14	14	31	⟶	14
21	→	44				44	⟶	21
21	←	45	B	16	16	45	⟶	21
29	→	60				60	⟶	29
29	←	61	A	14	14	61	⟶	29
36	→	74				74	⟶	36
36	←	75	B	14	14	75	⟶	36
43	→	88				88	⟶	43
43	←	89	A	14	14	89	⟶	43
50	→	102				102	⟶	50
50	←	103	B	16	16	103	⟶	50
58	→	118				118	⟶	58
58	←	119	A	14	14	119	⟶	58
65	→	132				132	⟶	65
65	←	133	B	14	14	133	⟶	65
72	→	146				146	⟶	72
72	←	147		14	7	147	⟶	72
						153	⟶	75
					7	154	⟶	76
79	←	160				160	⟶	79

Table — Example 2.

$\Delta^* = 29$ since the Δ^*-value is equal to the increase in the number of bights which $P/B = 11/80$ receives when subjected to a 1st-order Method II enlargement. Say that

during the construction process we want to pass through the 2-parts Column-coded Regular Knot, hence through $p/b = 2/15$ as a Column-coded Regular Knot. The last half-cycle of this knot is half-cycle #30 which has the bight-number $i = \frac{30-2}{2} = 14$. Hence in the algorithm diagram we must give the bight-numbers $i \leq 14$ a coding which is compatible with a Column-coded Regular Knot. However, since the continued enlargement of the $p/b = 2/15$ Regular Knot is according to Enlargement Method II, the bight-number $i = 14$ associated with half-cycle 30 is not associated with a crossing, but with the point where the Working End meets the Standing End and so completes the string-run of the $p/b = 2/15$ Regular Knot. Consequently, only the points of the algorithm diagram associated with the bight-numbers $i < 14$ should be given a coding which is compatible with a Column-coded Regular Knot.

As usual, the bight-numbers above the points of the algorithm diagram are associated with the half-cycles which run from the left-hand bight-boundary to the right-hand bight-boundary and the bight-numbers below the points of the algorithm diagram are associated with the half-cycles which run from the right-hand bight-boundary to the left-hand bight-boundary.

If the first half-cycle runs from left to right, then the *even* numbered half-cycles run from right to left and are associated with the bight-numbers below the points in the algorithm diagram.

At the right-hand side of Fig. 258 are depicted two algorithm diagrams. In the lowermost algorithm diagram the points associated with the bight-number $i < 14$ are given a Column-coding as shown for the appropriate braiding direction by the solid black codings above and below the bight-numbers. The remaining non-coded inner $P - 1 - 2 = 11 - 3 = 8$ points may be given any coding we desire. In Fig. 258 these points have been given a coding such that the overall coding is a 2-pass Headhunter's coding. Note that the $p/b = 4/29$ Regular Knot does **not** have a Casa-coding.

Since the total required string-length for a Band Knot is very long, we again braid the knot by using each string-end alternatively. First we braid the $p/b = 2/15$ Column-coded Regular Knot, which involves the first 30 half-cycles. Then we braid the remaining 130 half-cycles, from $p/b = 2/15$ to $P/B = 11/80$, by means of a series of braiding sequences each of which consists of two sub-sequences.

From the algorithm diagram of the $P/B = 11/80$ 2-pass Headhunter's-coded Regular Band Knot (upper right-hand side of Fig. 258) we compile the left-hand side of the Table (pg. 288). This table is associated with braiding this Band Knot from $p/b = 2/15$ to $P/B = 11/80$. Note that all braiding sequences, except the last one, consist of two sub-sequences. However, in order to ensure that all braiding sequences consist of two sub-sequences, we divide the last braiding sequence into two equal parts. This results in the braiding sequences and their sub-sequences shown on the right-hand side of the Table (pg. 288). In this table the consecutive braiding sequences are again separated by thick horizontal line segments, while the two sub-sequences of each braiding sequence are separated by a shorter thin line segment.

Since the sum of the half-cycles in the sub-sequences marked 'B' is four more than the sum of the half-cycles in the sub-sequences marked 'A', the braiding of the $p/b = 2/15$ Column-coded Regular Knot does **not** start at the mid-length position of the string, but at a position which is $78/82 = 39/41$ of the string-length from one end. If the shorter end is braided upwards and the longer end downwards, then the sub-sequences 'A' are braided upwards and the sub-sequences 'B' downwards. Note that the last two sub-sequences make each only about half a revolution of the Band Knot.

For the braiding process from the $p/b = 2/15$ Column-coded Regular Knot to the $P/B = 11/80$ 2-pass Headhunter's-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of the Table (pg. 288) we obtain from the algorithm diagram (upper right-hand side of Fig. 258) the following half-cycle algorithms:

$(i = 14)$	31 ↗	:	$o - u.$
$(i = 15)$	32 ↖	:	$u - o.$
			↗ half-cycles as half-cycle #31.
			↖ half-cycles as half-cycle #32.
$(i = 21)$	**44 ↖**	:	$2u - (1, s)2o.$
$(i = 21)$	45 ↘	:	$u - 2o.$
$(i = 22)$	46 ↙	:	$o - 2u.$
			↘ half-cycles as half-cycle #45.
			↙ half-cycles as half-cycle #46.
$(i = 29)$	**60 ↙**	:	$(s)u - o - 2u.$
$(i = 29)$	61 ↗	:	$u - o - 2u.$
$(i = 30)$	62 ↖	:	$o - u - 2o.$
			↗ half-cycles as half-cycle #61.
			↖ half-cycles as half-cycle #62.
$(i = 36)$	**74 ↖**	:	$o - (1, s)2u - 2o.$
$(i = 36)$	75 ↘	:	$o - 2u - 2o.$
$(i = 37)$	76 ↙	:	$u - 2o - 2u.$
			↘ half-cycles as half-cycle #75.
			↙ half-cycles as half-cycle #76.
$(i = 43)$	**88 ↙**	:	$u - 2o - u - (s)o - u.$
$(i = 43)$	89 ↗	:	$u - 2o - u - o - u.$
$(i = 44)$	90 ↖	:	$o - 2u - o - u - o.$
			↗ half-cycles as half-cycle #89.
			↖ half-cycles as half-cycle #90.
$(i = 50)$	**102 ↖**	:	$o - 2u - o - u - (1, s)2o.$
$(i = 50)$	103 ↘	:	$o - 2u - o - u - 2o.$
$(i = 51)$	104 ↙	:	$u - 2o - u - o - 2u.$
			↘ half-cycles as half-cycle #103.
			↙ half-cycles as half-cycle #104.
$(i = 58)$	**118 ↙**	:	$(1, s)2u - 2o - u - o - 2u.$
$(i = 58)$	119 ↗	:	$2u - 2o - u - o - 2u.$
$(i = 59)$	120 ↖	:	$2o - 2u - o - u - 2o.$
			↗ half-cycles as half-cycle #119.
			↖ half-cycles as half-cycle #120.
$(i = 65)$	**132 ↖**	:	$2o - 2u - (s, 1)2o - u - 2o.$
$(i = 65)$	133 ↘	:	$2o - 2u - 2o - u - 2o.$
$(i = 66)$	134 ↙	:	$2u - 2o - 2u - o - 2u.$
			↘ half-cycles as half-cycle #133.

$\swarrow$ half-cycles as half-cycle #134.

$(i = 72)$ 146 $\swarrow$: $2u - 2o - 2u - (1, s)2o - 2u.$

$(i = 72)$ 147 $\nearrow$: $2u - 2o - 2u - 2o - 2u.$

$(i = 73)$ 148 $\nwarrow$: $2o - 2u - 2o - 2u - 2o.$

$\nearrow$ half-cycles as half-cycle #147.

$\nwarrow$ half-cycles as half-cycle #148.

$(i = 75)$ 153 $\nearrow$: $2u - 2o - 2u - 2o - 2u.$

$(i = 76)$ 154 $\searrow$: $2o - 2u - 2o - 2u - 2o.$

$(i = 76)$ 155 $\swarrow$: $2u - 2o - 2u - 2o - 2u.$

$\searrow$ half-cycles as half-cycle #154.

$\swarrow$ half-cycles as half-cycle #155.

$(i = 79)$ 160 $\searrow$: $2o - 2u - 2o - 2u - 2o.$

Example 3 :

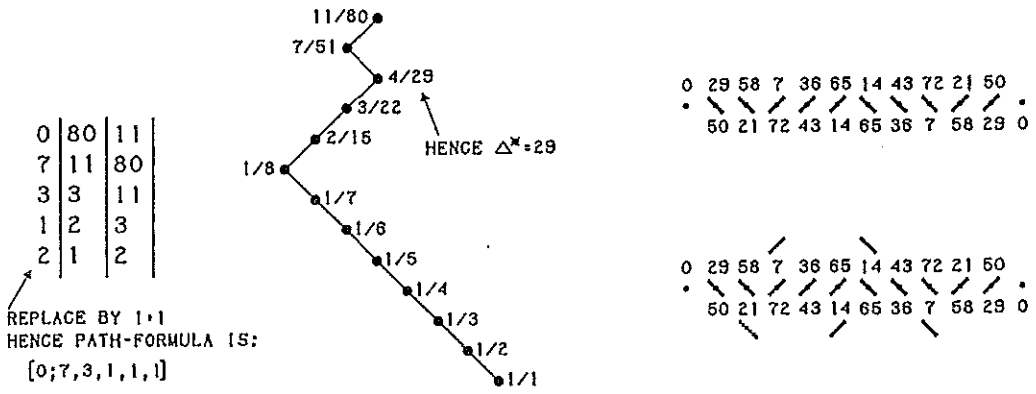


Fig. 259 — $P/B = 11/80$; Euclid's algorithm ; path in the RKT ; algorithm diagram.

BIGHT-NUMBER i	⟷ HALF-CYCLE		NUMBER OF HALF-CYCLES		HALF-CYCLE ⟶	BIGHT-NUMBER i
21 ←	44				44 ⟶	21
21 ←	45	A	16	16	45 ⟶	21
29 →	60				60 ⟶	29
29 ←	61	B	14	14	61 ⟶	29
36 →	74				74 ⟶	36
36 ←	75	A	14	14	75 ⟶	36
43 →	88				88 ⟶	43
43 ←	89	B	14	14	89 ⟶	43
50 →	102				102 ⟶	50
50 ←	103	A	16	16	103 ⟶	50
58 →	118				118 ⟶	58
58 ←	119	B	14	14	119 ⟶	58
65 →	132				132 ⟶	65
65 ←	133	A	14	14	133 ⟶	65
72 →	146				146 ⟶	72
72 ←	147	B	14	14	147 ⟶	72
79 ←	160				160 ⟶	79

Table — Example 3.

Apart from the coding, the algorithm diagram for this Example is the same as the one for Example 2 since its the path in the RKT is that of Example 2. But in contrast to Example 2, say that during the construction process we want to pass through the 3-parts Casa-coded Regular Knot, hence through $p/b = 3/22$ as a Casa-coded Regular Knot. The last half-cycle of this knot is half-cycle #44 which has the bight-number $i = \frac{44-2}{2} = 21$. Hence in the algorithm diagram we must give the bight-numbers $i \leq 21$ a coding which is compatible with a Casa-coded Regular Knot. However, since the continued enlargement of the $p/b = 3/22$ Regular Knot is according to Enlargement Method II, the bight-number $i = 21$ associated with half-cycle 44 is not associated with a crossing, but with the point where the Working End meets the Standing End and so completes the string-run of the $p/b = 3/22$ Regular Knot. Consequently, only the points of the algorithm diagram associated with the bight-numbers $i < 21$ should be given a coding which is compatible with a Casa-coded Regular Knot.

At the right-hand side of Fig. 259 are depicted two algorithm diagrams. In the lowermost algorithm diagram the points associated with the bight-number $i < 21$ are given a Casa-coding as shown for the appropriate braiding direction by the solid black codings above and below the bight-numbers. The remaining non-coded inner $P - 1 - 4 = 11 - 5 = 6$ points may be given any coding we desire. In Fig. 259 these points have been given a coding such that the overall coding is a Fan-coding.

First we braid the $p/b = 3/22$ Casa-coded Regular Knot, which involves the first 44 half-cycles. Then we braid the remaining 116 half-cycles, from $p/b = 3/22$ to $P/B = 11/80$, by means of a series of braiding sequences each of which consists of two sub-sequences.

From the algorithm diagram of the $P/B = 11/80$ Fan-coded Regular Band Knot (upper right-hand side of Fig. 259) we compile the left-hand side of the Table (pg. 291). This table is associated with braiding this Band Knot from $p/b = 3/22$ to $P/B = 11/80$. Note that all braiding sequences consist of two sub-sequences. Thus the braiding sequences and their sub-sequences shown on the right-hand side of the Table (pg. 291) can be identical to those on the left-hand side of the Table.

Since the sum of the half-cycles in the sub-sequences marked 'A' is four more than the sum of the half-cycles in the sub-sequences marked 'B', the braiding of the $p/b = 3/22$ Casa-coded Regular Knot does **not** start at the mid-length position of the string, but at a position which is $78/82 = 39/41$ of the string-length from one end. If the longer end is braided upwards and the shorter end downwards, then the sub-sequences 'A' are braided upwards and the sub-sequences 'B' downwards.

For the braiding process from the $p/b = 3/22$ Casa-coded Regular Knot to the $P/B = 11/80$ Fan-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of the Table (pg. 291) we obtain from the algorithm diagram (upper right-hand side of Fig. 259) the following half-cycle algorithms:

$(i = 21)$	45 ↗	:	$o - u - o.$
$(i = 22)$	46 ↖	:	$o - u - o.$
			↗ half-cycles as half-cycle #45.
			↖ half-cycles as half-cycle #46.
$(i = 29)$	60 ↖	:	$(s)u - o - u - o.$
$(i = 29)$	61 ↘	:	$o - u - o - u.$
$(i = 30)$	62 ↙	:	$o - u - o - u.$
			↘ half-cycles as half-cycle #61.

			↙ half-cycles as half-cycle #62.
$(i = 36)$	**74** ↙	:	$o - (1, s)2u - o - u.$
$(i = 36)$	75 ↗	:	$u - 2o - u - o.$
$(i = 37)$	76 ↖	:	$u - 2o - 2u.$
			↗ half-cycles as half-cycle #75.
			↖ half-cycles as half-cycle #76.
$(i = 43)$	**88** ↖	:	$u - 2o - (1, s)2u - o.$
$(i = 43)$	89 ↘	:	$o - 2u - 2o - u.$
$(i = 44)$	90 ↙	:	$o - 2u - 2o - u.$
			↘ half-cycles as half-cycle #89.
			↙ half-cycles as half-cycle #90.
$(i = 50)$	**102** ↙	:	$o - 2u - 2o - (1, s)2u.$
$(i = 50)$	103 ↗	:	$u - 2o - 2u - 2o.$
$(i = 51)$	104 ↖	:	$u - 2o - 2u - 2o.$
			↗ half-cycles as half-cycle #103.
			↖ half-cycles as half-cycle #104.
$(i = 58)$	**118** ↖	:	$(1, s)2u - 2o - 2u - 2o.$
$(i = 58)$	119 ↘	:	$2o - 2u - 2o - 2u.$
$(i = 59)$	120 ↙	:	$2o - 2u - 2o - 2u.$
			↘ half-cycles as half-cycle #119.
			↙ half-cycles as half-cycle #120.
$(i = 65)$	**132** ↙	:	$2o - (2, s)3u - 2o - 2u.$
$(i = 65)$	133 ↗	:	$2u - 3o - 2u - 2o.$
$(i = 66)$	134 ↖	:	$2u - 3o - 2u - 2o.$
			↗ half-cycles as half-cycle #133.
			↖ half-cycles as half-cycle #134.
$(i = 72)$	**146** ↖	:	$2u - 3o - (2, s)3u - 2o.$
$(i = 72)$	147 ↘	:	$2o - 3u - 3o - 2u.$
$(i = 73)$	148 ↙	:	$2o - 3u - 3o - 2u.$
			↘ half-cycles as half-cycle #147.
			↙ half-cycles as half-cycle #148.
$(i = 79)$	160 ↙	:	$2o - 3u - 3o - 2u.$

Example 4:

The most convenient braiding procedure is the one in which not only each sub-sequence ends with a half-cycle which intersects the Standing End half-cycle, but which in addition starts at the mid-length position of the string. Such a braiding procedure can in many cases be obtained with a suitable arrangement of the braiding sequences and sub-sequences. For this Example such a preferred braiding process, from the $p/b = 3/13$ Casa-coded Regular Knot to the $P/B = 19/85$ 3-pass Gaucho-coded Regular Band Knot, is shown on the right-hand side of the Table on pg. 294. On the left-hand side

of this Table are shown the braiding sequences and sub-sequences obtained from the algorithm diagram at the upper right-hand side of Fig. 260.

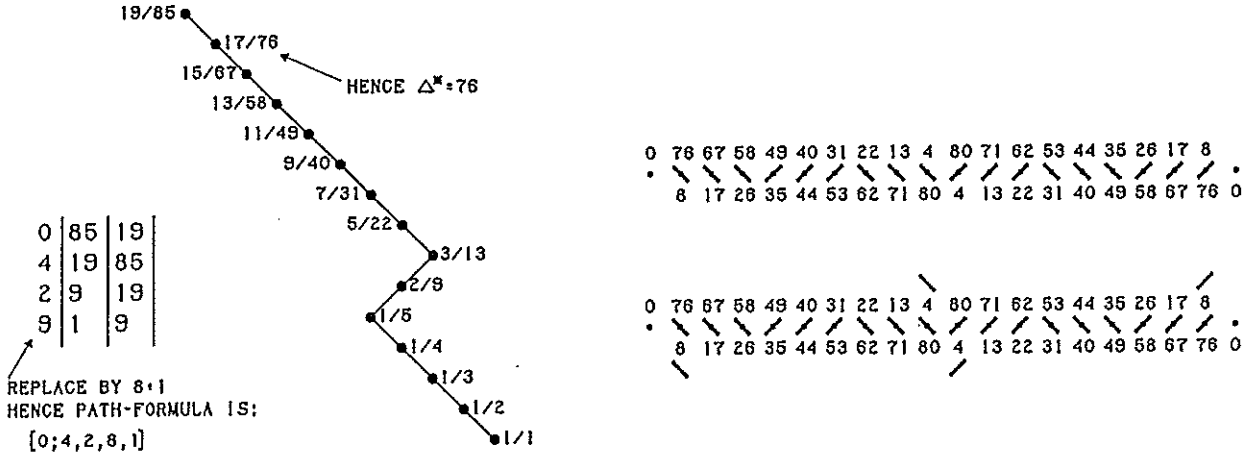


Fig. 260 — $P/B = 19/85$; Euclid's algorithm; path in the RKT; algorithm diagram.

BIGHT-NUMBER ↓	⇄ HALF-CYCLE		NUMBER OF HALF-CYCLES		HALF-CYCLE →	BIGHT-NUMBER ↓
12	←	26			26	12
12	←	27			27	12
13	→	28	A	10	28	13
17	→	36		10	36	17
17	←	37			37	17
22	→	46	B	10	46	22
22	←	47			47	22
26	→	54	A	8	54	26
26	←	55		8	55	26
31	→	64	B	10	64	31
31	←	65			65	31
35	→	72	A	8	72	35
35	←	73		8	73	35
40	→	82	B	10	82	40
40	←	83			83	40
44	→	90	A	8	90	44
44	←	91		8	91	44
49	→	100	B	10	100	49
49	←	101			101	49
53	→	108	A	8	108	53
53	←	109		8	109	53
58	→	118	B	10	118	58
58	←	119			119	58
62	→	126	A	8	126	62
62	←	127		8	127	62
67	→	136	B	10	136	67
67	←	137			137	67
71	→	144	A	8	144	71
71	←	145		8	145	71
76	→	154	B	10	154	76
76	←	155			155	76
80	→	162	A	8	162	80
80	←	163		8	163	80
84	←	170	B	8	170	84

Table — Example 4.

For the braiding process from the $p/b = 3/13$ Casa-coded Regular Knot to the $P/B = 19/85$ 3-pass Gaucho-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of the Table (pg. 294) we obtain from the algorithm diagram (upper right-hand side of Fig. 260) the following half-cycle algorithms:

$(i = 12)$	27 ↗	:	$u - o.$
$(i = \mathbf{13})$	**28** ↖	:	$(s,1)2u - o.$
$(i = 13)$	29 ↗	:	$2u - o.$
$(i = 14)$	30 ↖	:	$2u - o.$
			↗ half-cycles as half-cycle #29.
			↖ half-cycles as half-cycle #30.
$(i = \mathbf{17})$	**36** ↖	:	$2u - (s,1)2o.$
$(i = 17)$	37 ↘	:	$2o - 2u.$
$(i = 18)$	38 ↙	:	$2o - 2u.$
			↘ half-cycles as half-cycle #37.
			↙ half-cycles as half-cycle #38.
$(i = \mathbf{22})$	**46** ↙	:	$(s,2)3o - 2u.$
$(i = 22)$	47 ↗	:	$3u - 2o.$
$(i = 23)$	48 ↖	:	$3u - 2o.$
			↗ half-cycles as half-cycle #47.
			↖ half-cycles as half-cycle #48.
$(i = \mathbf{26})$	**54** ↖	:	$3u - (s,2)3o.$
$(i = 26)$	55 ↗	:	$3u - 3o.$
$(i = 27)$	56 ↖	:	$3u - 3o.$
			↗ half-cycles as half-cycle #55.
			↖ half-cycles as half-cycle #56.
$(i = \mathbf{31})$	**64** ↖	:	$(s)o - 3u - 3o.$
$(i = 31)$	65 ↘	:	$u - 3o - 3u.$
$(i = 32)$	66 ↙	:	$u - 3o - 3u.$
			↘ half-cycles as half-cycle #65.
			↙ half-cycles as half-cycle #66.
$(i = \mathbf{35})$	**72** ↙	:	$u - (3,s)4o - 3u.$
$(i = 35)$	73 ↘	:	$u - 4o - 3u.$
$(i = 36)$	74 ↙	:	$u - 4o - 3u.$
			↘ half-cycles as half-cycle #73.
			↙ half-cycles as half-cycle #74.
$(i = 40)$	**82** ↙	:	$(s,1)2u - 4o - 3u.$
$(i = 40)$	83 ↗	:	$2o - 4u - 3o.$
$(i = 41)$	84 ↖	:	$2o - 4u - 3o.$
			↗ half-cycles as half-cycle #83.
			↖ half-cycles as half-cycle #84.
$(i = \mathbf{44})$	**90** ↖	:	$2o - (3,s,1)5u - 3o.$
$(i = 44)$	91 ↗	:	$2o - 5u - 3o.$
$(i = 45)$	92 ↖	:	$2o - 5u - 3o.$
			↗ half-cycles as half-cycle #91.

			$\nwarrow$ half-cycles as half-cycle #92.
$(i = 49)$	**100** $\nwarrow$	:	$(s,2)3o - 5u - 3o.$
$(i = 49)$	101 $\searrow$	:	$3u - 5o - 3u.$
$(i = 50)$	102 $\swarrow$	:	$3u - 5o - 3u.$
			$\searrow$ half-cycles as half-cycle #101.
			$\swarrow$ half-cycles as half-cycle #102.
$(i = 53)$	**108** $\swarrow$	:	$3u - (3,s,2)6o - 3u.$
$(i = 53)$	109 $\searrow$	:	$3u - 6o - 3u.$
$(i = 54)$	110 $\swarrow$	:	$3u - 6o - 3u.$
			$\searrow$ half-cycles as half-cycle #109.
			$\swarrow$ half-cycles as half-cycle #110.
$(i = 58)$	**118** $\swarrow$	:	$(s)o - 3u - 6o - 3u.$
$(i = 58)$	119 $\nearrow$	:	$u - 3o - 6u - 3o.$
$(i = 59)$	120 $\nwarrow$	:	$u - 3o - 6u - 3o.$
			$\nearrow$ half-cycles as half-cycle #119.
			$\nwarrow$ half-cycles as half-cycle #120.
$(i = 62)$	**126** $\nwarrow$	:	$u - 3o - 3u - (s)o - 3u - 3o.$
$(i = 62)$	127 $\nearrow$	:	$u - 3o - 3u - o - 3u - 3o.$
$(i = 63)$	128 $\nwarrow$	:	$u - 3o - 3u - o - 3u - 3o.$
			$\nearrow$ half-cycles as half-cycle #127.
			$\nwarrow$ half-cycles as half-cycle #128.
$(i = 67)$	**136** $\nwarrow$	:	$(s,1)2u - 3o - 3u - o - 3u - 3o.$
$(i = 67)$	137 $\searrow$	:	$2o - 3u - 3o - u - 3o - 3u.$
$(i = 68)$	138 $\swarrow$	:	$2o - 3u - 3o - u - 3o - 3u.$
			$\searrow$ half-cycles as half-cycle #137.
			$\swarrow$ half-cycles as half-cycle #138.
$(i = 71)$	**144** $\swarrow$	:	$2o - 3u - 3o - (s,1)2u - 3o - 3u.$
$(i = 71)$	145 $\searrow$	:	$2o - 3u - 3o - 2u - 3o - 3u.$
$(i = 72)$	146 $\swarrow$	:	$2o - 3u - 3o - 2u - 3o - 3u.$
			$\searrow$ half-cycles as half-cycle #145.
			$\swarrow$ half-cycles as half-cycle #146.
$(i = 76)$	**154** $\swarrow$	:	$(s,2)3o - 3u - 3o - 2u - 3o - 3u.$
$(i = 76)$	155 $\nearrow$	:	$3u - 3o - 3u - 2o - 3u - 3o.$
$(i = 77)$	156 $\nwarrow$	:	$3u - 3o - 3u - 2o - 3u - 3o.$
			$\nearrow$ half-cycles as half-cycle #155.
			$\nwarrow$ half-cycles as half-cycle #156.
$(i = 80)$	**162** $\nwarrow$	:	$3u - 3o - 3u - (s,2)3o - 3u - 3o.$
$(i = 80)$	163 $\searrow$	:	$3o - 3u - 3o - 3u - 3o - 3u.$
$(i = 81)$	164 $\swarrow$	:	$2o - 3u - 3o - 3u - 3o - 3u.$
			$\searrow$ half-cycles as half-cycle #163.
			$\swarrow$ half-cycles as half-cycle #164.
$(i = 84)$	170 $\swarrow$	:	$3o - 3u - 3o - 3u - 3o - 3u.$

One important facet of Regular Band Knots which we haven't discussed yet is the

length of string required. This length depends on the number of parts, the number of bights, the width of the string, the thickness of the string, the bevelling of the string-edges, the coding-pattern, the stretchability of the string, the flexibility of the string, and the tightness of the braid. It will therefore be evident that a simple exact formula, for calculating the length of string required, cannot be given. However, a fair approximation for the length of string in the Regular Band Knot may be obtained from the following empirical formulae (the required string-length should be taken somewhat longer than the calculated length of string in the Band Knot):

For an *over - under* coding throughout:

$$\text{Length of string} = 2 \times P \times B \times w^*,$$

$$w^* = w + 1.2 \times t, \quad \text{where } w = \text{width of string; } t = \text{thickness of string,}$$

$$B = \frac{\text{circumference of object}}{1.9 \times w}.$$

For a *two over - two under* coding throughout:

$$\text{Length of string} = 2 \times P \times B \times w^*,$$

$$w^* = w + 0.6 \times t, \quad \text{where } w = \text{width of string; } t = \text{thickness of string,}$$

$$B = \frac{\text{circumference of object}}{1.4 \times w}.$$

In order to make the braiding process easier, the size of the intermediate 2-parts Column-coded Regular Knot or 3-parts Casa-coded Regular Knot should be adjusted so that the length of string in these knots is respectively $4 \times B \times w^*$ and $6 \times B \times w^*$.

Explorers in the Braiding Universe

Exploring in the *Braiding Universe* constitutes to a large extent a mental activity. Explorers in this field who lose sight of reality are bound to become residents of cloud-cuckoo-land. Unfortunately there are already a large number of such characters, luckily mainly confined at the moment to the arena of **topological knot theory**. Fortunately, nature has seen to it that *topological knot theory* is as far removed from real knots and braids as the planet Pluto is from the Sun.† However, the danger is that being in the same *'solar system'*, the topological nutty knot virus may spread to those explorers who are engaged with real knots and braids, and have lost their natural immunity through some academic brainwashing.

Every braider should therefore be very careful, and examine published material for **direct** or **indirect** practical applicability, not only relating to the practical usefulness of the braidform in question, but also to the actual construction procedure of it. If the material lacks any one of these vitally important properties, then the rubbish bin is the place to file it as far as real knots and braids are concerned. **The proof of the pudding is in the eating!**

† The two most important reasons that the *topological knot theory* is of no relevance to real knots and braids are its projection method, and its failure to recognise that the fundamental properties of knots and braids are related to their string-run and are not influenced by the type of crossings.

A Pineapple Knot

The following Pineapple Knot is an interweave of three Casa Knots, each of the same even number of parts.

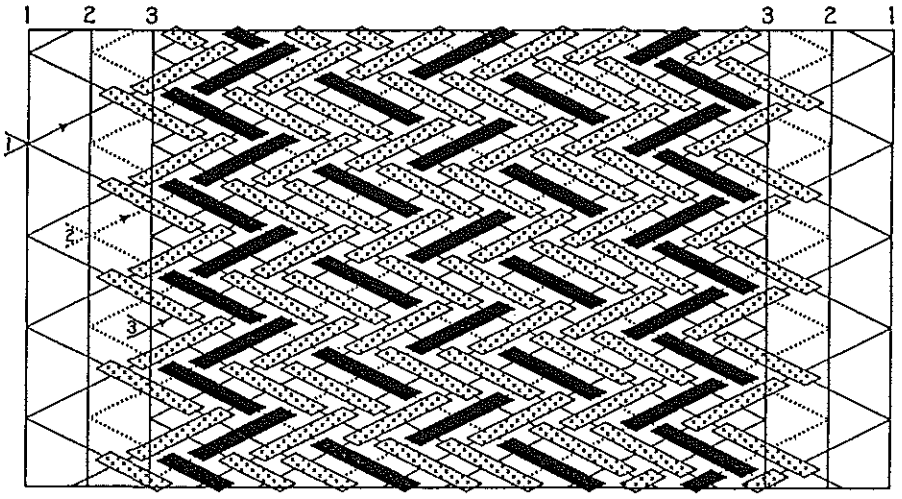


Fig. 261 — $A = 3$, $B^* = 5$, $P = 24$, $B = 15$, $P_{c_1} = P_{c_2} = P_{c_3} = 8$.

1st component; 1st colour; between bight-boundaries 1 (left) and 3 (right).

1. $L \longrightarrow R$: Free Run.
2. $R \longrightarrow L$: o.
3. $L \longrightarrow R$: u.
4. $R \longrightarrow L$: $u - 2o$.
5. $L \longrightarrow R$: $o - 2u$.
6. $R \longrightarrow L$: $2u - 2o$.
7. $L \longrightarrow R$: $2o - 2u$.
8. $R \longrightarrow L$: $o - 2u - o - u - o$.
9. $L \longrightarrow R$: $u - 2o - u - o - u$.
10. $R \longrightarrow L$: $o - u - o - u - o - u - o$.

2nd component; 2nd colour; between bight-boundaries 2 (left) and 2 (right).

1. $L \longrightarrow R$: $u - o - u - o - 2u - o$.
2. $R \longrightarrow L$: $o - u - o - u - 3o - u$.
3. $L \longrightarrow R$: $u - o - u - o - 3u - o$.
4. $R \longrightarrow L$: $o - 2u - o - u - 4o - u$.
5. $L \longrightarrow R$: $u - 2o - u - o - 3u - o - u$.
6. $R \longrightarrow L$: $o - 2u - o - 2u - 4o - u$.
7. $L \longrightarrow R$: $u - 2o - u - 2o - 3u - o - u$.
8. $R \longrightarrow L$: $2o - 2u - o - 2u - 2o - u - 2o - u$.
9. $L \longrightarrow R$: $2u - 2o - u - 2o - 3u - 2o - u$.
10. $R \longrightarrow L$: $2o - 2u - 2o - 2u - 2o - u - 2o - u$.

3rd component; 1st colour; between bight-boundaries 3 (left) and 1 (right).

1. $L \longrightarrow R$: $2u - 2o - 3u - 2o - 3u - 2o$.
2. $R \longrightarrow L$: $2o - 2u - 2o - u - 3o - u - 2o - 2u$.
3. $L \longrightarrow R$: $2u - 2o - 3u - 2o - 4u - 2o$.
4. $R \longrightarrow L$: $2o - 3u - 2o - u - 3o - u - 3o - 2u$.
5. $L \longrightarrow R$: $2u - 3o - 3u - 2o - 4u - 2o - u$.
6. $R \longrightarrow L$: $2o - 3u - 2o - 2u - 3o - u - 3o - 2u$.
7. $L \longrightarrow R$: $2u - 3o - 3u - 3o - 4u - 2o - u$.
8. $R \longrightarrow L$: $3o - 3u - 2o - 2u - 3o - 2u - 3o - 2u$.
9. $L \longrightarrow R$: $3u - 3o - 3u - 3o - 4u - 3o - u$.
10. $R \longrightarrow L$: $3o - 3u - 3o - 2u - 3o - 2u - 3o - 2u$.

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for
the braiding artisan

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New Zealand.

Solution to the Question in Issue No. 13

Question on pg. 279.

When two Regular Rectangular Mats have identical c_{I_x} sequences, then they also have identical c_{II_x} sequences.

For two Regular Rectangular Mats to have identical c_{I_x} sequences, and hence identical c_{II_x} sequences, both mats must have identical b_v-values.

We obtained for Regular Rectangular Mats:[†]

$$\Delta = \Delta_{I,1} b_v \,,$$

$$\Delta = \frac{nb_v + 1}{|b_h|_{b_v}} \,; \quad n \text{ is the smallest positive integer such that } \Delta \text{ is an integer.}$$

$$\Delta^* = \Delta_{II,1} b_v \,,$$

$$\Delta^* = \frac{n^* b_v - 1}{|b_h|_{b_v}} \,; \quad n^* \text{ is the smallest positive integer such that } \Delta^* \text{ is an integer.}$$

$$\Delta^* = \frac{n^\bullet b_v + 1}{|-b_h|_{b_v}} \,; \quad n^\bullet \text{ is the smallest positive integer such that } \Delta^* \text{ is an integer.}$$

$$\Delta + \Delta^* = b_v \,.$$

Let ${b_h}' = m \cdot b_v + b_h$, where m is a positive integer or a negative integer such that $(m \cdot b_v + b_h) > 0$. Then:

$$\Delta' = \frac{n' b_v + 1}{|{b_h}'|_{b_v}} = \frac{n' b_v + 1}{|m b_v + b_h|_{b_v}} = \frac{n' b_v + 1}{|b_h|_{b_v}} \,; \quad \text{hence } n' = n \text{ and } \Delta' = \Delta \,.$$

$\Delta' + \Delta^{*\prime} = b_v$; hence $\Delta^{*\prime} = b_v - \Delta' = b_v - \Delta = \Delta^*$.

Thus the Regular Rectangular Mats $(mb_v + b_h)/b_v$, with m as an (**integer**[‡]) variable such that $(mb_v + b_h) > 0$, have identical c_{I_x} sequences and identical c_{II_x} sequences. For example, the Regular Rectangular Mats $b_h/b_v = 3/7$; $b_h/b_v = 10/7$; $b_h/b_v = 17/7$; $b_h/b_v = 24/7$; etc. all have the following c_{I_x} and c_{II_x} sequences:

bight-point 1 $\longrightarrow$ $c_{I_1} = 5 = \Delta$.	bight-point 1 $\longrightarrow$ $c_{II_1} = 7 = b_v$.
bight-point 2 $\longrightarrow$ $c_{I_2} = 3$.	bight-point 2 $\longrightarrow$ $c_{II_2} = 5$.
bight-point 3 $\longrightarrow$ $c_{I_3} = 1$.	bight-point 3 $\longrightarrow$ $c_{II_3} = 3$.
bight-point 4 $\longrightarrow$ $c_{I_4} = 6$.	bight-point 4 $\longrightarrow$ $c_{II_4} = 1$.
bight-point 5 $\longrightarrow$ $c_{I_5} = 4$.	bight-point 5 $\longrightarrow$ $c_{II_5} = 6$.
bight-point 6 $\longrightarrow$ $c_{I_6} = 2$.	bight-point 6 $\longrightarrow$ $c_{II_6} = 4$.
bight-point 7 $\longrightarrow$ $c_{I_7} = 7 = b_v$.	bight-point 7 $\longrightarrow$ $c_{II_7} = 2 = \Delta^*$.

Let $b_h < b_v$ and ${b_h}' = b_v - b_h$. Then:

$$\Delta' = \frac{n' b_v + 1}{|{b_h}'|_{b_v}} = \frac{n' b_v + 1}{|b_v - b_h|_{b_v}} = \frac{n' b_v + 1}{|-b_h|_{b_v}} \,; \quad \text{hence } n' = n^\bullet \text{ and } \Delta' = \Delta^* \,.$$

$\Delta' + \Delta^{*\prime} = b_v$; hence $\Delta^{*\prime} = b_v - \Delta' = b_v - \Delta^* = \Delta$.

[†] See *The Braider*, Issue No. 11, pg. 229; Issue No. 7, pg. 133; Issue No. 6, pg. 115.

[‡] **Integers** are the numbers $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$.

Thus the set of Regular Rectangular Mats $(m_1 b_v + b_h)/b_v$, with m_1 as a (**whole number**†) variable and $b_h < b_v$, have $c_{I_{x_1}}$ and $c_{II_{x_1}}$ sequences which are respectively complementary with the $c_{I_{x_2}}$ and $c_{II_{x_2}}$ sequences belonging to the set of Regular Rectangular Mats $(m_2 b_v - b_h)/b_v$, with m_2 as a (**natural number**‡) variable. For example, the Regular Rectangular Mats $b_h/b_v = 3/7$; $b_h/b_v = 10/7$; $b_h/b_v = 17/7$; $b_h/b_v = 24/7$; etc. all have $c_{I_{x_1}}$ and $c_{II_{x_1}}$ sequences which are respectively complementary to the following $c_{I_{x_2}}$ and $c_{II_{x_2}}$ sequences belonging to the Regular Rectangular Mats $b_h/b_v = 4/7$; $b_h/b_v = 11/7$; $b_h/b_v = 18/7$; $b_h/b_v = 25/7$; etc.:

bight-point 1 $\longrightarrow$	$c_{I_1} = 2 = \Delta$.	bight-point 1 $\longrightarrow$	$c_{II_1} = 7 = b_v$.
bight-point 2 $\longrightarrow$	$c_{I_2} = 4$.	bight-point 2 $\longrightarrow$	$c_{II_2} = 2$.
bight-point 3 $\longrightarrow$	$c_{I_3} = 6$.	bight-point 3 $\longrightarrow$	$c_{II_3} = 4$.
bight-point 4 $\longrightarrow$	$c_{I_4} = 1$.	bight-point 4 $\longrightarrow$	$c_{II_4} = 6$.
bight-point 5 $\longrightarrow$	$c_{I_5} = 3$.	bight-point 5 $\longrightarrow$	$c_{II_5} = 1$.
bight-point 6 $\longrightarrow$	$c_{I_6} = 5$.	bight-point 6 $\longrightarrow$	$c_{II_6} = 3$.
bight-point 7 $\longrightarrow$	$c_{I_7} = 7 = b_v$.	bight-point 7 $\longrightarrow$	$c_{II_7} = 5 = \Delta^*$.

We have seen earlier how Regular Rectangular Mats are related to Regular Knots.†† The tree in which the enlargement-paths of Regular Knots are depicted (the RKT) also depicts the enlargement-paths of the Regular Rectangular Mats after replacing 'p' and 'b' by respectively 'b_h' and 'b_v'; this gives us the RRMT (the Regular Rectangular Mat Tree). The RRMT not only enables us to comprehend the relationships between different Regular Rectangular Mats (see Fig. 262), but also the relationships between a Regular Rectangular Mat b_h/b_v and its associated 'Δ' and 'Δ^*' values (see Fig. 263).

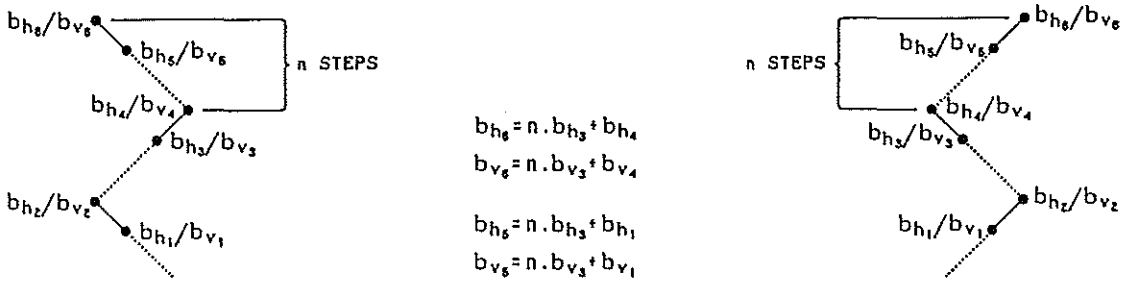


Fig. 262 — Relationships between Regular Rectangular Mats in the RRMT.

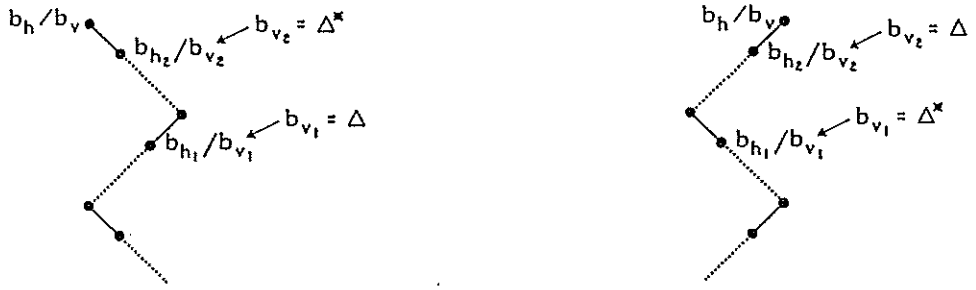


Fig. 263 — A Regular Rectangular Mat b_h/b_v and its associated Δ and Δ^* values.

† **Whole numbers** are numbers 0, 1, 2, 3, ...
‡ **Natural numbers** or **positive integers** are the numbers 1, 2, 3, ...
†† See *The Braider*, Issue No. 10, pg. 200.

Euclid's algorithms and the path formulae associated with the Regular Rectangular Mats b_h/b_v and $(m \cdot b_v + b_h)/b_v$, with m as a (natural number) variable and $b_h < b_v$, are shown in Fig. 264. Their paths in the RRMT are depicted in Fig. 265.

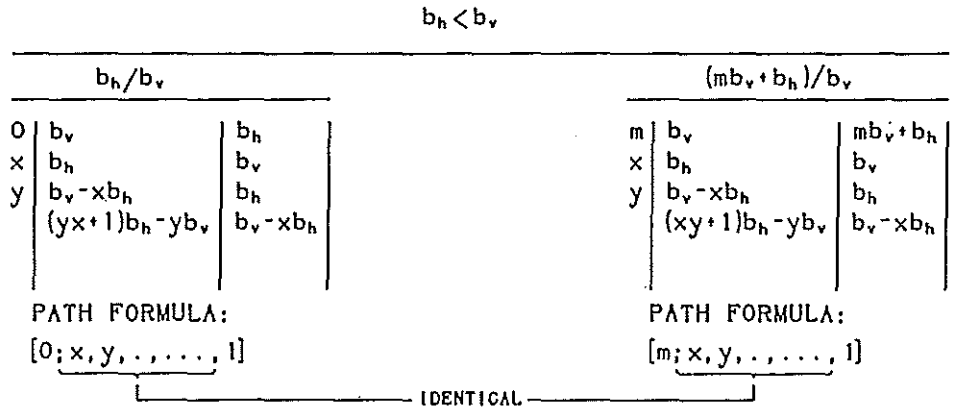

$b_h < b_v$
b_h/b_v
$(mb_v + b_h)/b_v$
0 | b_v | b_h
x | b_h | b_v
y | $b_v - xb_h$ | b_h
$(yx+1)b_h - yb_v$ | $b_v - xb_h$
m | b_v | $mb_v + b_h$
x | b_h | b_v
y | $b_v - xb_h$ | b_h
$(xy+1)b_h - yb_v$ | $b_v - xb_h$
PATH FORMULA:
[0; x, y, . , . . . , 1]
PATH FORMULA:
[m; x, y, . , . . . , 1]
IDENTICAL

Fig. 264 — Euclid's algorithms and path formulae for b_h/b_v and $(mb_v + b_h)/b_v$, with $b_h < b_v$.



Fig. 265 — The paths in the RRMT for b_h/b_v and $(mb_v + b_h)/b_v$, with $b_h < b_v$.

Euclid's algorithms and the path formulae associated with the Regular Rectangular Mats b_h/b_v and $(b_v - b_h)/b_v$, where $\frac{b_v}{2} < b_h < b_v$, are shown in Fig. 266. Their paths in the RRMT are depicted in Fig. 267.

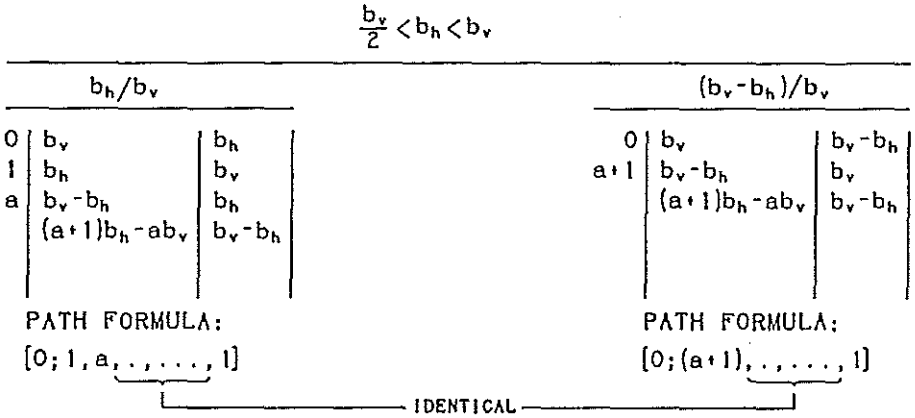

$\frac{b_v}{2} < b_h < b_v$
b_h/b_v
$(b_v - b_h)/b_v$
0 | b_v | b_h
1 | b_h | b_v
a | $b_v - b_h$ | b_h
$(a+1)b_h - ab_v$ | $b_v - b_h$
0 | b_v | $b_v - b_h$
a+1 | $b_v - b_h$ | b_v
$(a+1)b_h - ab_v$ | $b_v - b_h$
PATH FORMULA:
[0; 1, a, . , . . . , 1]
PATH FORMULA:
[0; (a+1), . , . . . , 1]
IDENTICAL

Fig. 266 — Euclid's algorithms and path formulae for b_h/b_v and $(b_v - b_h)/b_v$, with $\frac{b_v}{2} < b_h < b_v$.

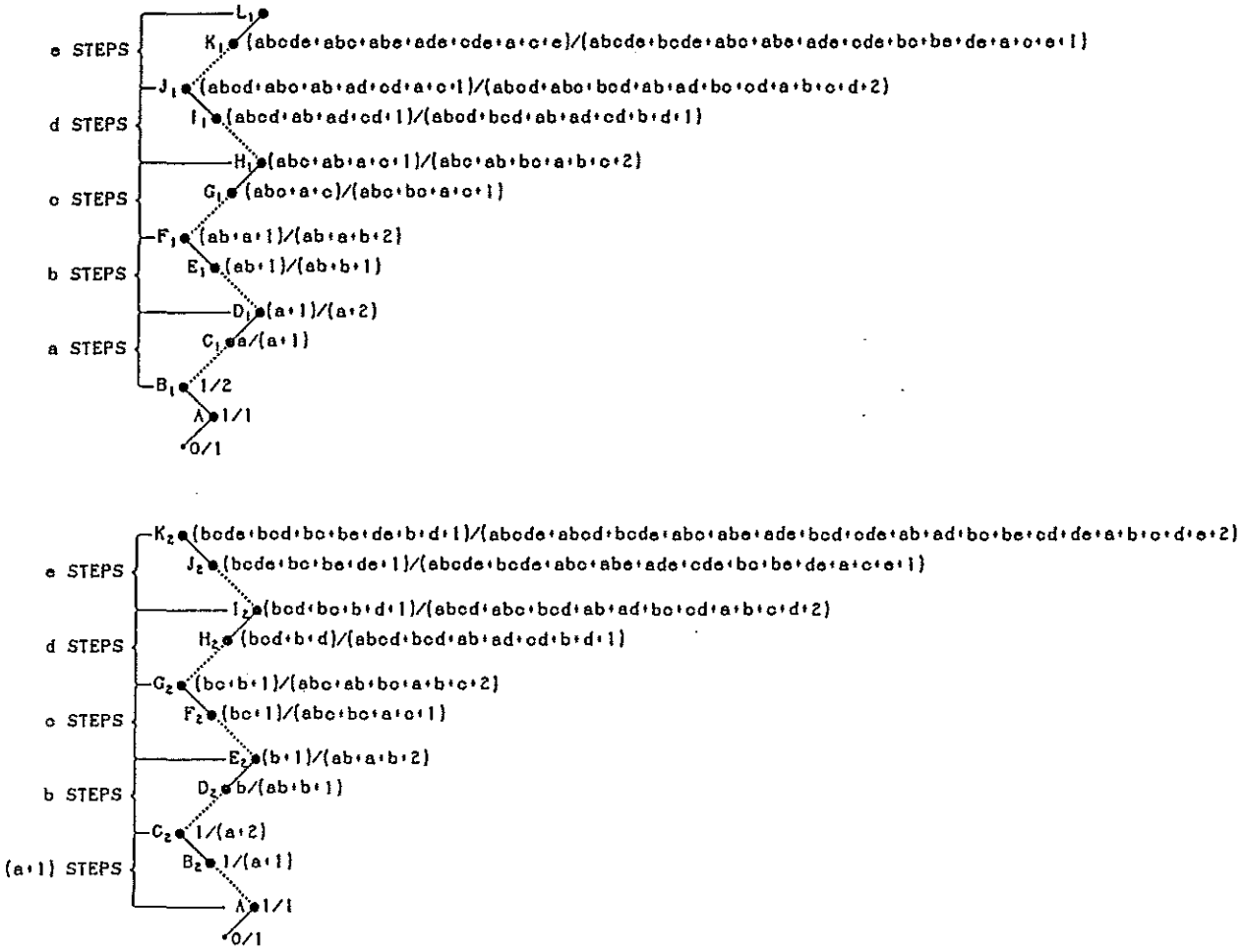


Fig. 267 — The paths in the RRMT for b_h/b_v and $(b_v - b_h)/b_v$, with $\frac{b_v}{2} < b_h < b_v$.

The sequences $F_n/F_{(n+1)}$ with $n \geq 1$ and $F_n/F_{(n+2)}$ with $n \geq 1$ are respectively the same as the sequences $F_{(n-1)}/F_n$ with $n \geq 2$ and $F_{(n-2)}/F_n$ with $n \geq 3$.†

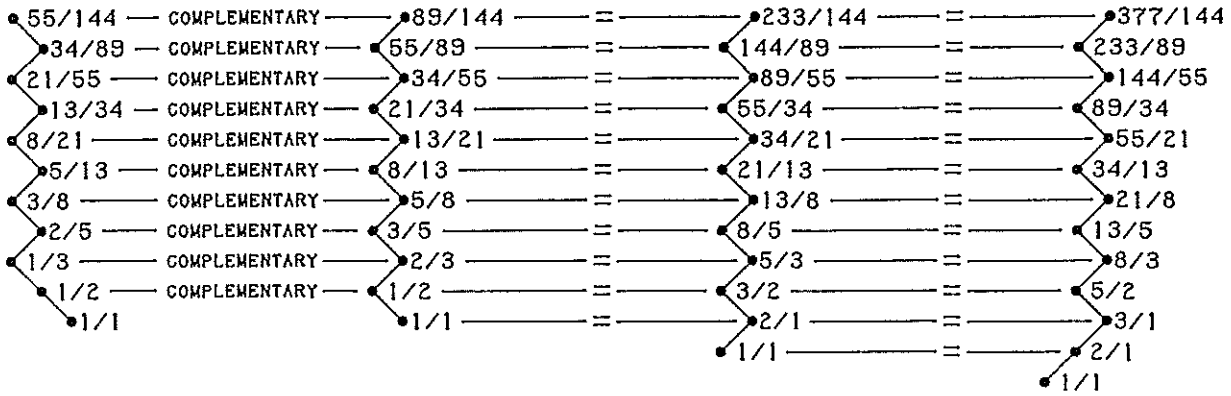


Fig. 268 — The relationships between the nodes on the sequence paths $F_{(n-1)}/F_n$; $F_{(n+1)}/F_n$; $F_{(n+2)}/F_n$ and $F_{(n-2)}/F_n$.

For the Regular Rectangular Mats $b_h/b_v = F_{(n-1)}/F_n$; $b_h/b_v = F_{(n+1)}/F_n$; $b_h/b_v = F_{(n+2)}/F_n$ and $b_h/b_v = F_{(n-2)}/F_n$, Fig. 268 illustrates the relationships between the

† See *The Braider*, Issue No. 8, pg. 168.

nodes on the sequence paths $F_{(n-1)}/F_n$; $F_{(n+1)}/F_n$; $F_{(n+2)}/F_n$ and their complementary sequence path $F_{(n-2)}/F_n$.

From the more general case, depicted on the right-hand side of Fig. 269, it will be seen that the sequence paths $F_{(n-1)}/F_n$ and $F_{(n-2)}/F_n$ are each other's most basic complementary ones (see left-hand side of Fig. 269 where $m = 2$).

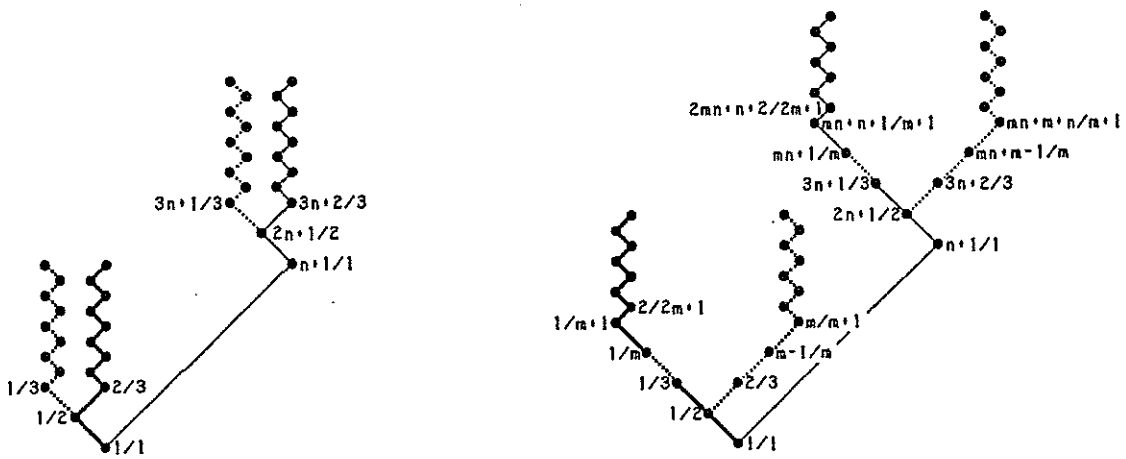


Fig. 269 — Fibonacci sequence paths in the RRMT.

Braiding Regular Rectangular Mats.

We mentioned earlier† that the braiding of Regular Rectangular Mats via enlargement processes is, to a lesser or greater extent, always a somewhat cumbersome method. Hence in practice we use a different process; a process which we shall discuss with the aid of a few Examples.

It should be noted that although the string-run diagrams of the Regular Rectangular Mats $b_h/b_v = s/t$ and $b_h/b_v = t/s$ are in essence identical (one is rotated 90° relative to the other), their coding and their braiding procedures are different. Since some coding arrangements are easier to braid than others (for example a column-coding is much easier to braid than a row-coding), we place the grid-diagram of a Regular Rectangular Mat so that its left-hand bight-boundary and the coding arrangement in relation to this boundary gives us the easiest construction procedure.

In *The Braider*, Issue No. 10, pg. 200, we have seen how a Regular Rectangular Mat is related to a Regular Knot. Hence it will be obvious that the Regular Rectangular Mat $b_h/b_v = s/t$ can be related to the two Regular Knots $p/b = s/t$ and $p/b = t/s$. Consequently the string-run of a Regular Rectangular Mat can be seen as the set of half-cycles associated with the Regular Knot $p/b = s/t$, but can also be seen as the set of half-cycles associated with the Regular Knot $p/b = t/s$.

When $b_v = t$, then the set of half-cycles associated with the Regular Knot $p/b = s/t$ is the primary set of half-cycles, and the set of half-cycles associated with the Regular Knot $p/b = t/s$ is the secondary set of half-cycles.

When $b_v = s$, then the set of half-cycles associated with the Regular Knot $p/b = t/s$ is the primary set of half-cycles, and the set of half-cycles associated with the Regular

† See *The Braider*, Issue No. 12, pg. 256.

Knot $p/b = s/t$ is the secondary set of half-cycles.

The Regular Rectangular Mat will be braided in accordance with its primary set of half-cycles.

The string-run of the Regular Rectangular Mat with $s = 13$ and $t = 4$ is depicted in Fig. 270. Hence from the above it follows that we use the left-hand diagram when $b_v = 4$ (consequently $b_h = 13$), and the right-hand diagram when $b_v = 13$ (consequently $b_h = 4$).

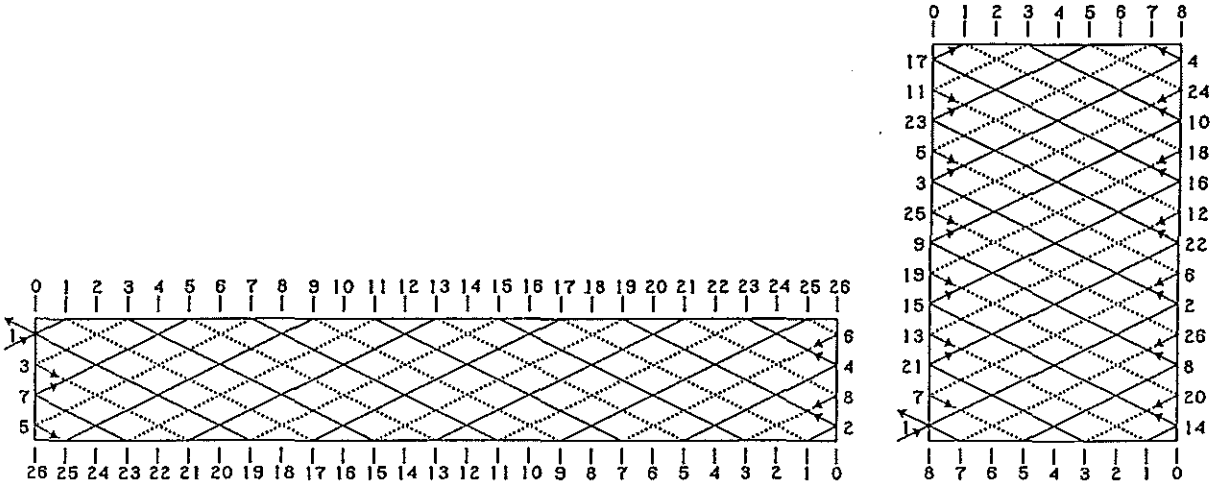


Fig. 270 — The Regular Rectangular Mat with $s = 13$ and $t = 4$.

The crossings on the **even**-numbered intersection-columns are associated with the set of **primary** half-cycles, while the crossing on the **odd**-numbered intersection-columns are associated with the set of **secondary** half-cycles. See Figs. 270, 271, 272. We call the **even**-numbered intersection-columns the **primary** intersection-columns, and the **odd**-numbered intersection-columns the **secondary** intersection-columns of the Regular Rectangular Mat.

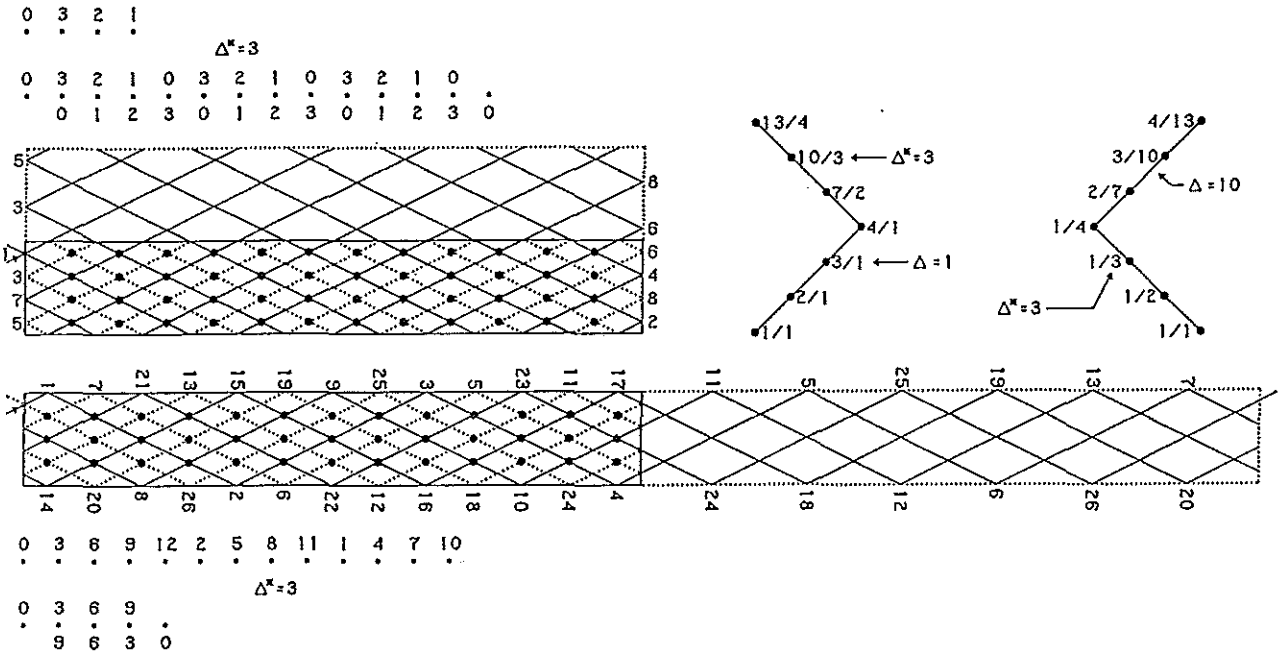


Fig. 271 — The Regular Rectangular Mat $b_h/b_v = 13/4$.

Thus for the Regular Rectangular Mat $b_h/b_v = 13/4$, the crossing-points on the

primary intersection-columns (the *even*-numbered intersection-columns) are associated with the Regular Knot $p/b = 13/4$, while the crossing-points on the secondary intersection-columns (the *odd*-numbered intersection-columns) are associated with the Regular Knot $p/b = 4/13$. For the Regular Rectangular Mat $b_h/b_v = 4/13$, the crossing-points on the primary intersection-columns (the *even*-numbered intersection-columns) are associated with the Regular Knot $p/b = 4/13$, while the crossing-points on the secondary intersection-columns (the *odd*-numbered intersection-columns) are associated with the Regular Knot $p/b = 13/4$.

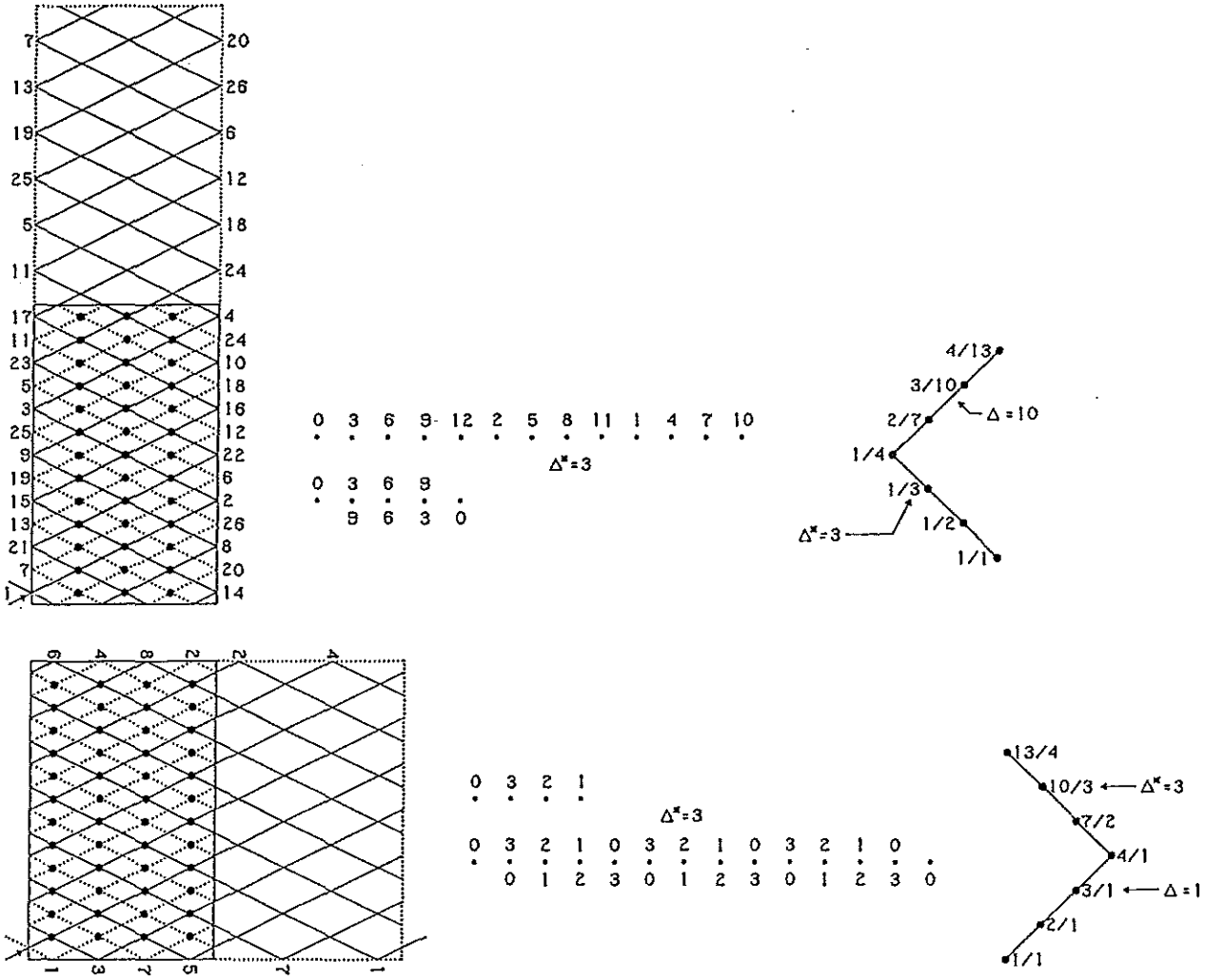


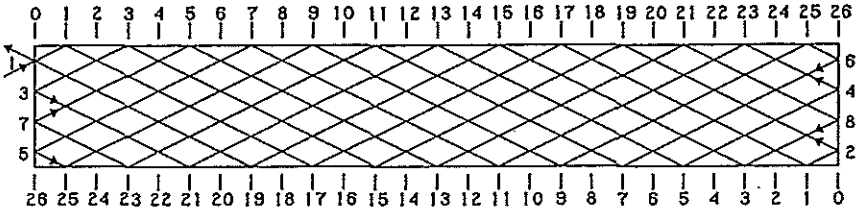
Fig. 272 — The Regular Rectangular Mat $b_h/b_v = 4/13$.

In *The Braider*, Issue No. 13, pg. 275 we have shown how a bight-index was assigned to each bight-point on the bight-boundary of the Standing End. The half-cycle which leaves the bight-point with bight-index I_a and bight-number $i = (a-1)$ on the bight-boundary of the Standing End is half-cycle $h_o = (2a+1)$. The bight-point on the opposite bight-boundary in which this half-cycle ends has also the bight-index I_a but has the bight-number $i = a$; the half-cycle which leaves this bight-point is half-cycle $h_e = (2a+2)$.† In Fig. 270 the primary half-cycles for respectively $b_h/b_v = 13/4$ and $b_h/b_v = 4/13$ are indicated. In the uppermost string-run diagram of Fig. 271 the primary half-cycles are indicated while in the lowermost string-run diagram the secondary

† See also *The Braider*, Issue No. 4, pg. 80, and Issue No. 5, pg. 90.

half-cycles are indicated. The complementary cyclic bight-number scheme and basic algorithm diagram are shown respectively above and below their associated diagram. Similarly in Fig. 272 the primary half-cycles are indicated in the uppermost string-run diagram while the secondary half-cycles are indicated in the lowermost string-run diagram; the complementary cyclic bight-number scheme and basic algorithm diagram are shown to the right of their associated diagram.

For the Regular Rectangular Mat $b_h/b_v = 13/4$, the basic algorithm table for the string-run depicted in Figs. 270 and 271 is shown in Fig. 273. The table contains $(2b_h + 1) = 2 \cdot 13 + 1 = 27$ columns, the first and last of which represent respectively the left-hand and right-hand bight-boundaries. The inner $(2b_h - 1) = 25$ columns represent the intersection-columns of the mat; the odd-numbered columns represent the secondary intersection columns and the even-numbered columns represent the primary intersection columns.



4/13	0	3	2	1	0	3	2	1	0	3	2	1	0	4/13													
		0	1	2	3	0	1	2	3	0	1	2	3	0													
1→	1					2								3										4			
		•	7						•	6						•	5								←2		
3→					8		✳	•				9		✳	•				10		✳	•					
		•		•	✳		13			•		•	✳		12			•		•	✳		11			←4		
5→	14		✳		✳	•		•	15		✳		✳	•		•	16		✳		✳	•		•	17		
	✳	•	20	•		•	✳		✳	•	19	•		•	✳		✳	•	18	•		•	✳		✳	←6		
7→	✳		✳	•	21	•	✳	•	✳		✳	•	22	•	✳	•	✳		✳	•	23	•	✳	•	✳			
	✳	•	✳	•	✳	•	26	•	✳	•	✳	•	✳	•	25	•	✳	•	✳	•	✳	•	24	•	✳	←8		
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	

Fig. 273 — A string-run with its basic algorithm table for the Regular Rectangular Mat $b_h/b_v = 13/4$.

The numbers with arrows in the leftmost and rightmost columns indicate the (primary) half-cycles according to which the mat will be braided. The two header lines contain the basic algorithm diagram entries for the primary half-cycles; the uppermost line contains the entries for the odd-numbered primary half-cycles and the second line contains the entries for the even-numbered primary half-cycles.

The basic table is constructed in three stages. In the first stage (see Fig. 274) the primary intersections, indicated by a dot, are determined in the usual manner† with the aid of the basic algorithm diagram displayed in the two header lines.

† See *The Braider*, Issue No. 5, pp. 93 - 95.

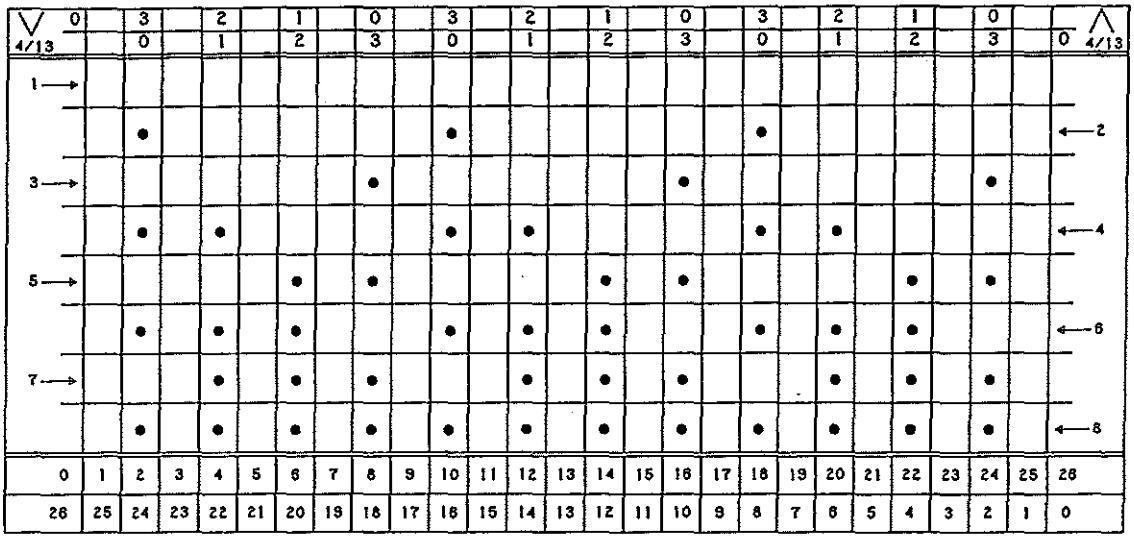


Fig. 274 — The first construction stage of the algorithm table.

In the second stage (see Fig. 275) we enter the bight-points which are situated on the upper and lower horizontal bight-boundaries. Each such bight-point is represented by a bight-point indicator with the number of the secondary half-cycle which leaves this bight-point. The consecutive bight-point indicators on the string-run are $2b_v = 2 \cdot 4 = 8$ columns in total apart. The indicator for bight-point $\#1$ is on the 1st (primary) half-cycle in the 1st secondary intersection-column from the left-hand bight-boundary (hence in the 2nd column from the left since the left-hand and the right-hand bight-boundary count each as 1 column).

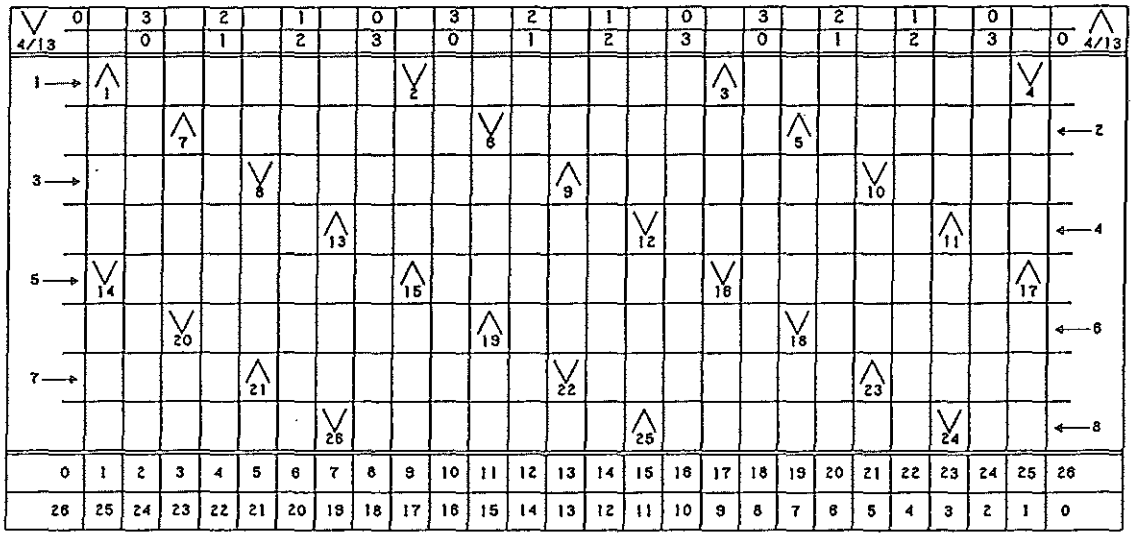


Fig. 275 — The second construction stage of the algorithm table.

In the third stage (see Fig. 276) the position of the secondary intersections (indicated by a star) are determined in the usual manner for each secondary half-cycle in relation to the bight-point from which the secondary half-cycle leaves. These intersections are in the secondary intersection-columns.

The intersection-points in the basic algorithm table are finally replaced by their appropriate coding.

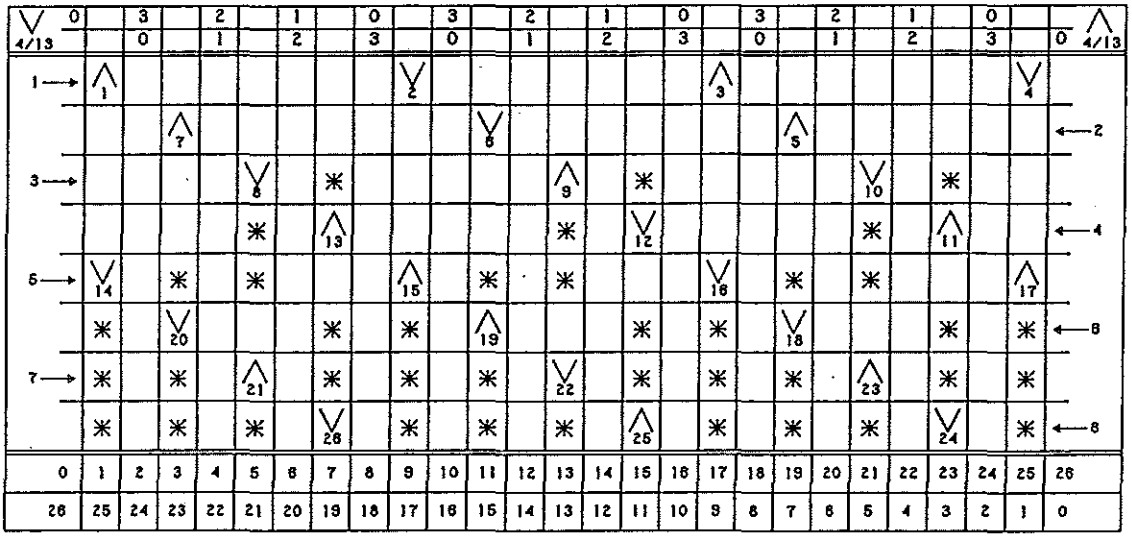


Fig. 276 — The third construction stage of the algorithm table.

The position of the Standing End in Fig. 273 enabled us to start the set of secondary half-cycles with its first half-cycle leaving the leftmost bight-point on the upper horizontal bight-boundary. In general, however, the Standing End will not be placed in the uppermost bight-point on the left-hand bight-boundary and consequently the first bight-point on the upper horizontal bight-boundary through which the string-run passes will not be the leftmost bight-point on this upper horizontal bight-boundary. Since each secondary half-cycle runs between the upper and lower horizontal bight-boundaries, it will be evident that the first of the secondary half-cycles starts in fact at a bight-point on the lower horizontal bight-boundary, and that this bight-point should receive the number 1. Hence in the case that the Standing End is positioned at the uppermost bight-point on the left-hand bight-boundary, our previous number 1 bight-point on the upper bight-boundary becomes bight-point number 2 (see Fig. 277).

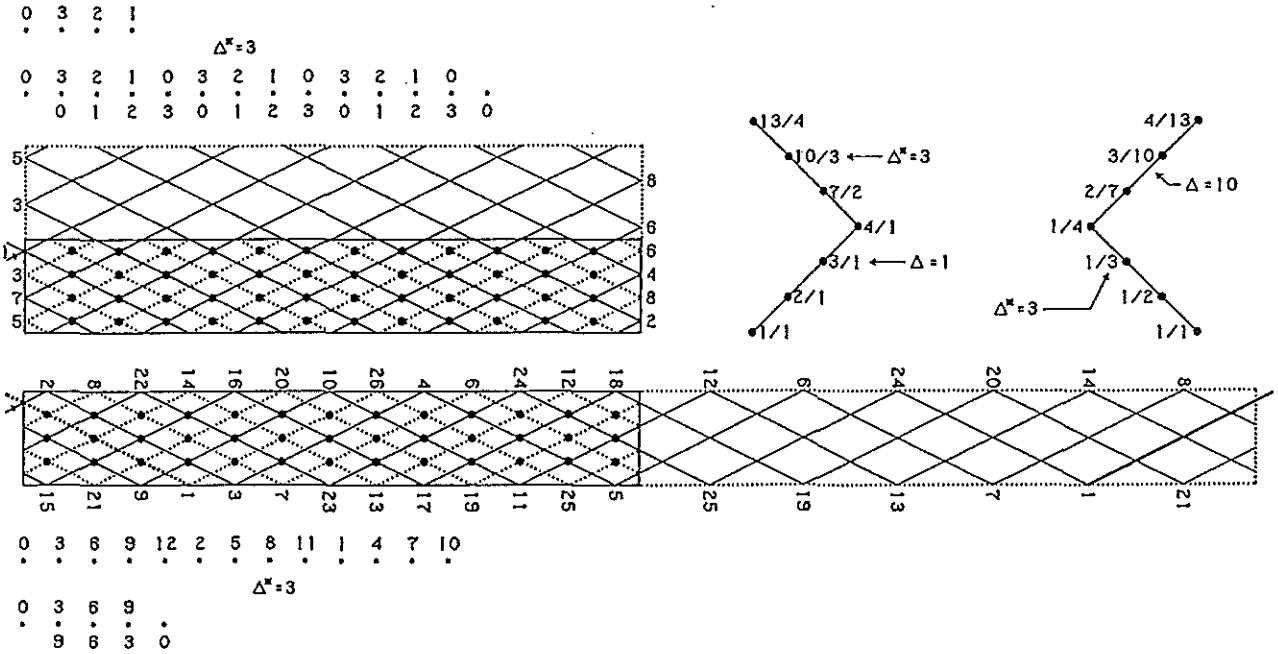


Fig. 277 — The Regular Rectangular Mat $b_h/b_v = 13/4$.

The first section of the first secondary half-cycle (the section which runs from bight-point #1 on the lower bight-boundary to the left-hand bight-boundary where the Standing End (of the primary half-cycles) is positioned) will not be laid down first, but instead will be the very last part of the total string-run to be laid down. Hence in the basic algorithm table we have to ensure, by blocking out the relevant positions on the secondary intersection-columns, that we cannot enter intersection points on this secondary half-cycle section till it is being laid down. Thus the basic algorithm table is constructed in the following three stages.

In the first stage the primary intersections, indicated by a dot, are determined with the aid of the basic algorithm diagram displayed in the two header lines (see Fig. 274).

In the second stage (see Fig. 278) we enter the bight-points which are situated on the upper and lower horizontal bight-boundaries. Each such bight-point is represented by a bight-point indicator with the number of the secondary half-cycle which leaves this bight-point. The consecutive bight-point indicators on the string-run are $2b_v = 2 \cdot 4 = 8$ columns in total apart. The indicator for bight-point #1 is in the $n^{th} = 4^{th}$ secondary intersection-column from the left-hand bight-boundary (hence in the $2n^{th} = 8^{th}$ column from the left) on the last (primary) half-cycle ($= 2b_v{}^{th} = 8^{th}$ half-cycle) when the Standing End on the left-hand bight-boundary (on the first primary half-cycle) is positioned at the $n^{th} = 4^{th}$ bight-point from the **lower** horizontal bight-boundary.

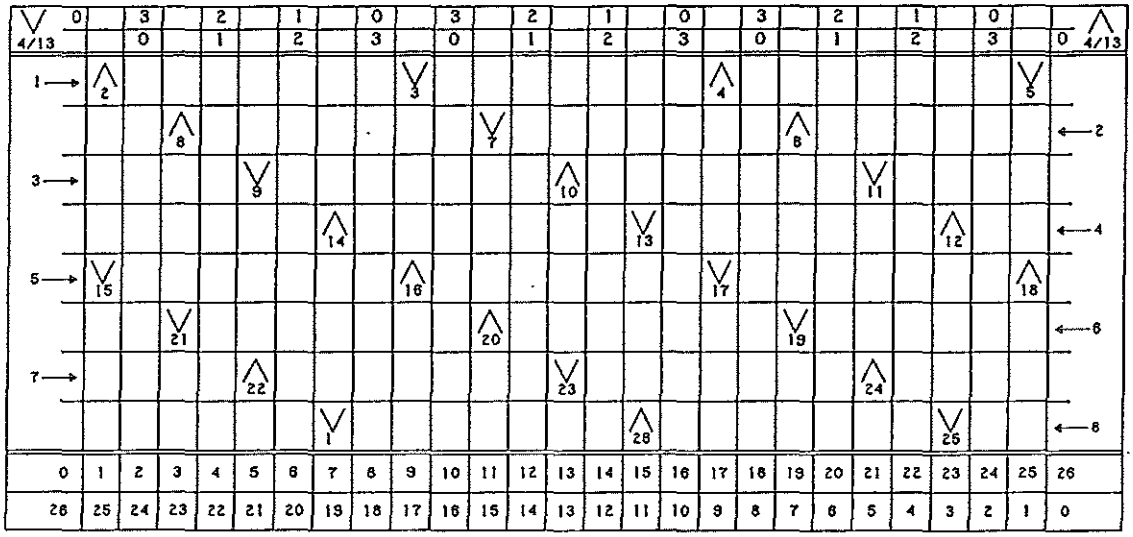


Fig. 278 — The second construction stage of the algorithm table.

The indicator for bight-point #2 is in the $m^{th} = 1^{st}$ secondary intersection-column from the left-hand bight-boundary (hence in the $2m^{th} = 2^{nd}$ column from the left) on the first (primary) half-cycle when the Standing End on the left-hand bight-boundary (on the first primary half-cycle) is positioned at the $m^{th} = 1^{st}$ bight-point from the **upper** horizontal bight-boundary.

In the third stage (see Fig. 279) we first determine which cells in the basic algorithm table have to be blocked out. For this we determine the number of each (primary) half-cycle which enters a bight-point on the left-hand bight-boundary. The process is similar to the assignment of bight-indices to the bight-points on the Standing End's bight-boundary (see *The Braider*, Issue No. 13, pg. 275), but instead of Δ and modulus b_v we use 2Δ and modulus $2b_v$. This gives with $\Delta = 1$ and modulus $b_v = 4$ the sequence 0, 2, 4, 6 (see Fig. 279). The first number **below** 0 is entered in the first secondary intersection-column from the left-hand bight-boundary as shown in Fig. 279.

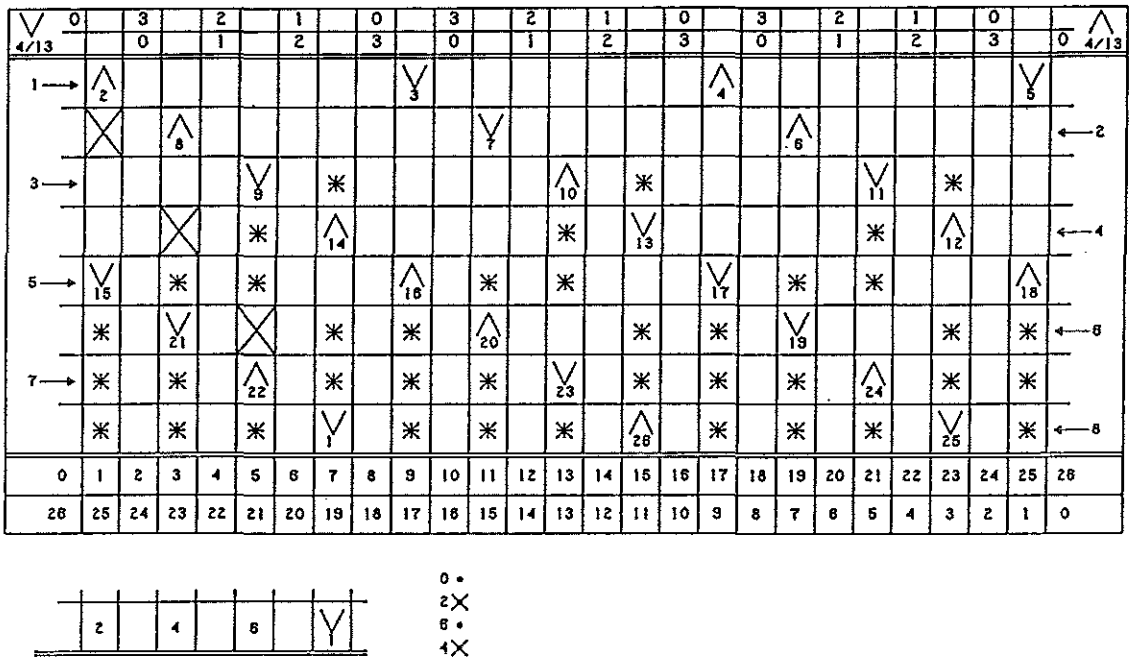


Fig. 279 — The third construction stage of the algorithm table.

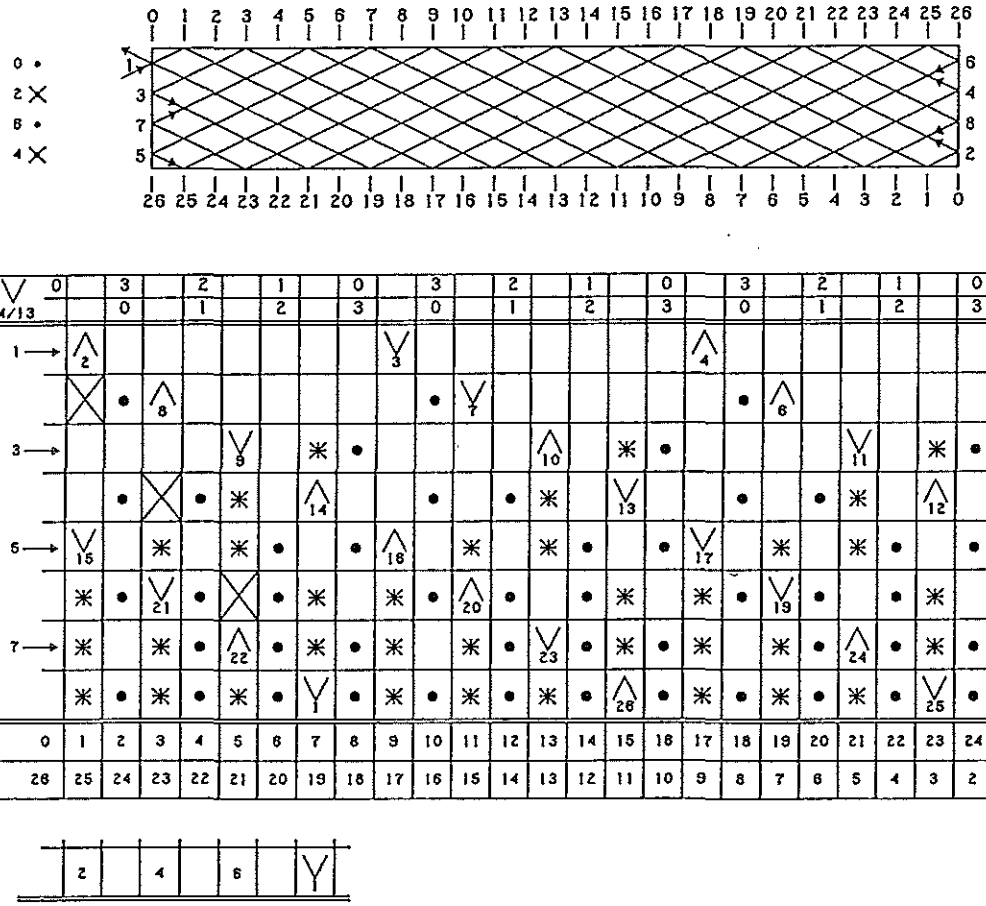


Fig. 280 — The final basic algorithm table.

The remaining numbers below 0 are set off in order in the sequential secondary intersection-columns towards the right (see Fig. 279). Note that each successive number

is derived from its preceding one by adding 2Δ and calculating the mod $2b_v$ value. Each number indicates the (primary) half-cycle in which the corresponding secondary intersection-column cell must be blocked by crossing it out. The position of the secondary intersections (indicated by a star) in the secondary intersection-columns are then determined in the usual manner for each secondary half-cycle in relation to the bight-point from which it leaves.

In the final basic algorithm table (see Fig. 280) the intersection-points are replaced by their appropriate coding.

The string-run of the same Regular Rectangular Mat, but with a different position for the Standing End, is shown in Fig. 281.

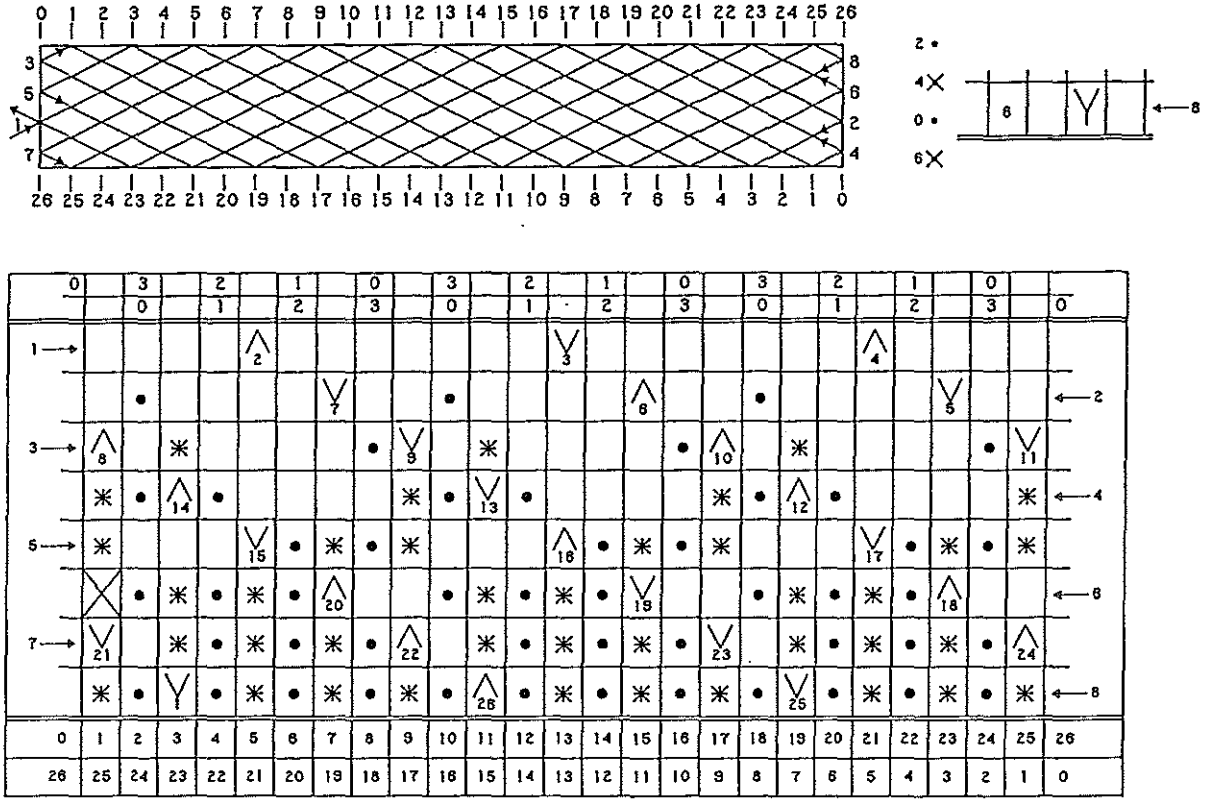
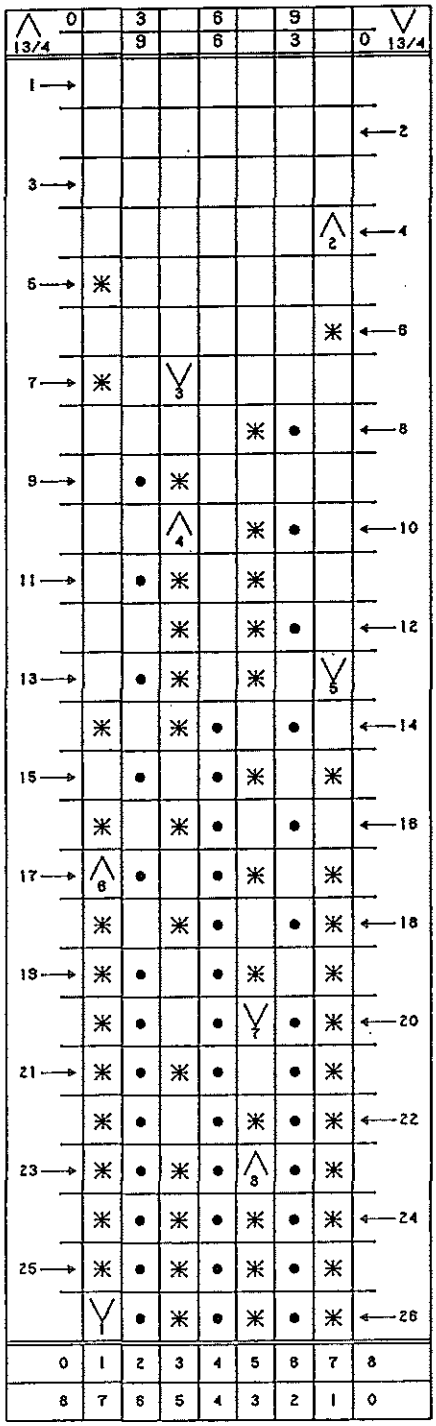


Fig. 281 — A string-run with its basic algorithm table for the Regular Rectangular Mat $b_h/b_v = 13/4$.

We mentioned earlier that although the string-run diagrams of the Regular Rectangular Mats $b_h/b_v = s/t$ and $b_h/b_v = t/s$ are in essence identical (one is rotated 90° relative to the other), their coding and their braiding procedures are different. This enables us to select the most convenient orientation with respect to its construction process.

Fig. 282 shows a string-run and its associated basic algorithm table for the Regular Rectangular Mat $b_h/b_v = 4/13$. The relevant complementary cyclic bight-number schemes and basic algorithm diagrams associated with its primary and secondary set of half-cycles were shown in Fig. 272, pg. 305, and discussed on pg. 306.

The position of the Standing End on the left-hand bight-boundary is such that there are no blocked out cells in the basic algorithm table.



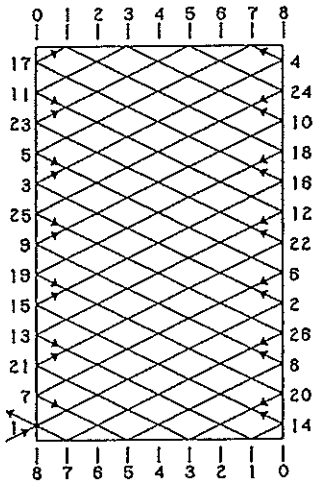


Fig. 282 — A string-run with its basic algorithm table for the Regular Rectangular Mat $b_h/b_v = 4/13$.

Two further string-runs with their associated basic algorithm tables for this Regular Rectangular Mat are shown in Figs. 283 and 284.

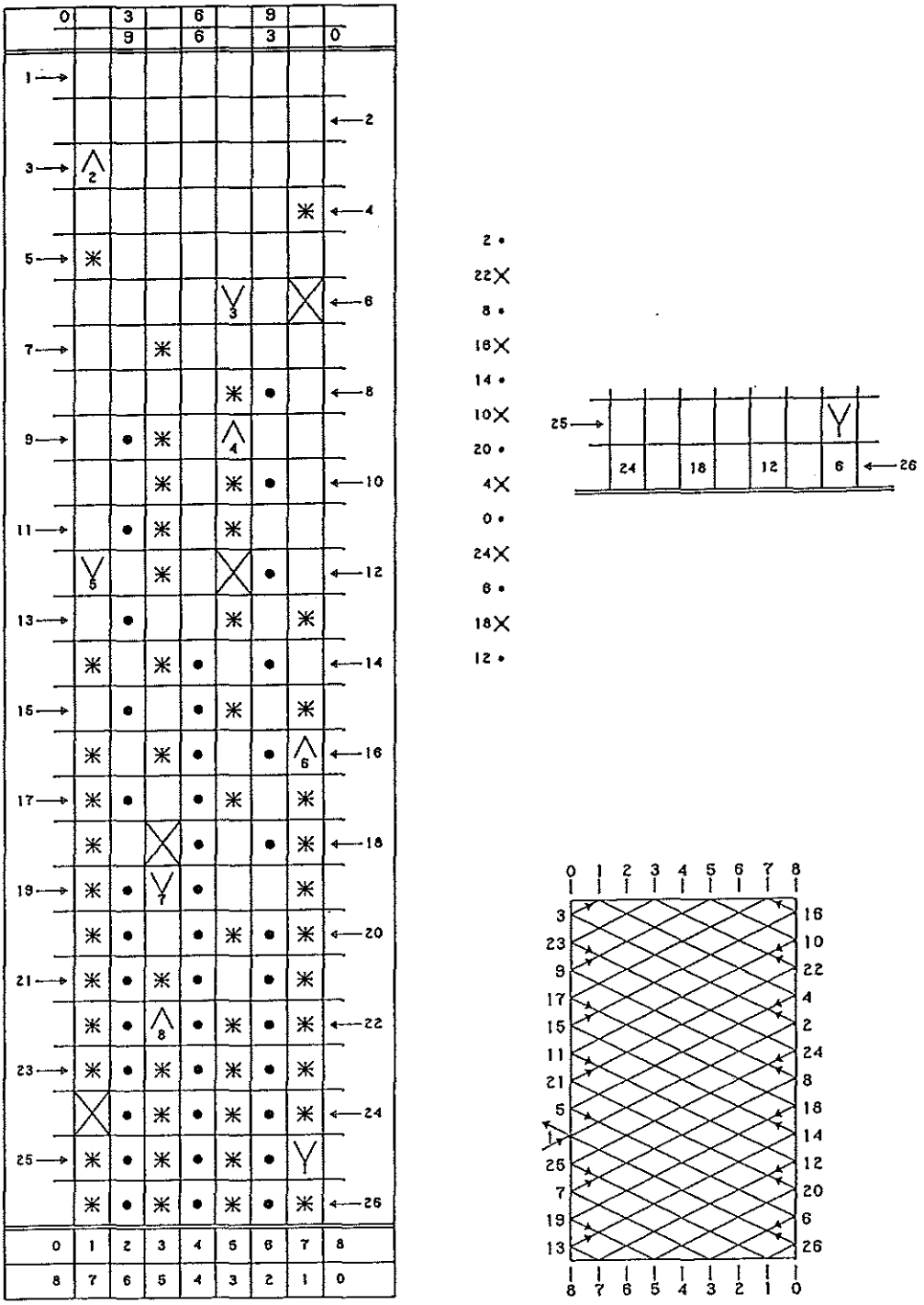


Fig. 283 — A string-run with its basic algorithm table for the Regular Rectangular Mat $b_h/b_v = 4/13$.

The Δ-value used in the calculation of the cells to be blocked out in the basic algorithm table is equal to 10 (see Fig. 272 upper diagram and associated path in the RKT). Hence $2\Delta = 20$ with modulus $2b_v = 26$.

For the string-run depicted in Fig. 284, the initial not to be laid down section of the 1st secondary half-cycle covers more than two (primary) half-cycles. For determining

the cells which have to be blocked out, we enter in the bottom row the sequence of values as usual. The first number in a following row is 1 more than the first number in its preceding row. Note that the sequential direction is opposite to the direction of the arrows.

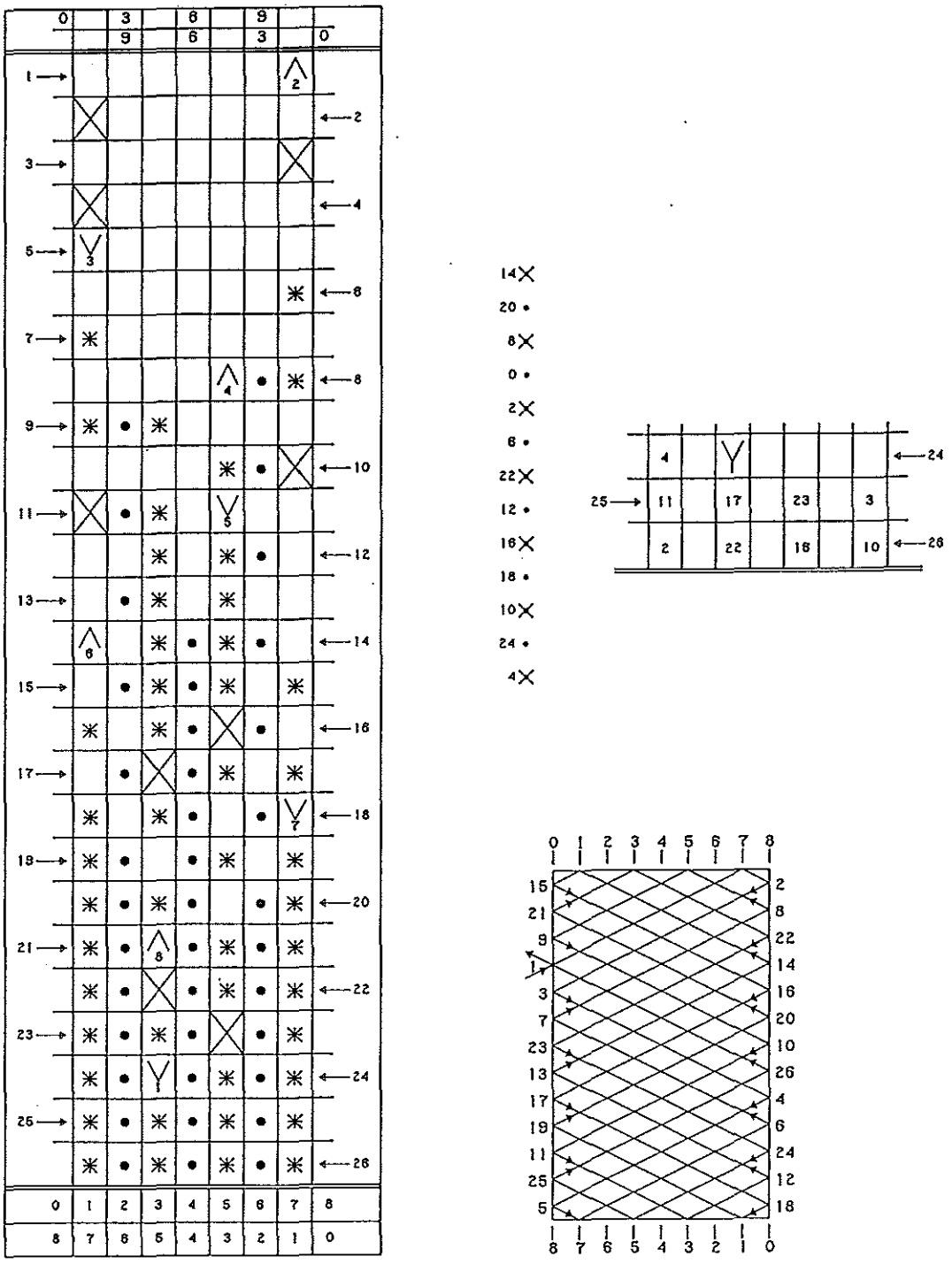


Fig. 284 — A string-run with its basic algorithm table for the Regular Rectangular Mat $b_h/b_v = 4/13$.

The algorithm tables are most conveniently computer generated by means of suitable software.

(Regular) Band Knots

In this Issue of *The Braider* we shall start with revisiting the Regular Band Knots discussed in Issue No. 11 and Issue No. 12 to see where and how they fit into the general method discussed in Issue No. 13.

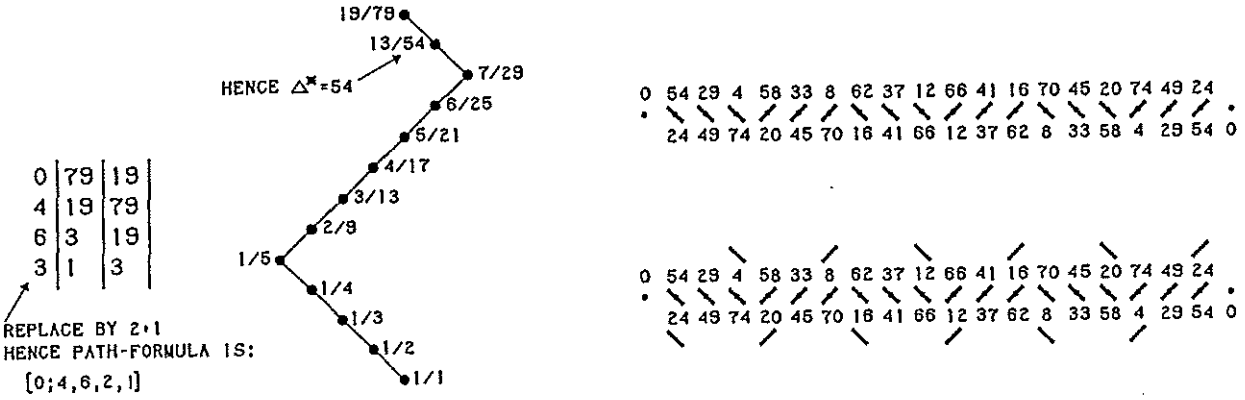


Fig. 285 — Regular Band Knot $p/b = 19/79$ with a 3-pass Gaucho-coding.

BIGHT-NUMBER i	→ ←	HALF-CYCLE		NUMBER OF HALF-CYCLES		HALF-CYCLE	→	BIGHT-NUMBER i
28	←	58				58	→	28
28	←	59				59	→	28
29	→	60	A	10	9	60	→	29
						67	→	32
33	→	68				68	→	33
33	←	69	B	8	9			
37	→	76				76	→	37
37	←	77	A	8	8	77	→	37
41	→	84				84	→	41
41	←	85	B	8	8	85	→	41
45	→	92				92	→	45
45	←	93	A	8	8	93	→	45
49	→	100				100	→	49
49	←	101			8	101	→	49
			B	10		108	→	53
						109	→	53
54	→	110			9	110	→	54
54	←	111						
			A	8		117	→	57
58	→	118				118	→	58
58	←	119	B	8	9			
62	→	126				126	→	62
62	←	127	A	8	8	127	→	62
66	→	134				134	→	66
66	←	135	B	8	8	135	→	66
70	→	142				142	→	70
70	←	143	A	8	8	143	→	70
74	→	150				150	→	74
74	←	151	B	8	8	151	→	74
78	←	158				158	→	78

Table for Fig. 285.

The Regular Band Knot in Fig. 285 is an example of the Gaucho-coded knots discussed in *The Braider*, Issue No. 11, pp. 238 - 240, hence of those $(\alpha+1)$-pass Gaucho-coded Regular Band Knots with $[2m(\alpha+1)+1]$-parts and $[\{2m(\alpha+1)+1\}N+(\alpha+1)]$-bights. For the example in Fig. 285 $\alpha = 2$, $m = 3$ and $N = 4$. This 3-pass Gaucho knot can be evolved from the 2-pass Gaucho knot $p/b = 13/54$ which in turn can be evolved from the 1-pass Gaucho knot (the Casa knot) $p/b = 7/29$ as the algorithm diagram in Fig. 285 shows. The path of the Casa knot $p/b = 7/29$ in the RKT shows that this Casa knot can be enlarged from the Casa knot $p/b = 3/13$ via the Casa knot $p/b = 5/21$. From the braiding sequences and their sub-sequences on the right-hand side of the table associated with the $P/B = 19/79$ 3-pass Gaucho-coded Band Knot the two evolutions, each of which requires six sub-sequences $(= 2m)$, can readily be seen. Each of the two sub-sequences in the first braiding sequence of each evolution consist of nine $(= 2N + 1)$ half-cycles. All further sub-sequences in an evolution consist of eight $(= 2N)$ half-cycles.

For the braiding process from the $p/b = 7/29$ Casa-coded Regular Knot to the $P/B = 19/79$ 3-pass Gaucho-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of the table on pg. 315, we obtain from the algorithm diagram (upper right-hand side of Fig. 285) the following half-cycle algorithms:

$(i = 28)$	59 ↗	:	$u - o - u - o - u - o.$
$(i = \mathbf{29})$	**60** ↖	:	$(s,1)2u - o - u - o - u - o.$
$(i = 29)$	61 ↗	:	$2u - o - u - o - u - o.$
$(i = 30)$	62 ↖	:	$2u - o - u - o - u - o.$
			↗ half-cycles as half-cycle #61.
			↖ half-cycles as half-cycle #62.
$(i = 32)$	67 ↗	:	$2u - o - u - o - u - o.$
$(i = \mathbf{33})$	**68** ↘	:	$2o - (s,1)2u - o - u - o - u.$
$(i = 33)$	69 ↙	:	$2o - 2u - o - u - o - u.$
$(i = 34)$	70 ↘	:	$2o - 2u - o - u - o - u.$
			↙ half-cycles as half-cycle #69.
			↘ half-cycles as half-cycle #70.
$(i = \mathbf{37})$	**76** ↘	:	$2o - 2u - (s,1)2o - u - o - u.$
$(i = 37)$	77 ↖	:	$2u - 2o - 2u - o - u - o.$
$(i = 38)$	78 ↗	:	$2u - 2o - 2u - o - u - o.$
			↖ half-cycles as half-cycle #77.
			↗ half-cycles as half-cycle #78.
$(i = \mathbf{41})$	**84** ↗	:	$2u - 2o - 2u - (s,1)2o - u - o.$
$(i = 41)$	85 ↙	:	$2o - 2u - 2o - 2u - o - u.$
$(i = 42)$	86 ↘	:	$2o - 2u - 2o - 2u - o - u.$
			↙ half-cycles as half-cycle #85.
			↘ half-cycles as half-cycle #86.
$(i = \mathbf{45})$	**92** ↘	:	$2o - 2u - 2o - 2u - (s,1)2o - u.$
$(i = 45)$	93 ↖	:	$2u - 2o - 2u - 2o - 2u - o.$
$(i = 46)$	94 ↗	:	$2u - 2o - 2u - 2o - 2u - o.$
			↖ half-cycles as half-cycle #93.

↗ half-cycles as half-cycle #94.

$(i = 49)$	**100** ↗	:	$2u - 2o - 2u - 2o - 2u - (s,1)2o.$

$(i = 49)$	101 ↙	:	$2o - 2u - 2o - 2u - 2o - 2u.$
$(i = 50)$	102 ↘	:	$2o - 2u - 2o - 2u - 2o - 2u.$
			↙ half-cycles as half-cycle #101.
			↘ half-cycles as half-cycle #102.
$(i = 53)$	108 ↘	:	$2o - 2u - 2o - 2u - 2o - 2u.$

$(i = 53)$	109 ↖	:	$2u - 2o - 2u - 2o - 2u - 2o.$
$(i = 54)$	**110** ↗	:	$(s,2)3u - 2o - 2u - 2o - 2u - 2o.$
$(i = 54)$	111 ↖	:	$3u - 2o - 2u - 2o - 2u - 2o.$
$(i = 55)$	112 ↗	:	$3u - 2o - 2u - 2o - 2u - 2o.$
			↖ half-cycles as half-cycle #111.
			↗ half-cycles as half-cycle #112.
$(i = 57)$	117 ↖	:	$3u - 2o - 2u - 2o - 2u - 2o.$

$(i = 58)$	**118** ↙	:	$3o - (s,2)3u - 2o - 2u - 2o - 2u.$
$(i = 58)$	119 ↘	:	$3o - 3u - 2o - 2u - 2o - 2u.$
$(i = 59)$	120 ↙	:	$3o - 3u - 2o - 2u - 2o - 2u.$
			↘ half-cycles as half-cycle #119.
			↙ half-cycles as half-cycle #120.
$(i = 62)$	**126** ↙	:	$3o - 3u - (s,2)3o - 2u - 2o - 2u.$

$(i = 62)$	127 ↗	:	$3u - 3o - 3u - 2o - 2u - 2o.$
$(i = 63)$	128 ↖	:	$3u - 3o - 3u - 2o - 2u - 2o.$
			↗ half-cycles as half-cycle #127.
			↖ half-cycles as half-cycle #128.
$(i = 66)$	**134** ↖	:	$3u - 3o - 3u - (s,2)3o - 2u - 2o.$

$(i = 66)$	135 ↘	:	$3o - 3u - 3o - 3u - 2o - 2u.$
$(i = 67)$	136 ↙	:	$3o - 3u - 3o - 3u - 2o - 2u.$
			↘ half-cycles as half-cycle #135.
			↙ half-cycles as half-cycle #136.
$(i = 70)$	**142** ↙	:	$3o - 3u - 3o - 3u - (s,2)3o - 2u.$

$(i = 70)$	143 ↗	:	$3u - 3o - 3u - 3o - 3u - 2o.$
$(i = 71)$	144 ↖	:	$3u - 3o - 3u - 3o - 3u - 2o.$
			↗ half-cycles as half-cycle #143.
			↖ half-cycles as half-cycle #144.
$(i = 74)$	**150** ↖	:	$3u - 3o - 3u - 3o - 3u - (s,2)3o.$

$(i = 74)$	151 ↘	:	$3o - 3u - 3o - 3u - 3o - 3u.$
$(i = 75)$	152 ↙	:	$3o - 3u - 3o - 3u - 3o - 3u.$
			↘ half-cycles as half-cycle #151.
			↙ half-cycles as half-cycle #152.
$(i = 78)$	158 ↙	:	$3o - 3u - 3o - 3u - 3o - 3u.$

The Regular Band Knot in Fig. 286 is an example of the Gaucho-coded knots discussed in *The Braider*, Issue No. 11, pp. 242 - 244, hence of those $(\alpha+1)$-pass Gaucho-coded Regular Band Knots with $[2m(\alpha+1)+1]$-parts and $[\{2m(\alpha+1)+1\}N-(\alpha+1)]$-bights. For the example in Fig. 286 $\alpha = 2$, $m = 3$ and $N = 4$. This 3-pass Gaucho knot can be evolved from the 2-pass Gaucho knot $p/b = 13/50$ which in turn can be evolved from the 1-pass Gaucho knot (the Casa knot) $p/b = 7/27$ as the algorithm diagram in Fig. 286 shows. The path of the Casa knot $p/b = 7/27$ in the RKT shows that it can be enlarged from the Casa knot $p/b = 3/11$ via the Casa knot $p/b = 5/19$.

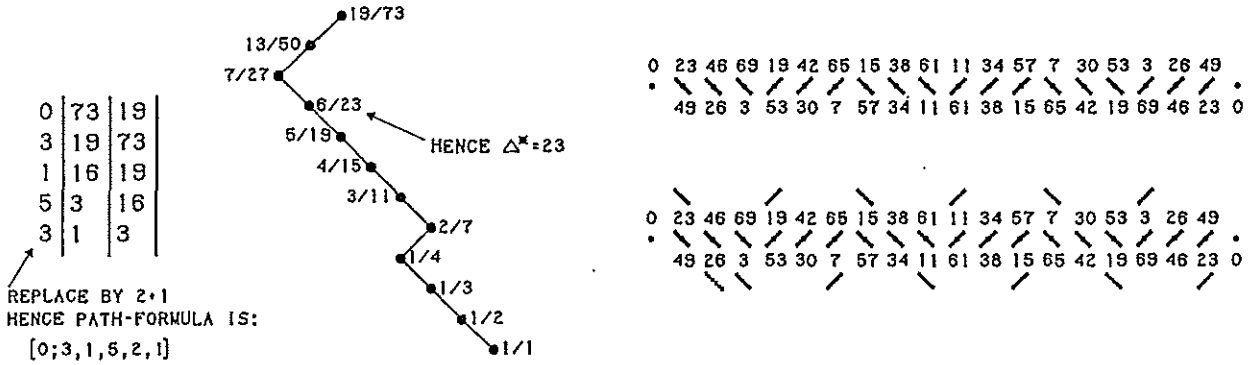


Fig. 286 — Regular Band Knot $p/b = 19/73$ with a 3-pass Gaucho-coding.

BIGHT-NUMBER 1	⇄ HALF-CYCLE		NUMBER OF HALF-CYCLES		HALF-CYCLE →	BIGHT-NUMBER 1
26	←	54			54 →	26
26	←	55	A	8	55 →	26
30	→	62		8	62 →	30
30	←	63	B	8	63 →	30
34	→	70		8	70 →	34
34	←	71	A	8	71 →	34
38	→	78		8	78 →	38
38	←	79	B	8	79 →	38
42	→	86		8	86 →	42
42	←	87	A	8	87 →	42
				7	93 →	45
46	→	94			94 →	46
46	←	95	B	6		
49	→	100		7	100 →	49
49	←	101	A	8	101 →	49
53	→	108		8	108 →	53
53	←	109	B	8	109 →	53
57	→	116		8	116 →	57
57	←	117	A	8	117 →	57
61	→	124		8	124 →	61
61	←	125	B	8	125 →	61
65	→	132		8	132 →	65
65	←	133	A	8	133 →	65
				7	139 →	68
69	→	140			140 →	69
69	←	141	B	6		
72	←	146		7	146 →	72

Table 1 for Fig. 286.

From the braiding sequences and their sub-sequences on the right-hand side of Table 1 associated with the $P/B = 19/73$ 3-pass Gaucho-coded Band Knot the two evolutions,

each of which requires six sub-sequences ($= 2m$), can readily be seen. Each of the two sub-sequences in the **last** braiding sequence of each evolution consist of seven ($= 2N-1$) half-cycles. All other preceding sub-sequences in an evolution consist of eight ($= 2N$) half-cycles.

For the braiding process from the $p/b = 7/27$ Casa-coded Regular Knot to the $P/B = 19/73$ 3-pass Gaucho-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of the table on pg. 318, we obtain from the algorithm diagram (upper right-hand side of Fig. 286) the following half-cycle algorithms:

($i = 26$)	**54** ↖	:	$u - o - u - o - u - (1,s)2o.$
($i = 26$)	55 ↗	:	$u - o - u - o - u - 2o.$
($i = 27$)	56 ↖	:	$u - o - u - o - u - 2o.$
			↗ half-cycles as half-cycle #55.
			↖ half-cycles as half-cycle #56.
($i = 30$)	**62** ↖	:	$u - o - u - o - (1,s)2u - 2o.$
($i = 30$)	63 ↘	:	$o - u - o - u - 2o - 2u.$
($i = 31$)	64 ↙	:	$o - u - o - u - 2o - 2u.$
			↘ half-cycles as half-cycle #63.
			↙ half-cycles as half-cycle #64.
($i = 34$)	**70** ↙	:	$o - u - o - (1,s)2u - 2o - 2u.$
($i = 34$)	71 ↗	:	$u - o - u - 2o - 2u - 2o.$
($i = 35$)	72 ↖	:	$u - o - u - 2o - 2u - 2o.$
			↗ half-cycles as half-cycle #71.
			↖ half-cycles as half-cycle #72.
($i = 38$)	**78** ↖	:	$u - o - (1,s)2u - 2o - 2u - 2o.$
($i = 38$)	79 ↘	:	$o - u - 2o - 2u - 2o - 2u.$
($i = 39$)	80 ↙	:	$o - u - 2o - 2u - 2o - 2u.$
			↘ half-cycles as half-cycle #79.
			↙ half-cycles as half-cycle #80.
($i = 42$)	**86** ↙	:	$o - (1,s)2u - 2o - 2u - 2o - 2u.$
($i = 42$)	87 ↗	:	$u - 2o - 2u - 2o - 2u - 2o.$
($i = 43$)	88 ↖	:	$u - 2o - 2u - 2o - 2u - 2o.$
			↗ half-cycles as half-cycle #87.
			↖ half-cycles as half-cycle #88.
($i = 45$)	**93** ↗	:	$u - 2o - 2u - 2o - 2u - 2o.$
($i = 46$)	94 ↘	:	$(1,s)2o - 2u - 2o - 2u - 2o - 2u.$
($i = 46$)	95 ↙	:	$2o - 2u - 2o - 2u - 2o - 2u.$
($i = 47$)	96 ↘	:	$2o - 2u - 2o - 2u - 2o - 2u.$
			↙ half-cycles as half-cycle #95.
			↘ half-cycles as half-cycle #96.
($i = 49$)	**100** ↘	:	$2o - 2u - 2o - 2u - 2o - 2u.$

($i = 49$)	**100 ↘**	:	$2o - 2u - 2o - 2u - 2o - (2,s)3u.$
($i = 49$)	101 ↖	:	$2u - 2o - 2u - 2o - 2u - 3o.$
($i = 50$)	102 ↗	:	$2u - 2o - 2u - 2o - 2u - 3o.$
			↖ half-cycles as half-cycle #101.
			↗ half-cycles as half-cycle #102.
($i = 53$)	**108 ↗**	:	$2u - 2o - 2u - 2o - (2,s)3u - 3o.$
($i = 53$)	109 ↙	:	$2o - 2u - 2o - 2u - 3o - 3u.$
($i = 54$)	110 ↘	:	$2o - 2u - 2o - 2u - 3o - 3u.$
			↙ half-cycles as half-cycle #109.
			↘ half-cycles as half-cycle #110.
($i = 57$)	**116 ↘**	:	$2o - 2u - 2o - (2,s)3u - 3o - 3u.$
($i = 57$)	117 ↖	:	$2u - 2o - 2u - 3o - 3u - 3o.$
($i = 58$)	118 ↗	:	$2u - 2o - 2u - 3o - 3u - 3o.$
			↖ half-cycles as half-cycle #117.
			↗ half-cycles as half-cycle #118.
($i = 61$)	**124 ↗**	:	$2u - 2o - (2,s)3u - 3o - 3u - 3o.$
($i = 61$)	125 ↙	:	$2o - 2u - 3o - 3u - 3o - 3u.$
($i = 62$)	126 ↘	:	$2o - 2u - 3o - 3u - 3o - 3u.$
			↙ half-cycles as half-cycle #125.
			↘ half-cycles as half-cycle #126.
($i = 65$)	**132 ↘**	:	$2o - (2,s)3u - 3o - 3u - 3o - 3u.$
($i = 65$)	133 ↖	:	$2u - 3o - 3u - 3o - 3u - 3o.$
($i = 66$)	134 ↗	:	$2u - 3o - 3u - 3o - 3u - 3o.$
			↖ half-cycles as half-cycle #133.
			↗ half-cycles as half-cycle #134.
($i = 68$)	139 ↖	:	$2u - 3o - 3u - 3o - 3u - 3o.$
($i = 69$)	**140 ↙**	:	$(2,s)3o - 3u - 3o - 3u - 3o - 3u.$
($i = 69$)	141 ↘	:	$3o - 3u - 3o - 3u - 3o - 3u.$
($i = 70$)	142 ↙	:	$3o - 3u - 3o - 3u - 3o - 3u.$
			↘ half-cycles as half-cycle #141.
			↙ half-cycles as half-cycle #142.
($i = 72$)	146 ↙	:	$3o - 3u - 3o - 3u - 3o - 3u.$

In this braiding process the braiding sequence in which the two sub-sequences each have $(2N - 1)$ half-cycles is braided last, whereas in the '*braiding to pattern*' method, discussed in *The Braider*, Issue No. 11, pp. 242 - 244, the braiding sequence in which the two sub-sequences each have $(2N - 1)$ half-cycles was braided first. Hence the braiding processes are quite different. In *The Braider*, Issue no. 12, pg. 271, we mentioned that '*braiding to pattern*' methods are always very restricted, hence the procedure discussed here should be the preferred one since it not only results more logically from the general procedure, but is much better attuned to the Standing End referencing process.

The appropriate braiding scheme for the '*braiding to pattern*' method, discussed in *The Braider*, Issue No. 11, pp. 242 - 244, can readily be obtained with the general procedure by a suitable arrangement of the braiding sequences. This braiding scheme is shown on the right-hand side of Table 2 below.

BIGHT-NUMBER		HALF-CYCLE		NUMBER OF HALF-CYCLES	HALF-CYCLE		BIGHT-NUMBER
26	←	54			54	→	26
26	←	55		7	55	→	26
			A 8		61	→	29
30	→	62		7	62	→	30
30	←	63					
			B 8		68	→	33
34	→	70		8	69	→	33
34	←	71			70	→	34
			A 8		76	→	37
38	→	78		8	77	→	37
38	←	79			78	→	38
			B 8		84	→	41
42	→	86		8	85	→	41
42	←	87			86	→	42
			A 8		92	→	45
46	→	94		8	93	→	45
46	←	95	B 6		94	→	46
49	→	100			100	→	49
49	←	101		7	101	→	49
			A 8		107	→	52
53	→	108		7	108	→	53
53	←	109					
			B 8		114	→	56
57	→	116		8	115	→	56
57	←	117			116	→	57
			A 8		122	→	60
61	→	124		8	123	→	60
61	←	125			124	→	61
			B 8		130	→	64
65	→	132		8	131	→	64
65	←	133			132	→	65
			A 8		138	→	68
69	→	140		8	139	→	68
69	←	141			140	→	69
72	←	146	B 6		146	→	72

Table 2 for Fig. 286.

For the braiding process from the $p/b = 7/27$ Casa-coded Regular Knot to the $P/B = 19/73$ 3-pass Gaucho-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of the above Table 2, we obtain from the algorithm diagram (upper right-hand side of Fig. 286) the following half-cycle algorithms:

$(i = \mathbf{26})$ **54** ↖ : $u - o - u - o - u - (1, s)2o.$

$(i = 26)$ 55 ↗ : $u - o - u - o - u - 2o.$
$(i = 27)$ 56 ↖ : $u - o - u - o - u - 2o.$
↗ half-cycles as half-cycle #55.
↖ half-cycles as half-cycle #56.
$(i = 29)$ 61 ↗ : $u - o - u - o - u - 2o.$

$(i = \mathbf{30})$ **62** ↘ : $o - u - o - u - (1, s)2o - 2u.$
$(i = 30)$ 63 ↙ : $o - u - o - u - 2o - 2u.$
$(i = 31)$ 64 ↘ : $o - u - o - u - 2o - 2u.$
↙ half-cycles as half-cycle #63.
↘ half-cycles as half-cycle #64.
$(i = 33)$ 68 ↘ : $o - u - o - u - 2o - 2u.$

$(i = 33)$ 69 ↖ : $u - o - u - o - 2u - 2o.$
$(i = \mathbf{34})$ **70** ↗ : $u - o - u - (1, s)2o - 2u - 2o.$
$(i = 34)$ 71 ↖ : $u - o - u - 2o - 2u - 2o.$
$(i = 35)$ 72 ↗ : $u - o - u - 2o - 2u - 2o.$
↖ half-cycles as half-cycle #71.
↗ half-cycles as half-cycle #72.
$(i = 37)$ 76 ↗ : $u - o - u - 2o - 2u - 2o.$

$(i = 37)$ 77 ↙ : $o - u - o - 2u - 2o - 2u.$
$(i = \mathbf{38})$ **78** ↘ : $o - u - (1, s)2o - 2u - 2o - 2u.$
$(i = 38)$ 79 ↙ : $o - u - o - 2u - 2o - 2u.$
$(i = 39)$ 80 ↘ : $o - u - o - 2u - 2o - 2u.$
↙ half-cycles as half-cycle #79.
↘ half-cycles as half-cycle #80.
$(i = 41)$ 84 ↘ : $o - u - 2o - 2u - 2o - 2u.$

$(i = 41)$ 85 ↖ : $u - o - 2u - 2o - 2u - 2o.$
$(i = \mathbf{42})$ **86** ↗ : $u - (1, s)2o - 2u - 2o - 2u - 2o.$
$(i = 42)$ 87 ↖ : $u - 2o - 2u - 2o - 2u - 2o.$
$(i = 43)$ 88 ↗ : $u - 2o - 2u - 2o - 2u - 2o.$
↖ half-cycles as half-cycle #87.
↗ half-cycles as half-cycle #88.
$(i = 45)$ 92 ↗ : $u - 2o - 2u - 2o - 2u - 2o.$

$(i = 45)$ 93 ↙ : $o - 2u - 2o - 2u - 2o - 2u.$
$(i = \mathbf{46})$ **94** ↘ : $(1, s)2o - 2u - 2o - 2u - 2o - 2u.$
$(i = 46)$ 95 ↙ : $2o - 2u - 2o - 2u - 2o - 2u.$
$(i = 47)$ 96 ↘ : $2o - 2u - 2o - 2u - 2o - 2u.$
↙ half-cycles as half-cycle #95.
↘ half-cycles as half-cycle #96.
$(i = 49)$ 100 ↘ : $2o - 2u - 2o - 2u - 2o - 2u.$

$(i = 49)$ **100** $\searrow$: $2o - 2u - 2o - 2u - 2o - (2, s)3u.$

$(i = 49)$ 101 $\nwarrow$: $2u - 2o - 2u - 2o - 2u - 3o.$
$(i = 50)$ 102 $\nearrow$: $2u - 2o - 2u - 2o - 2u - 3o.$
$\nwarrow$ half-cycles as half-cycle #101.
$\nearrow$ half-cycles as half-cycle #102.
$(i = 52)$ 107 $\nwarrow$: $2u - 2o - 2u - 2o - 2u - 3o.$

$(i = 53)$ **108** $\swarrow$: $2o - 2u - 2o - 2u - (2, s)3o - 3u.$
$(i = 53)$ 109 $\searrow$: $2o - 2u - 2o - 2u - 3o - 3u.$
$(i = 54)$ 110 $\swarrow$: $2o - 2u - 2o - 2u - 3o - 3u.$
$\searrow$ half-cycles as half-cycle #109.
$\swarrow$ half-cycles as half-cycle #110.
$(i = 56)$ 114 $\swarrow$: $2o - 2u - 2o - 2u - 3o - 3u.$

$(i = 56)$ 115 $\nearrow$: $2u - 2o - 2u - 2o - 3u - 3o.$
$(i = 57)$ **116** $\nwarrow$: $2u - 2o - 2u - (2, s)3o - 3u - 3o.$
$(i = 57)$ 117 $\nearrow$: $2u - 2o - 2u - 3o - 3u - 3o.$
$(i = 58)$ 118 $\nwarrow$: $2u - 2o - 2u - 3o - 3u - 3o.$
$\nearrow$ half-cycles as half-cycle #117.
$\nwarrow$ half-cycles as half-cycle #118.
$(i = 60)$ 122 $\nwarrow$: $2u - 2o - 2u - 3o - 3u - 3o.$

$(i = 60)$ 123 $\searrow$: $2o - 2u - 2o - 3u - 3o - 3u.$
$(i = 61)$ **124** $\swarrow$: $2o - 2u - (2, s)3o - 3u - 3o - 3u.$
$(i = 61)$ 125 $\searrow$: $2o - 2u - 3o - 3u - 3o - 3u.$
$(i = 62)$ 126 $\swarrow$: $2o - 2u - 3o - 3u - 3o - 3u.$
$\searrow$ half-cycles as half-cycle #125.
$\swarrow$ half-cycles as half-cycle #126.
$(i = 64)$ 130 $\searrow$: $2o - 2u - 3o - 3u - 3o - 3u.$

$(i = 64)$ 131 $\nearrow$: $2u - 2o - 3u - 3o - 3u - 3o.$
$(i = 65)$ **132** $\nwarrow$: $2u - (2, s)3o - 3u - 3o - 3u - 3o.$
$(i = 65)$ 133 $\nearrow$: $2u - 3o - 3u - 3o - 3u - 3o.$
$(i = 66)$ 134 $\nwarrow$: $2u - 3o - 3u - 3o - 3u - 3o.$
$\nearrow$ half-cycles as half-cycle #133.
$\nwarrow$ half-cycles as half-cycle #134.
$(i = 68)$ 138 $\nwarrow$: $2u - 3o - 3u - 3o - 3u - 3o.$

$(i = 68)$ 139 $\searrow$: $2o - 3u - 3o - 3u - 3o - 3u.$
$(i = 69)$ **140** $\swarrow$: $(2, s)3o - 3u - 3o - 3u - 3o - 3u.$
$(i = 69)$ 141 $\searrow$: $3o - 3u - 3o - 3u - 3o - 3u.$
$(i = 70)$ 142 $\swarrow$: $3o - 3u - 3o - 3u - 3o - 3u.$
$\searrow$ half-cycles as half-cycle #141.
$\swarrow$ half-cycles as half-cycle #142.
$(i = 72)$ 146 $\swarrow$: $3o - 3u - 3o - 3u - 3o - 3u.$

A Pineapple Knot

The following Pineapple Knot is an interweave of two Perfect Pineapple Knots.

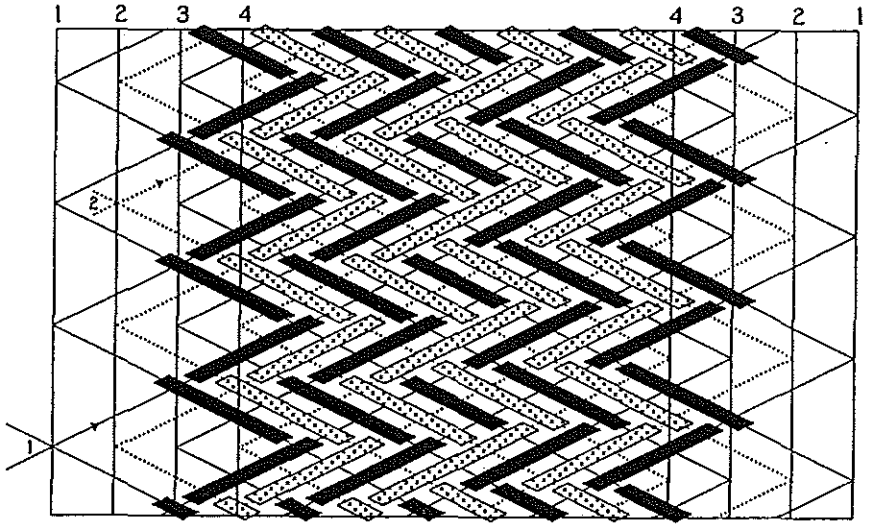


Fig. 287 — $A = 4$, $B^* = 4$, $P = 20$, $B = 16$, $P_{c_1} = 11$, $P_{c_2} = 9$.

1st component; 1st colour; between bight-boundaries 1 & 3 (left) and 1 & 3 (right).

1. $L \longrightarrow R$: Free Run.
2. $R \longrightarrow L$: u.
3. $L \longrightarrow R$: o.
4. $R \longrightarrow L$: $o - u$.
5. $L \longrightarrow R$: $u - o$.
6. $R \longrightarrow L$: $2o - u$.
7. $L \longrightarrow R$: $2u - o$.
8. $R \longrightarrow L$: $2o - 2u - o$.
9. $L \longrightarrow R$: $2u - 2o - u$.
10. $R \longrightarrow L$: $2o - 3u$.
11. $L \longrightarrow R$: $u - o - u - 2o$.
12. $R \longrightarrow L$: $2o - u - o - 2u - 2o$.
13. $L \longrightarrow R$: $u - o - u - 2o - 2u$.
14. $R \longrightarrow L$: $2o - 2u - 2o - u$.
15. $L \longrightarrow R$: $2u - o - 2u - 2o - u$.
16. $R \longrightarrow L$: $2o - 2u - 2o - 2u - 2o$.

2nd component; 2nd colour; between bight-boundaries 2 & 4 (left) and 2 & 4 (right).

1. $L \longrightarrow R$: $u - 2o - 3u - 2o - u$.
2. $R \longrightarrow L$: $u - 2o - u - 2o - 3u$.
3. $L \longrightarrow R$: $2u - 3o - 2u$.
4. $R \longrightarrow L$: $2u - 2o - u - 3o - u$.
5. $L \longrightarrow R$: $u - 2o - 3u - 3o - 2u$.
6. $R \longrightarrow L$: $u - 2o - u - 4o - 3u$.
7. $L \longrightarrow R$: $2u - 3o - 4u$.
8. $R \longrightarrow L$: $2u - 2o - 2u - 4o - u$.
9. $L \longrightarrow R$: $u - 2o - 4u - 4o - 2u$.
10. $R \longrightarrow L$: $u - 2o - 3u - 4o - 3u$.
11. $L \longrightarrow R$: $2u - 5o - 4u$.
12. $R \longrightarrow L$: $2u - 3o - 3u - 4o - u$.
13. $L \longrightarrow R$: $u - 3o - 5u - 4o - 2u$.
14. $R \longrightarrow L$: $u - 4o - 3u - 4o - 3u$.
15. $L \longrightarrow R$: $4u - 5o - 4u$.
16. $R \longrightarrow L$: $3u - 4o - 3u - 4o - u$.

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(Regular) Band Knots

We shall continue with our revisit to the Regular Band Knots discussed in Issue No. 12 to see where and how they fit into the general method discussed in Issue No. 13.

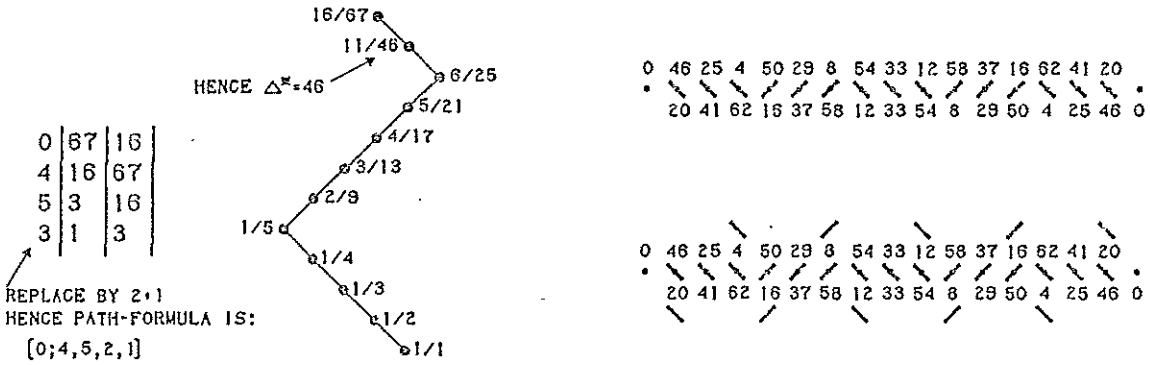


Fig. 288 — Regular Band Knot $p/b = 16/67$ with a 3-pass Headhunter's-coding.

BIGHT-NUMBER 1	⟷	HALF-CYCLE		NUMBER OF HALF-CYCLES		HALF-CYCLE	⟶	BIGHT-NUMBER 1
24	←	50				50	→	24
24	←	51	A	10	9	51	→	24
25	→	52				52	→	25
						59	→	28
29	→	60			9	60	→	29
29	←	61	B	8				
33	→	68				68	→	33
33	←	69	A	8	8	69	→	33
37	→	76				76	→	37
37	←	77	B	8	8	77	→	37
41	→	84				84	→	41
41	←	85			4	85	→	41
			A	10		88	→	43
					4	89	→	43
						92	→	45
					9	93	→	45
46	→	94				94	→	46
46	←	95	B	8				
50	→	102				101	→	49
50	←	103	A	8	9	102	→	50
54	→	110				110	→	54
54	←	111	B	8	8	111	→	54
58	→	118				118	→	58
58	←	119	A	8	8	119	→	58
62	→	126				126	→	62
62	←	127	B	8	4	127	→	62
						130	→	64
					4	131	→	64
66	←	134				134	→	66

Table for Fig. 288.

The Regular Band Knot in Fig. 288 is an example of the Headhunter's-coded knots discussed in *The Braider*, Issue No. 12, pp. 259-261, hence of those $(\alpha + 1)$-pass Headhunter's-coded Regular Band Knots with $[(2m + 1)(\alpha + 1) + 1]$-parts and

$[\{(2m+1)(\alpha+1)+1\}N+(\alpha+1)]$-bights. For the example in Fig. 288 $\alpha = 2$, $m = 2$ and $N = 4$. This 3-pass Headhunter's knot can be evolved from the 2-pass Headhunter's knot $p/b = 11/46$ which in turn can be evolved from the 1-pass Headhunter's knot (the Casa knot) $p/b = 6/25$ as the algorithm diagram in Fig. 288 shows. The path of the Casa knot $p/b = 6/25$ in the RKT shows that this Casa knot can be enlarged from the Column-coded Regular Knot $p/b = 2/9$ via the Casa knot $p/b = 4/17$. From the braiding sequences and their sub-sequences on the right-hand side of the table associated with the $P/B = 16/67$ 3-pass Headhunter's-coded Band Knot the two evolutions, each of which requires six sub-sequences $(= 2m + 2)$, can readily be seen. Each of the two sub-sequences in the first braiding sequence of each evolution consists of nine $(= 2N+1)$ half-cycles. Then follow $(m-1)$ braiding sequences in which each sub-sequence consists of $2N$ half-cycles. Each of the two sub-sequences in the final braiding sequence of each evolution consists of N half-cycles.

For the braiding process from the $p/b = 6/25$ Casa-coded Regular Knot to the $P/B = 16/67$ 3-pass Headhunter's-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of the table on pg. 325, we obtain from the algorithm diagram (upper right-hand side of Fig. 288) the following half-cycle algorithms:

$(i = 24)$	51 ↗	:	$u - o - u - o - u.$
$(i = 25)$	52 ↖	:	$(s,1)2o - u - o - u - o.$
$(i = 25)$	53 ↗	:	$2u - o - u - o - u.$
$(i = 26)$	54 ↖	:	$2o - u - o - u - o.$
			↗ half-cycles as half-cycle #53.
			↖ half-cycles as half-cycle #54.
$(i = 28)$	59 ↗	:	$2u - o - u - o - u.$
$(i = 29)$	60 ↘	:	$2o - (s,1)2u - o - u - o.$
$(i = 29)$	61 ↙	:	$2u - 2o - u - o - u.$
$(i = 30)$	62 ↘	:	$2o - 2u - o - u - o.$
			↙ half-cycles as half-cycle #61.
			↘ half-cycles as half-cycle #62.
$(i = 33)$	68 ↘	:	$2o - 2u - (s,1)2o - u - o.$
$(i = 33)$	69 ↖	:	$2o - 2u - 2o - u - o.$
$(i = 34)$	70 ↗	:	$2u - 2o - 2u - o - u.$
			↖ half-cycles as half-cycle #69.
			↗ half-cycles as half-cycle #70.
$(i = 37)$	76 ↗	:	$2u - 2o - 2u - (s,1)2o - u.$
$(i = 37)$	77 ↙	:	$2u - 2o - 2u - 2o - u.$
$(i = 38)$	78 ↘	:	$2o - 2u - 2o - 2u - o.$
			↙ half-cycles as half-cycle #77.
			↘ half-cycles as half-cycle #78.
$(i = 41)$	84 ↘	:	$2o - 2u - 2o - 2u - (s,1)2o.$
$(i = 41)$	85 ↖	:	$2o - 2u - 2o - 2u - 2o.$
$(i = 42)$	86 ↗	:	$2u - 2o - 2u - 2o - 2u.$
			↖ half-cycles as half-cycle #85.

			↗ half-cycles as half-cycle #86.
$(i = 43)$	88 ↗	:	$2u - 2o - 2u - 2o - 2u.$
$(i = 43)$	89 ↙	:	$2u - 2o - 2u - 2o - 2u.$
$(i = 44)$	90 ↘	:	$2o - 2u - 2o - 2u - 2o.$
			↙ half-cycles as half-cycle #89.
			↘ half-cycles as half-cycle #90.
$(i = 45)$	92 ↘	:	$2o - 2u - 2o - 2u - 2o.$
$(i = 45)$	93 ↖	:	$2o - 2u - 2o - 2u - 2o.$
$(i = 46)$	94 ↗	:	$(s, 2)3u - 2o - 2u - 2o - 2u.$
$(i = 46)$	95 ↖	:	$3o - 2u - 2o - 2u - 2o.$
$(i = 47)$	96 ↗	:	$3u - 2o - 2u - 2o - 2u.$
			↖ half-cycles as half-cycle #95.
			↗ half-cycles as half-cycle #96.
$(i = 49)$	101 ↖	:	$3o - 2u - 2o - 2u - 2o.$
$(i = 50)$	102 ↙	:	$3u - (s, 2)3o - 2u - 2o - 2u.$
$(i = 50)$	103 ↘	:	$3o - 3u - 2o - 2u - 2o.$
$(i = 51)$	104 ↙	:	$3u - 3o - 2u - 2o - 2u.$
			↘ half-cycles as half-cycle #103.
			↙ half-cycles as half-cycle #104.
$(i = 54)$	110 ↙	:	$3u - 3o - (s, 2)3u - 2o - 2u.$
$(i = 54)$	111 ↗	:	$3u - 3o - 3u - 2o - 2u.$
$(i = 55)$	112 ↖	:	$3o - 3u - 3o - 2u - 2o.$
			↗ half-cycles as half-cycle #111.
			↖ half-cycles as half-cycle #112.
$(i = 58)$	118 ↖	:	$3o - 3u - 3o - (s, 2)3u - 2o.$
$(i = 58)$	119 ↘	:	$3o - 3u - 3o - 3u - 2o.$
$(i = 67)$	120 ↙	:	$3u - 3o - 3u - 3o - 2u.$
			↘ half-cycles as half-cycle #119.
			↙ half-cycles as half-cycle #120.
$(i = 62)$	126 ↙	:	$3u - 3o - 3u - 3o - (s, 2)3u.$
$(i = 62)$	127 ↗	:	$3u - 3o - 3u - 3o - 3u.$
$(i = 63)$	128 ↖	:	$3o - 3u - 3o - 3u - 3o.$
			↗ half-cycles as half-cycle #127.
			↖ half-cycles as half-cycle #128.
$(i = 64)$	130 ↖	:	$3o - 3u - 3o - 3u - 3o.$
$(i = 64)$	131 ↘	:	$3o - 3u - 3o - 3u - 3o.$
$(i = 65)$	132 ↙	:	$3u - 3o - 3u - 3o - 3u.$
			↘ half-cycles as half-cycle #131.
			↙ half-cycles as half-cycle #132.
$(i = 66)$	134 ↙	:	$3u - 3o - 3u - 3o - 3u.$

The Regular Band Knot in Fig. 289 is an example of the Headhunter's-coded knots discussed in *The Braider*, Issue No. 12, pp. 262-264, hence of those $(\alpha + 1)$-pass Headhunter's-coded Regular Band Knots with $[(2m + 1)(\alpha + 1) + 1]$-parts and $[\{2m+1)(\alpha+1)+1\}N-(\alpha+1)]$-bights. For the example in Fig. 289 $\alpha = 2$, $m = 2$ and $N = 4$. This 3-pass Headhunter's knot can be evolved from the 2-pass Headhunter's knot $p/b = 11/42$ which in turn can be evolved from the 1-pass Headhunter's knot (the Casa knot) $p/b = 6/23$ as the algorithm diagram in Fig. 289 shows. The path of the Casa knot $p/b = 6/23$ in the RKT shows that it can be enlarged from the Column-coded Regular Knot $p/b = 2/7$ via the Casa knot $p/b = 4/15$.

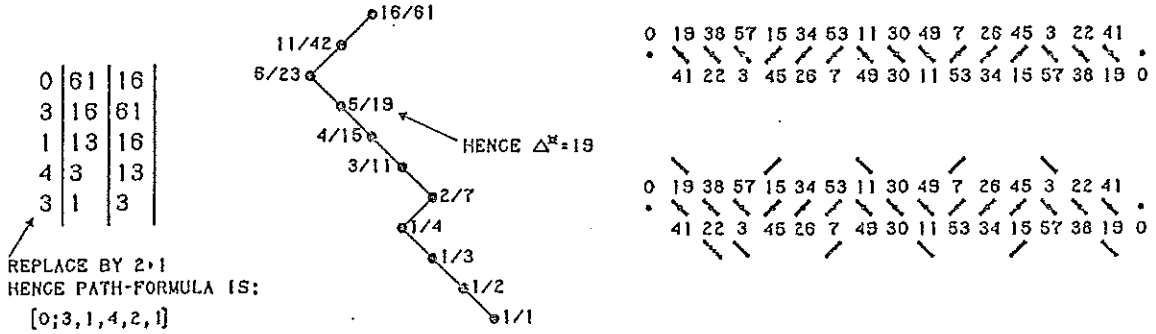


Fig. 289 — Regular Band Knot $p/b = 16/61$ with a 3-pass Headhunter's-coding.

BIGHT-NUMBER i	⟷ HALF-CYCLE		NUMBER OF HALF-CYCLES		HALF-CYCLE → BIGHT-NUMBER i
22	← 46				46 → 22
22	← 47	A	8	8	47 → 22
26	→ 54				54 → 26
26	← 55	B	8	8	55 → 26
30	→ 62				62 → 30
30	← 63	A	8	8	63 → 30
34	→ 70				70 → 34
34	← 71	B	8	8	71 → 34
38	→ 78				78 → 38
38	← 79			3	79 → 38
			6		81 → 39
				3	82 → 40
41	→ 84				84 → 41
41	← 85	A	8	8	85 → 41
45	→ 92				92 → 45
45	← 93	B	8	8	93 → 45
49	→ 100				100 → 49
49	← 101	A	8	8	101 → 49
53	→ 108				108 → 53
53	← 109	B	8	8	109 → 53
57	→ 116				116 → 57
57	← 117			3	117 → 57
			6		119 → 58
				3	120 → 59
60	← 122				122 → 60

Table 1 for Fig. 289.

From the braiding sequences and their sub-sequences on the right-hand side of Table 1 associated with the $P/B = 16/61$ 3-pass Headhunter's-coded Band Knot the two evolutions, each of which requires six sub-sequences $(= 2m + 2)$, can readily be seen.

Each of the two sub-sequences in the **last** braiding sequence of each evolution consists of three ($= N-1$) half-cycles. All other preceding sub-sequences in an evolution consist of eight ($= 2N$) half-cycles.

For the braiding process from the $p/b = 6/23$ Casa-coded Regular Knot to the $P/B = 16/61$ 3-pass Headhunter's-coded Regular Band Knot, associated with the braiding scheme on the right-hand side of the table on pg. 328, we obtain from the algorithm diagram (upper right-hand side of Fig. 289) the following half-cycle algorithms:

($i = 22$)	**46 ↖**	:	$o - u - o - u - (1,s)2o.$
($i = 22$)	47 ↗	:	$u - o - u - o - 2u.$
($i = 23$)	48 ↖	:	$o - u - o - u - 2o.$
			↗ half-cycles as half-cycle #47.
			↖ half-cycles as half-cycle #48.
($i = 26$)	**54 ↖**	:	$o - u - o - (1,s)2u - 2o.$
($i = 26$)	55 ↘	:	$o - u - o - 2u - 2o.$
($i = 27$)	56 ↙	:	$u - o - u - 2o - 2u.$
			↘ half-cycles as half-cycle #55.
			↙ half-cycles as half-cycle #56.
($i = 30$)	**62 ↙**	:	$u - o - (1,s)2u - 2o - 2u.$
($i = 30$)	63 ↗	:	$u - o - 2u - 2o - 2u.$
($i = 31$)	64 ↖	:	$o - u - 2o - 2u - 2o.$
			↗ half-cycles as half-cycle #63.
			↖ half-cycles as half-cycle #64.
($i = 34$)	**70 ↖**	:	$o - (1,s)2u - 2o - 2u - 2o.$
($i = 34$)	71 ↘	:	$o - 2u - 2o - 2u - 2o.$
($i = 35$)	72 ↙	:	$u - 2o - 2u - 2o - 2u.$
			↘ half-cycles as half-cycle #71.
			↙ half-cycles as half-cycle #72.
($i = 38$)	**78 ↙**	:	$(1,s)2u - 2o - 2u - 2o - 2u.$
($i = 38$)	79 ↗	:	$2u - 2o - 2u - 2o - 2u.$
($i = 39$)	80 ↖	:	$2o - 2u - 2o - 2u - 2o.$
($i = 39$)	81 ↗	:	$2u - 2o - 2u - 2o - 2u.$
($i = 40$)	82 ↘	:	$2o - 2u - 2o - 2u - 2o.$
($i = 40$)	83 ↙	:	$2u - 2o - 2u - 2o - 2u.$
($i = 41$)	84 ↘	:	$2o - 2u - 2o - 2u - 2o.$
($i = 41$)	**84 ↘**	:	$2o - 2u - 2o - 2u - (2,s)3o.$
($i = 41$)	85 ↖	:	$2o - 2u - 2o - 2u - 3o.$
($i = 42$)	86 ↗	:	$2u - 2o - 2u - 2o - 3u.$
			↖ half-cycles as half-cycle #85.

			↗ half-cycles as half-cycle #86.
$(i = 45)$	**92** ↗	:	$2u - 2o - 2u - (2,s)3o - 3u.$
$(i = 45)$	93 ↙	:	$2u - 2o - 2u - 3o - 3u.$
$(i = 46)$	94 ↘	:	$2o - 2u - 2o - 3u - 3o.$
			↙ half-cycles as half-cycle #93.
			↘ half-cycles as half-cycle #94.
$(i = 49)$	**100** ↘	:	$2o - 2u - (2,s)3o - 3u - 3o.$
$(i = 49)$	101 ↖	:	$2o - 2u - 3o - 3u - 3o.$
$(i = 50)$	102 ↗	:	$2u - 2o - 3u - 3o - 3u.$
			↖ half-cycles as half-cycle #101.
			↗ half-cycles as half-cycle #102.
$(i = 53)$	**108** ↗	:	$2u - (2,s)3o - 3u - 3o - 3u.$
$(i = 53)$	109 ↙	:	$2u - 3o - 3u - 3o - 3u.$
$(i = 54)$	110 ↘	:	$2o - 3u - 3o - 3u - 3o.$
			↙ half-cycles as half-cycle #109.
			↘ half-cycles as half-cycle #110.
$(i = 57)$	**116** ↘	:	$(2,s)3o - 3u - 3o - 3u - 3o.$
$(i = 57)$	117 ↖	:	$3o - 3u - 3o - 3u - 3o.$
$(i = 58)$	118 ↗	:	$3u - 3o - 3u - 3o - 3u.$
$(i = 58)$	119 ↖	:	$3o - 3u - 3o - 3u - 3o.$
$(i = 59)$	120 ↙	:	$3u - 3o - 3u - 3o - 3u.$
$(i = 59)$	121 ↘	:	$3o - 3u - 3o - 3u - 3o.$
$(i = 60)$	122 ↙	:	$3u - 3o - 3u - 3o - 3u.$

The reader will observe that this braiding processes is quite different to the one associated with the '*braiding to pattern*' method discussed in *The Braider*, Issue No. 12, pp. 262 - 264 .

The procedure discussed here should be the preferred one since it not only results more logically from the general procedure, but is much better attuned to the Standing End referencing process.

The appropriate braiding scheme for the '*braiding to pattern*' method, discussed in *The Braider*, Issue No. 12, pp. 262 - 264 , can readily be obtained with the general procedure by a suitable arrangement of the braiding sequences. Such an arrangement of braiding sequences is shown by the braiding scheme on the right-hand side of Table 2 on pg. 331.

The half-cycle algorithms associated with this braiding scheme and the algorithm diagram at the upper right-hand side of Fig. 289, for the braiding process from the $p/b = 6/23$ Casa-coded Regular Knot to the $P/B = 16/61$ 3-pass Headhunter's-coded Regular Band Knot, are shown below the table on pp. 331 - 333 .

EIGHT-NUMBER ⇄ HALF-CYCLE				NUMBER OF HALF-CYCLES	HALF-CYCLE → EIGHT-NUMBER		
1							1
22	←	46			46	→	22
22	←	47	A 8	7	47	→	22
					53	→	25
26	→	54		7	54	→	26
26	←	55	B 8		60	→	29
				8	61	→	29
30	→	62			62	→	30
30	←	63	A 8		68	→	33
				8	69	→	33
34	→	70			70	→	34
34	←	71	B 8		76	→	37
				4	77	→	37
38	→	78			78	→	38
38	←	79	A 6		80	→	39
				4	81	→	39
41	→	84			84	→	41
41	←	85	B 8	7	85	→	41
					91	→	44
45	→	92		7	92	→	45
45	←	93	A 8		98	→	48
				8	99	→	48
49	→	100			100	→	49
49	←	101	B 8		106	→	52
				8	107	→	52
53	→	108			108	→	53
53	←	109	A 8		114	→	56
				4	115	→	56
57	→	116			116	→	57
57	←	117	B 6		118	→	58
				4	119	→	58
60	←	122			122	→	60

Table 2 for Fig. 289.

$(i = 22)$ $46 \nwarrow$: $o - u - o - u - (1, s)2o.$

$(i = 22)$ $47 \nearrow$: $u - o - u - o - 2u.$

$(i = 23)$ $48 \nwarrow$: $o - u - o - u - 2o.$

$\nearrow$ half-cycles as half-cycle #47.

$\nwarrow$ half-cycles as half-cycle #48.

$(i = 25)$ $53 \nearrow$: $u - o - u - o - 2u.$

$(i = 26)$ $54 \searrow$: $o - u - o - (1, s)2u - 2o.$

$(i = 26)$ $55 \swarrow$: $u - o - u - 2o - 2u.$

$(i = 27)$ $56 \searrow$: $o - u - o - 2u - 2o.$

$\swarrow$ half-cycles as half-cycle #55.

$\searrow$ half-cycles as half-cycle #56.

$(i = 29)$ 60 $\searrow$: $o - u - o - 2u - 2o.$

$(i = 29)$ 61 $\nwarrow$: $o - u - o - 2u - 2o.$
$(i = 30)$ **62** $\nearrow$: $u - o - (1, s)2u - 2o - 2u.$
$(i = 30)$ 63 $\nwarrow$: $o - u - 2o - 2u - 2o.$
$(i = 31)$ 64 $\nearrow$: $u - o - 2u - 2o - 2u.$
$\nwarrow$ half-cycles as half-cycle #63.
$\nearrow$ half-cycles as half-cycle #64.
$(i = 33)$ 68 $\nearrow$: $u - o - 2u - 2o - 2u.$

$(i = 33)$ 69 $\swarrow$: $u - o - 2u - 2o - 2u.$
$(i = 34)$ **70** $\searrow$: $o - (1, s)2u - 2o - 2u - 2o.$
$(i = 34)$ 71 $\swarrow$: $u - 2o - 2u - 2o - 2u.$
$(i = 35)$ 72 $\searrow$: $o - 2u - 2o - 2u - 2o.$
$\swarrow$ half-cycles as half-cycle #71.
$\searrow$ half-cycles as half-cycle #72.
$(i = 37)$ 76 $\searrow$: $o - 2u - 2o - 2u - 2o.$

$(i = 37)$ 77 $\nwarrow$: $o - 2u - 2o - 2u - 2o.$
$(i = 38)$ **78** $\nearrow$: $(1, s)2u - 2o - 2u - 2o - 2u.$
$(i = 38)$ 79 $\nwarrow$: $2o - 2u - 2o - 2u - 2o.$
$(i = 39)$ 80 $\nearrow$: $2u - 2o - 2u - 2o - 2u.$

$(i = 39)$ 81 $\swarrow$: $2u - 2o - 2u - 2o - 2u.$
$(i = 40)$ 82 $\searrow$: $2o - 2u - 2o - 2u - 2o.$
$\swarrow$ half-cycles as half-cycle #81.
$\searrow$ half-cycles as half-cycle #82.
$(i = 41)$ 84 $\searrow$: $2o - 2u - 2o - 2u - 2o.$

$(i = 41)$ 84 $\searrow$: $2o - 2u - 2o - 2u - (2, s)3o.$

$(i = 41)$ 85 $\nwarrow$: $2o - 2u - 2o - 2u - 3o.$
$(i = 42)$ 86 $\nearrow$: $2u - 2o - 2u - 2o - 3u.$
$\nwarrow$ half-cycles as half-cycle #85.
$\nearrow$ half-cycles as half-cycle #86.
$(i = 44)$ 91 $\nwarrow$: $2o - 2u - 2o - 2u - 3o.$

$(i = 45)$ **92** $\swarrow$: $2u - 2o - 2u - (2, s)3o - 3u.$
$(i = 45)$ 93 $\searrow$: $2o - 2u - 2o - 3u - 3o.$
$(i = 46)$ 94 $\swarrow$: $2u - 2o - 2u - 3o - 3u.$
$\searrow$ half-cycles as half-cycle #93.
$\swarrow$ half-cycles as half-cycle #94.
$(i = 48)$ 98 $\swarrow$: $2u - 2o - 2u - 3o - 3u.$

$(i=48)$	99 ↗	:	$2u-2o-2u-3o-3u.$
$(i=49)$	**100** ↖	:	$2o-2u-(2,s)3o-3u-3o.$
$(i=49)$	101 ↗	:	$2u-2o-3u-3o-3u.$
$(i=50)$	102 ↖	:	$2o-2u-3o-3u-3o.$
			↗ half-cycles as half-cycle #101.
			↖ half-cycles as half-cycle #102.
$(i=52)$	106 ↖	:	$2o-2u-3o-3u-3o.$
$(i=52)$	107 ↘	:	$2o-2u-3o-3u-3o.$
$(i=53)$	**108** ↙	:	$2u-(2,s)3o-3u-3o-3u.$
$(i=53)$	109 ↘	:	$2o-3u-3o-3u-3o.$
$(i=54)$	110 ↙	:	$2u-3o-3u-3o-3u.$
			↘ half-cycles as half-cycle #109.
			↙ half-cycles as half-cycle #110.
$(i=56)$	114 ↙	:	$2u-3o-3u-3o-3u.$
$(i=56)$	115 ↗	:	$2u-3o-3u-3o-3u.$
$(i=57)$	**116** ↖	:	$(2,s)3o-3u-3o-3u-3o.$
$(i=57)$	117 ↗	:	$3u-3o-3u-3o-3u.$
$(i=58)$	118 ↖	:	$3o-3u-3o-3u-3o.$
$(i=58)$	119 ↘	:	$3o-3u-3o-3u-3o.$
$(i=59)$	120 ↙	:	$3u-3o-3u-3o-3u.$
			↘ half-cycles as half-cycle #119.
			↙ half-cycles as half-cycle #120.
$(i=60)$	122 ↙	:	$3u-3o-3u-3o-3u.$

The Regular Band Knot in Fig. 290 is an example of the Fan-coded knots discussed in *The Braider*, Issue No. 12, pp. 266 - 268, hence of those $(\alpha+1)$-pass Fan-coded Regular Band Knots with $[(2m+2)(\alpha+1)-1]$-parts and $[\{(2m+2)(\alpha+1)-1\}N-(\alpha+1)]$-bights. For the example in Fig. 290 $\alpha=2$, $m=2$ and $N=4$.

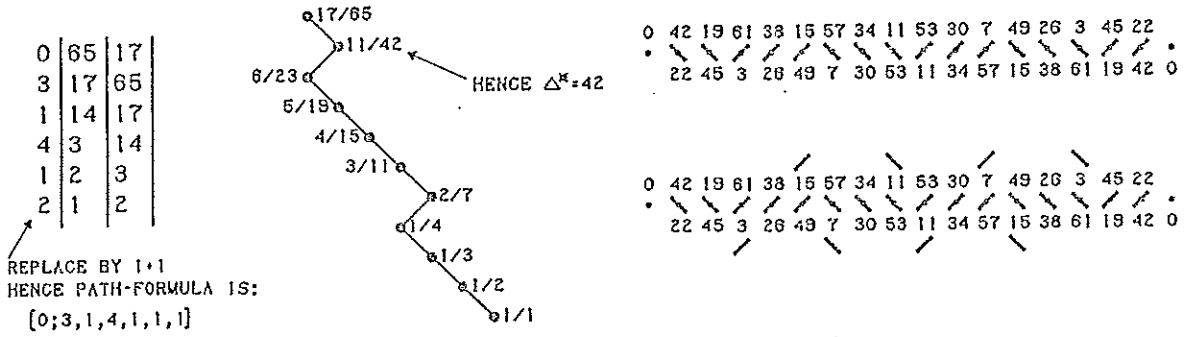


Fig. 290 — Regular Band Knot $p/b=17/65$ with a 3-pass Fan-coding.

This 3-pass Fan knot can be evolved from the 2-pass Fan knot $p/b=11/42$ which in turn can be evolved from the 1-pass Fan knot (the Casa knot) $p/b=5/19$ as the algorithm diagram in Fig. 290 shows. The path of the Casa knot $p/b=5/19$ in the RKT shows that this Casa knot can be enlarged from the Casa-coded Regular Knot $p/b=3/11$. From the braiding sequences and their sub-sequences on the right-hand

side of Table 1 associated with the $P/B = 17/65$ 3-pass Fan-coded Band Knot the two evolutions, each of which requires six sub-sequences ($= 2m + 2$), can readily be seen. Each of the two sub-sequences in the last braiding sequence of each evolution consists of seven ($= 2N - 1$) half-cycles. All other preceding sub-sequences in an evolution consist of eight ($= 2N$) half-cycles.

BIGHT-NUMBER	⇄ HALF-CYCLE		NUMBER OF HALF-CYCLES		HALF-CYCLE →	BIGHT-NUMBER
18	←	38			38 →	18
18	←	39	A	8	39 →	18
19	→	40			40 →	19
22	→	46		8	46 →	22
22	←	47	B	8	47 →	22
26	→	54			54 →	26
26	←	55	A	8	55 →	26
30	→	62		8	62 →	30
30	←	63	B	8	63 →	30
34	→	70			70 →	34
34	←	71	A	8	71 →	34
				7	77 →	37
38	→	78			78 →	38
38	←	79	B	8		
				7	84 →	41
42	→	86			85 →	41
42	←	87	A	6	86 →	42
45	→	92		8	92 →	45
45	←	93	B	8	93 →	45
49	→	100		8	100 →	49
49	←	101	A	8	101 →	49
53	→	108		8	108 →	53
53	←	109	B	8	109 →	53
57	→	116		8	116 →	57
57	←	117	A	8	117 →	57
				7	123 →	61
61	→	124			124 →	61
61	←	125	B	6		
64	←	130		7	130 →	64

Table 1 for Fig. 290.

The half-cycle algorithms for this braiding process are again obtained from its braiding sequences and sub-sequences shown on the right-hand side of Table 1 above, together with the algorithm diagram at the upper right-hand side of Fig. 290.

This braiding process, which for the earlier mentioned reasons to is to be preferred, is different to the process associated with the '*braiding to pattern*' method as discussed in *The Braider*, Issue No. 12, pp. 266 - 268. The half-cycle algorithms for the process associated with the '*braiding to pattern*' method is obtained from the braiding sequences and their sub-sequences shown on the right-hand side of Table 2 on pg. 335, together with the algorithm diagram at the upper right-hand side of Fig. 290.

The Regular Band Knot in Fig. 291 is an example of the Fan-coded knots discussed in *The Braider*, Issue No. 12, pp. 268 - 271, thus of the $(\alpha + 1)$-pass Fan-coded Regular Band Knots with $[(2m+2)(\alpha+1)-1]$-parts and $[\{(2m+2)(\alpha+1)-1\}N+(\alpha+1)]$-bights. For the example in Fig. 291 $\alpha = 2$, $m = 2$ and $N = 4$.

BIGHT-NUMBER		HALF-CYCLE		NUMBER OF HALF-CYCLES	HALF-CYCLE		BIGHT-NUMBER
18	←	38			38	→	18
18	←	39	A 8	7	39	→	18
19	→	40			40	→	19
22	→	46		7	45	→	21
22	←	47	B 8		46	→	22
26	→	54		8	52	→	25
26	←	55	A 8		53	→	25
30	→	62		8	54	→	26
30	←	63	B 8		60	→	29
34	→	70		8	61	→	29
34	←	71	A 8		62	→	30
38	→	78		8	68	→	33
38	←	79	B 8		69	→	33
42	→	86		7	70	→	34
42	←	87	A 6		76	→	37
45	→	92		7	77	→	37
45	←	93	B 8		78	→	38
49	→	100		8	84	→	41
49	←	101	A 8		85	→	41
53	→	108		8	86	→	42
53	←	109	B 8		91	→	44
57	→	116		8	92	→	45
57	←	117	A 8		98	→	48
61	→	124		8	99	→	48
61	←	125	B 6		100	→	49
64	←	130			106	→	52
					107	→	52
					108	→	53
					114	→	56
					115	→	56
					116	→	57
					122	→	60
					123	→	60
					124	→	61
					130	→	64

Table 2 for Fig. 290.

This 3-pass Fan knot can be evolved from the 2-pass Fan knot $p/b = 11/46$ which in turn can be evolved from the 1-pass Fan knot (the Casa knot) $p/b = 5/21$ as the algorithm diagram in Fig. 291 shows. The path of the Casa knot $p/b = 5/21$ in the RKT shows that this Casa knot can be enlarged from the Casa-coded Regular Knot $p/b = 3/13$. From the braiding sequences and their sub-sequences on the right-hand side of the table associated with the $P/B = 17/71$ 3-pass Fan-coded Band Knot the two evolutions, each of which requires six sub-sequences ($= 2m+2$), can readily be seen.

Each of the two sub-sequences in the first braiding sequence of each evolution consists of nine ($= 2N + 1$) half-cycles. All other sub-sequences in an evolution consist of eight ($= 2N$) half-cycles.

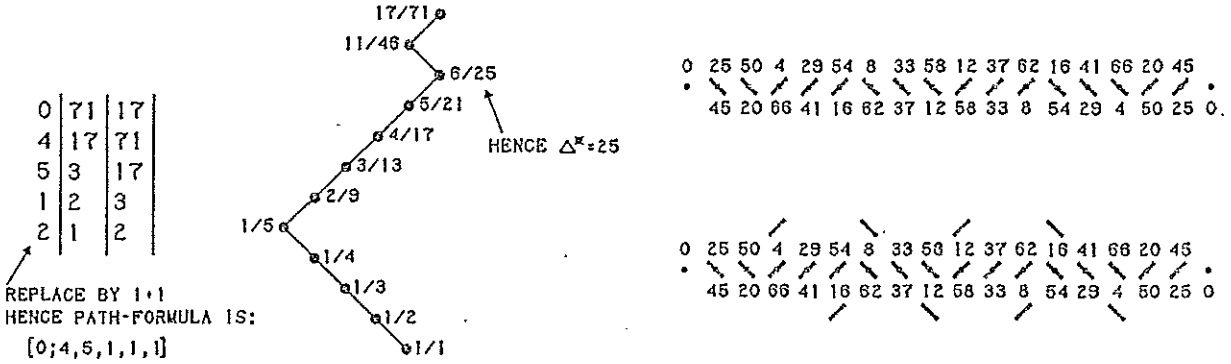


Fig. 291 — Regular Band Knot $p/b = 17/71$ with a 3-pass Fan-coding.

BIGHT-NUMBER ⟷ HALF-CYCLE				NUMBER OF HALF-CYCLES		HALF-CYCLE ⟶ BIGHT-NUMBER		
20	←	42				42	⟶	20
20	←	43	A	10	9	43	⟶	20
						51	⟶	24
25	→	52				52	⟶	25
25	←	53	B	8	9			
29	→	60				60	⟶	29
29	←	61	A	8	8	61	⟶	29
33	→	68				68	⟶	33
33	←	69	B	8	8	69	⟶	33
37	→	76				76	⟶	37
37	←	77	A	8	8	77	⟶	37
41	→	84				84	⟶	41
41	←	85	B	8	8	85	⟶	41
45	→	92				92	⟶	45
45	←	93	A	10	9	93	⟶	45
						101	⟶	49
50	→	102				102	⟶	50
50	←	103	B	8	9			
54	→	110				110	⟶	54
54	←	111	A	8	8	111	⟶	54
58	→	118				118	⟶	58
58	←	119	B	8	8	119	⟶	58
62	→	126				126	⟶	62
62	←	127	A	8	8	127	⟶	62
66	→	134				134	⟶	66
66	←	135	B	8	8	135	⟶	66
70	←	142				142	⟶	70

Table for Fig. 291.

The half-cycle algorithms for this braiding process are again obtained from its braiding sequences and sub-sequences shown on the right-hand side of the table above, together with the algorithm diagram at the upper right-hand side of Fig. 291.

This braiding process is identical to the process associated with the '*braiding to pattern*' method as discussed in *The Braider*, Issue No. 12, pp. 268 - 271.

** *We revisited the Regular Band Knots* $[2m(\alpha+1)+1]/[\{2m(\alpha+1)+1\}N+(\alpha+1)]$ *and* $[2m(\alpha+1)+1]/[\{2m(\alpha+1)+1\}N-(\alpha+1)]$ *with a Gaucho-coding, the*

Regular Band Knots $[(2m+1)(\alpha+1)+1]/[\{(2m+1)(\alpha+1)+1\}N+(\alpha+1)]$ *and* $[(2m+1)(\alpha+1)+1]/[\{(2m+1)(\alpha+1)+1\}N-(\alpha+1)]$ *with a Headhunter's-coding, and the Regular Band Knots* $[(2m+2)(\alpha+1)-1]/[\{(2m+2)(\alpha+1)-1\}N-(\alpha+1)]$ *and* $[(2m+2)(\alpha+1)-1]/[\{(2m+2)(\alpha+1)-1\}N+(\alpha+1)]$ *with a Fan-coding by looking at some numerical examples. The reader undoubtedly will have noticed in each example a property common to all its half-cycle sequences (other than possibly the first and/or the last depending on the coding type) between consecutive intersection half-cycles with the Standing End half-cycle, namely that in the evolution from an α-pass to an $(\alpha+1)$-pass knot the number of half-cycles in each such a half-cycle sequence are the same. For each coding type, prove for the general case of such half-cycle sequences that the P/B-formulae must be those specified above. Derive for each general P/B-formula the associated Δ^*-formula, without referring to its path in the RKT.*

Circular Appliqué

Braided appliqué is a form of braiding in which the braiding process takes place directly upon the surface to be decorated with the braid in question. Not only can it be used for a purely decorative purpose, but it is also a beautiful and very practical braiding process for joining leather parts together while at the same time creating a pleasing effect. It eliminates the use of thread or metal brads, and creates a harmonious relationship with the leather parts it joins and whose beauty it enhances. However, in order for this braiding process to be of practical value to the braider, it is of the utmost importance that he or she is familiar with the necessary methods which enable him or her to obtain in an easy way the braiding instructions required.

The **Cylindrical Braid** is the braidform used in circular appliqué; for this purpose the Cylindrical Braid is flattened out in a plane. The simplest cylindrical braids are the **Regular Braids**, to which we shall limit our discussion in this Issue of *The Braider*. Fig. 292 shows the string-run of an example of such a braid having 9-parts and 16-bights. Its usual string-run diagram of its cylindrical form is shown on the left-hand side and its related string-run diagram for its planar form is shown on the right-hand side.

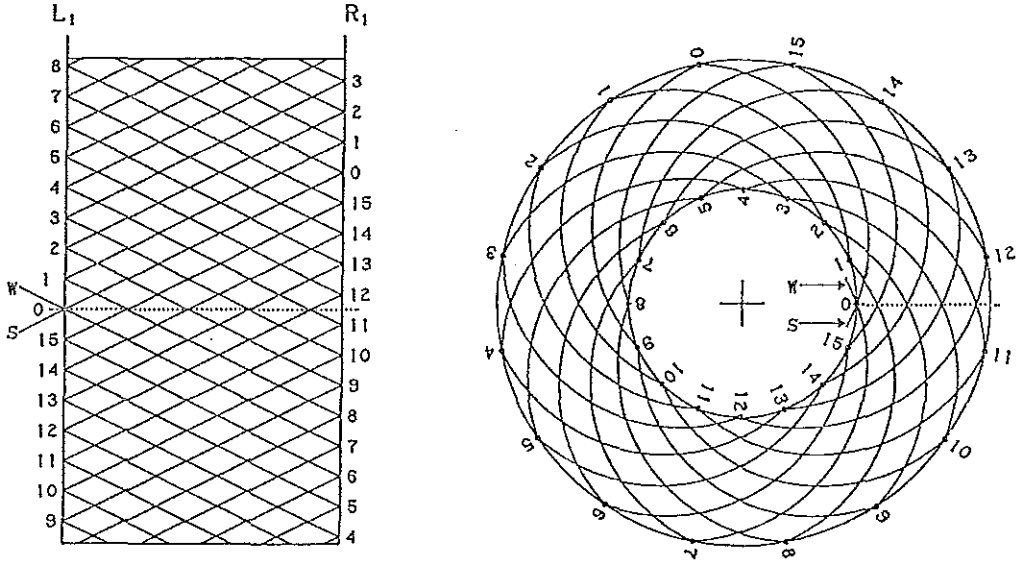


Fig. 292 — The string-run diagrams of a 9/16 Regular Cylindrical Braid.

We know that every braid consists of two independent parts:

(1). The string-run, which is the fundamental one, hence the one of prime importance, and

(2). The coding, which is only of secondary importance.

Hence when the planar form of a cylindrical braid is used as an appliqué braid we must fully understand its string-run in that application. The string-run of the planar form of the cylindrical braid is not identical to the string-run of its planar form as an appliqué braid since the **real bight-points** in the planar string-run of the cylindrical braid become **spurious bight-points** in the string-run of the appliqué braid. This in turn means that a particular appliqué braid can have many different string-runs.

Take for example the Regular Cylindrical Braid in Fig. 292. Here the string runs from bight-point 0 on L_1 (inner circle) to bight-point 0 on R_1 (outer circle), then to bight-point 9 on L_1 (inner circle), then to 9 on R_1 (outer circle), and so on.

When the planar form of this cylindrical braid is used as the circular appliqué braid depicted in Fig. 293, the string runs from spurious bight-point 0 on L_1 (inner circle) to spurious bight-point 0 on R_1 (outer circle), then to spurious bight-point 15 on R_1 (outer circle), then to spurious bight-point 8 on L_1 (inner circle), then to spurious bight-point 7 on L_1 (inner circle), then to spurious bight-point 7 on R_1 (outer circle), then to spurious bight-point 6 on R_1 (outer circle), and so on.

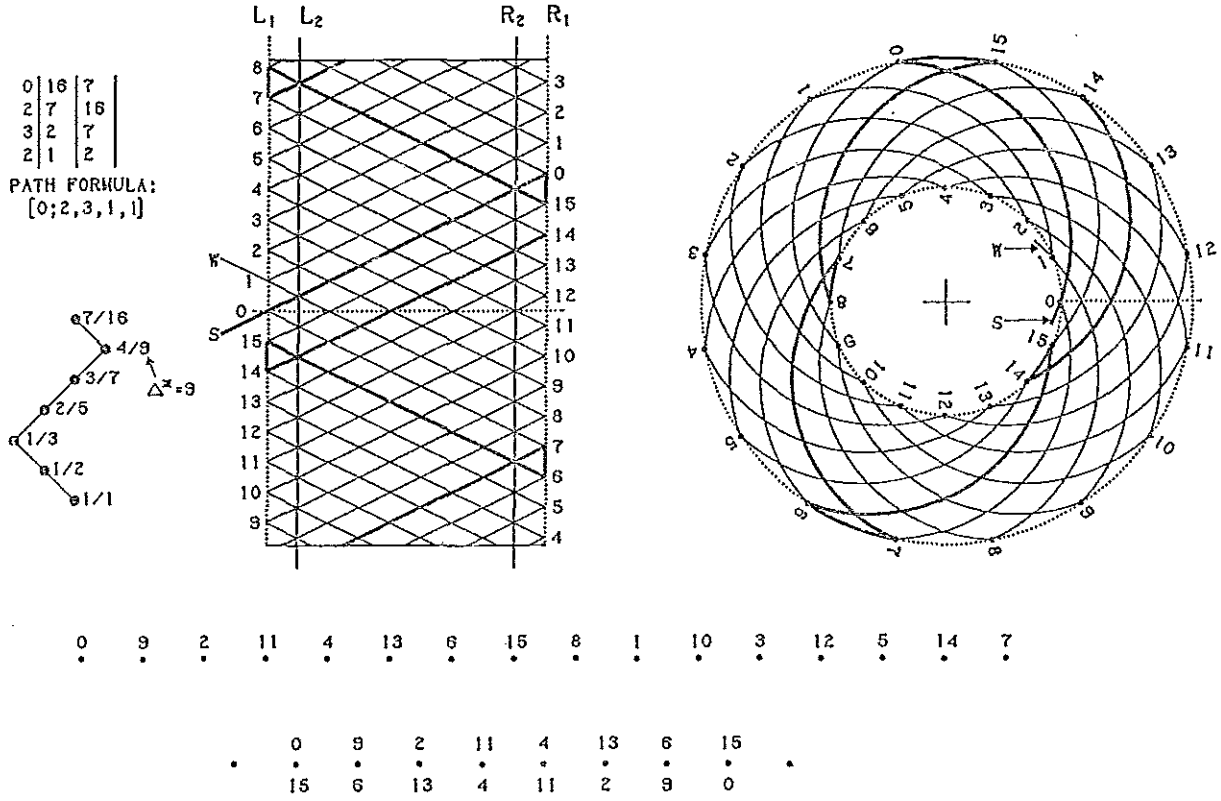


Fig. 293 — The string-run of a circular appliqué braid with $p/b = 9/16$.

However, when the planar form of this cylindrical braid is used as the circular appliqué braid depicted in Fig. 294, the string runs from spurious bight-point 0 on L_1 (inner circle) to spurious bight-point 0 on R_1 (outer circle), then to spurious bight-point 15 on R_1 (outer circle), then to spurious bight-point 8 on L_1 (inner circle), then to spurious bight-point 5 on L_1 (inner circle), then to spurious bight-point 5 on R_1 (outer circle), then to spurious bight-point 4 on R_1 (outer circle), and so on.

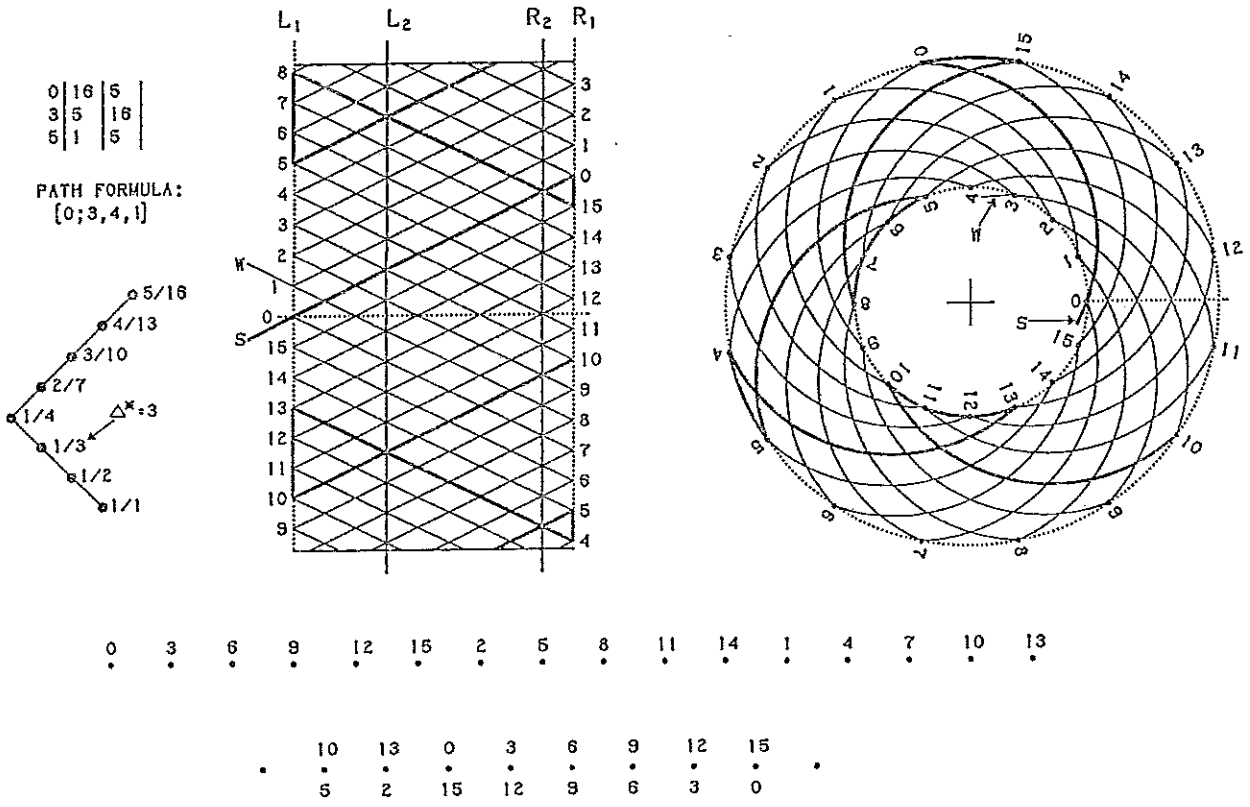


Fig. 294 — The string-run of a circular appliqué braid with $p/b = 9/16$.

The string-run of the circular appliqué which is based on the planar form of this cylindrical braid may also be as depicted in Fig. 295.

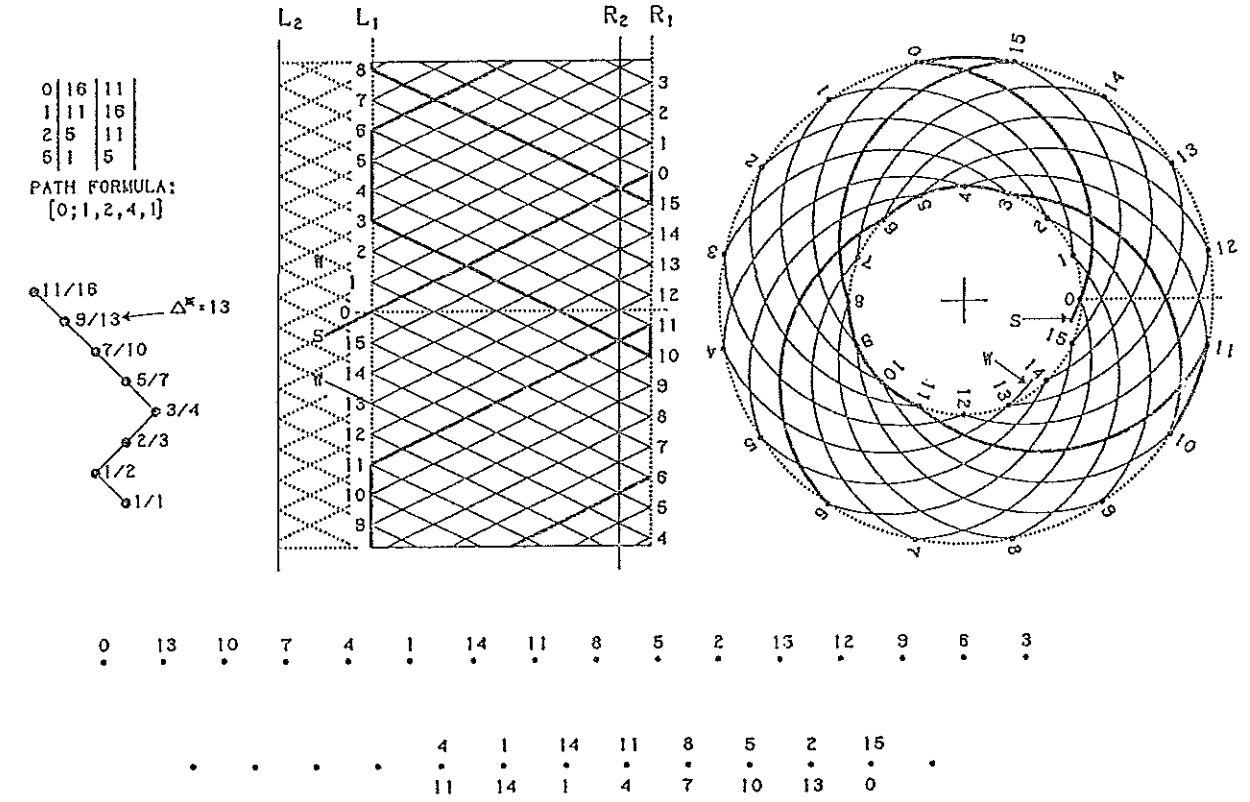


Fig. 295 — The string-run of a circular appliqué braid with $p/b = 9/16$.

Here the string-run runs from spurious bight-point 0 on L_1 (inner circle) to spurious bight-point 0 on R_1 (outer circle), then to spurious bight-point 15 on R_1 (outer circle), then to spurious bight-point 8 on L_1 (inner circle), then to spurious bight-point 11 on L_1 (inner circle), then to spurious bight-point 11 on R_1 (outer circle), then to spurious bight-point 10 on R_1 (outer circle), and so on.

Of course many other string-runs for this circular appliqué braid are possible. The choice is governed by the **number** of strings we want to use and by the **form** of the spurious bight-point required.

The string-run of each circular appliqué braid has besides its set of **spurious bight-points** a set of **virtual bight-points** which are formed by the string-crossings on the columns L_2 and R_2. Thus the string-run of the circular appliqué braid depicted in Fig. 293 is that of the planar form of the Regular Cylindrical Braid which has 7-parts and 16-bights. This string-run is extended beyond its virtual bight-boundaries on L_2 and R_2 to its spurious bight-boundaries L_1 and R_1. Its *complementary cyclic bight-number scheme* is that of the 7/16 Regular Cylindrical Braid (since 7 and 16 are coprime, it is a 1-string braid, hence a Regular Knot). As shown in Fig. 293, its Δ^*-value may be obtained from its path in the RKT, which path is obtained with the aid of Euclid's algorithm. The algorithm diagram of the circular appliqué braid has $(9 + 1) = 10$ column-points. The leftmost one is the spurious bight-boundary L_1, the second from the left is the virtual bight-boundary L_2, the ninth from the left is the virtual bight-boundary R_2, and the rightmost one is the spurious bight-boundary R_1. The column-points associated with the virtual Regular Cylindrical Braid 7/16 receive in the normal way their i-values. The i-values of any remaining column-points (none in the case of Fig. 293) are obtained by an extension in accordance with the complementary cyclic bight-number scheme for the virtual Regular Cylindrical Braid. This gives us the resulting basic algorithm diagram shown in Fig. 293.

The string-run of the circular appliqué braid depicted in Fig. 294 is that of the planar form of the Regular Cylindrical Braid which has 5-parts and 16-bights. This string-run is extended beyond its virtual bight-boundaries on L_2 and R_2 to its spurious bight-boundaries L_1 and R_1. Its *complementary cyclic bight-number scheme* is that of the 5/16 Regular Cylindrical Braid (since 5 and 16 are coprime, it is a 1-string braid, hence a Regular Knot). As shown in Fig. 294, its Δ^*-value may be obtained from its path in the RKT, which path is obtained with the aid of Euclid's algorithm. The algorithm diagram of the circular appliqué braid has $(9 + 1) = 10$ column-points. The leftmost one is the spurious bight-boundary L_1, the fourth from the left is the virtual bight-boundary L_2, the ninth from the left is the virtual bight-boundary R_2, and the rightmost one is the spurious bight-boundary R_1. The column-points associated with the virtual Regular Cylindrical Braid 5/16 receive in the normal way their i-values. The i-values of any remaining column-points (the second and third from the left in the case of Fig. 294) are obtained by an extension in accordance with the complementary cyclic bight-number scheme for the virtual Regular Cylindrical Braid. This gives us the resulting basic algorithm diagram shown in Fig. 294.

The string-run of the circular appliqué braid depicted in Fig. 295 is that of the planar form of the Regular Cylindrical Braid which has 11-parts and 16-bights. This string-run is extended on the right beyond its virtual bight-boundary on R_2 to its spurious bight-boundary R_1. Its *complementary cyclic bight-number scheme* is that of the 11/16 Regular Cylindrical Braid (since 11 and 16 are coprime, it is a 1-string braid, hence a

Regular Knot). As shown in Fig. 295, its Δ^*-value may be obtained from its path in the RKT, which path is obtained with the aid of Euclid's algorithm. The algorithm diagram of the circular appliqué braid has $(9+1)+3 = 13$ column-points. Its left spurious bight-boundary L_1 is the fourth column-point, and hence its leftmost intersection-column is the fifth column-point from the left. The twelfth column-point from the left is the virtual bight-boundary R_2, and the rightmost one is the spurious bight-boundary R_1. The column-points associated with the virtual Regular Cylindrical Braid 11/16 receive in the normal way their i-values. The i-values of any remaining column-points (none in the case of Fig. 295) are obtained by an extension in accordance with the complementary cyclic bight-number scheme for the virtual Regular Cylindrical Braid. However, the i-values of the column-points which are not associated with the intersection-columns of the spurious Regular Cylindrical Braid 9/16 are deleted. This gives us the resulting basic algorithm diagram shown in Fig. 295.

In accordance with the position of the bight-points on the bight-boundaries L_1 and R_1 of the Regular Cylindrical Braid, whose planar form is used for the circular appliqué braid, the spurious bight-points on the outer circle and inner circle of the circular appliqué braid lie on the same set of radial lines only when the circular appliqué braid has an **even** number of parts; otherwise the set of radial lines which belong to the spurious bight-points on the outer circle differ from the set of radial lines which belong to the spurious bight-points on the inner circle. Hence don't make the mistake which is made on pg. 463, plate 192, of the "*Encyclopedia of Rawhide and Leather Braiding*" by Bruce Grant in laying out the position of these spurious bight-points.

Since $p = 9$ for the circular appliqué braids in Figs. 293, 294 and 295, spurious bight-point 0 on the outer circle lies $\frac{p}{2} = 4\frac{1}{2}$ bights above the dotted radial line through spurious bight-point 0 on the inner circle. The spurious bight-index number on the outer circle at which the string enters has the same value as the spurious bight-index number on the inner circle from which the string exits. Let the column-distance from L_1 to L_2 be l (this distance is positive when L_2 is to the **left** of L_1, and negative when L_2 is to the **right** of L_1), and let the column-distance from R_1 to R_2 be r (this distance is positive when R_2 is to the **right** of R_1, and negative when R_2 is to the **left** of R_1). Then, when the string exits spurious bight-index number x on the inner circle, it enters spurious bight-index number x on the outer circle, and exits spurious bight-index number $|x+r|_b$ on the outer circle. Next the string enters spurious bight-index number $|x+r+p|_b$ on the inner circle, and exits spurious bight-index number $|x+r+p+l|_b$ on the inner circle.

For example, the circular appliqué braid in Fig. 294 has $p = 9$; $b = 16$; $l = -3$; $r = -1$. Hence the string exits spurious bight-index number 0 on inner circle, then enters spurious bight-index number 0 on outer circle and exits spurious bight-index number $|0-1|_{16} = 15$ on outer circle, then enters spurious bight-index number $|15+9|_{16} = 8$ on inner circle and exits spurious bight-index number $|8-3|_{16} = 5$ on inner circle, then enters spurious bight-index number 5 on outer circle and exits spurious bight-index number $|5-1|_{16} = 4$ on outer circle, then enters spurious bight-index number $|4+9|_{16} = 13$ on inner circle and exits spurious bight-index number $|13-3|_{16} = 10$ on inner circle, etc.

The circular appliqué braid in Fig. 295 has $p = 9$; $b = 16$; $l = +3$; $r = -1$. Hence the string exits spurious bight-index number 0 on inner circle, then enters spurious bight-index number 0 on outer circle and exits spurious bight-index number $|0-1|_{16} = 15$ on outer circle, then enters spurious bight-index number $|15+9|_{16} = 8$ on inner

circle and exits spurious bight-index number $|8+3|_{16} = 11$ on inner circle, then enters spurious bight-index number 11 on outer circle and exits spurious bight-index number $|11-1|_{16} = 10$ on outer circle, then enters spurious bight-index number $|10+9|_{16} = 3$ on inner circle and exits spurious bight-index number $|3+3|_{16} = 6$ on inner circle, etc.

When l and/or r are negative, the appearance of the spurious bights is as depicted at left in Fig. 296, and when l and/or r are positive, the appearance of the spurious bights is as depicted at right in Fig. 296.



Fig. 296 — The appearance of the spurious bights.

The string-run of the Casa-coded circular appliqué braid in Fig. 297 is that of Fig. 293. Its basic algorithm diagram completed with its column-coding is as shown.

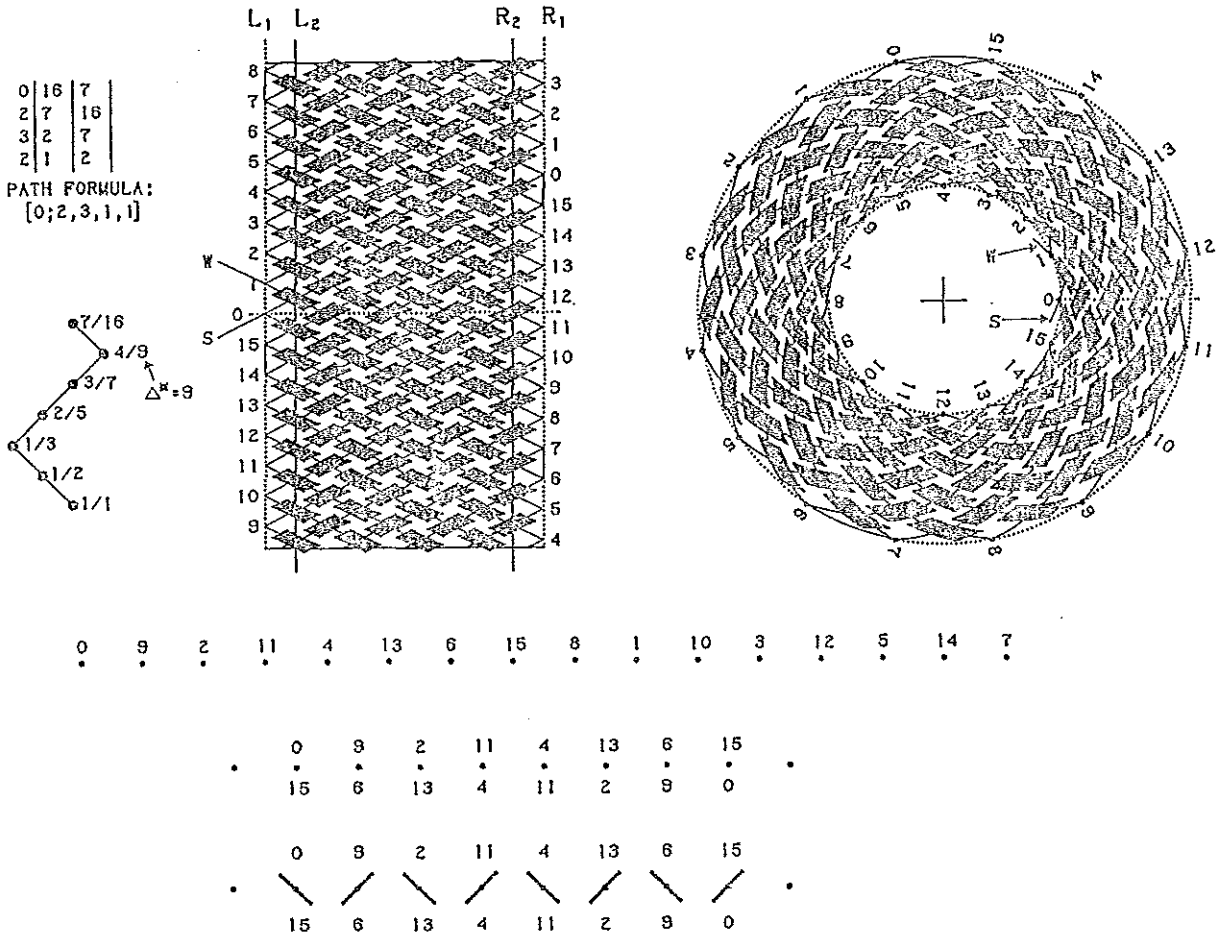


Fig. 297 — A Casa-coded $p/b = 9/16$ circular appliqué braid.

Its algorithm diagram gives the following half-cycle algorithms:

1. $0_{\text{inner}} \longrightarrow 0_{\text{outer}}$ Free run.
2. $(i = 0)$ $8_{\text{inner}} \longleftarrow 15_{\text{outer}}$ u.
3. $(i = 0)$ $7_{\text{inner}} \longrightarrow 7_{\text{outer}}$ u.

4.	$(i = 1)$	$15_{\text{inner}} \longleftarrow 6_{\text{outer}}$	$u.$
5.	$(i = 1)$	$14_{\text{inner}} \longrightarrow 14_{\text{outer}}$	$u.$
6.	$(i = 2)$	$6_{\text{inner}} \longleftarrow 13_{\text{outer}}$	$2u.$
7.	$(i = 2)$	$5_{\text{inner}} \longrightarrow 5_{\text{outer}}$	$2u.$
8.	$(i = 3)$	$13_{\text{inner}} \longleftarrow 4_{\text{outer}}$	$2u.$
9.	$(i = 3)$	$12_{\text{inner}} \longrightarrow 12_{\text{outer}}$	$2u.$
10.	$(i = 4)$	$4_{\text{inner}} \longleftarrow 11_{\text{outer}}$	$3u.$
11.	$(i = 4)$	$3_{\text{inner}} \longrightarrow 3_{\text{outer}}$	$3u.$
12.	$(i = 5)$	$11_{\text{inner}} \longleftarrow 2_{\text{outer}}$	$3u.$
13.	$(i = 5)$	$10_{\text{inner}} \longrightarrow 10_{\text{outer}}$	$3u.$
14.	$(i = 6)$	$2_{\text{inner}} \longleftarrow 9_{\text{outer}}$	$4u.$
15.	$(i = 6)$	$1_{\text{inner}} \longrightarrow 1_{\text{outer}}$	$4u.$
16.	$(i = 7)$	$9_{\text{inner}} \longleftarrow 0_{\text{outer}}$	$4u.$
17.	$(i = 7)$	$8_{\text{inner}} \longrightarrow 8_{\text{outer}}$	$4u.$
18.	$(i = 8)$	$0_{\text{inner}} \longleftarrow 7_{\text{outer}}$	$4u.$
19.	$(i = 8)$	$15_{\text{inner}} \longrightarrow 15_{\text{outer}}$	$4u.$
20.	$(i = 9)$	$7_{\text{inner}} \longleftarrow 14_{\text{outer}}$	$u - o - 3u.$
21.	$(i = 9)$	$6_{\text{inner}} \longrightarrow 6_{\text{outer}}$	$u - o - 3u.$
22.	$(i = 10)$	$14_{\text{inner}} \longleftarrow 5_{\text{outer}}$	$u - o - 3u.$
23.	$(i = 10)$	$13_{\text{inner}} \longrightarrow 13_{\text{outer}}$	$u - o - 3u.$
24.	$(i = 11)$	$5_{\text{inner}} \longleftarrow 12_{\text{outer}}$	$u - o - u - o - 2u.$
25.	$(i = 11)$	$4_{\text{inner}} \longrightarrow 4_{\text{outer}}$	$u - o - u - o - 2u.$
26.	$(i = 12)$	$12_{\text{inner}} \longleftarrow 3_{\text{outer}}$	$u - o - u - o - 2u.$
27.	$(i = 12)$	$11_{\text{inner}} \longrightarrow 11_{\text{outer}}$	$u - o - u - o - 2u.$
28.	$(i = 13)$	$3_{\text{inner}} \longleftarrow 10_{\text{outer}}$	$u - o - u - o - u - o - u.$
29.	$(i = 13)$	$2_{\text{inner}} \longrightarrow 2_{\text{outer}}$	$u - o - u - o - u - o - u.$
30.	$(i = 14)$	$10_{\text{inner}} \longleftarrow 1_{\text{outer}}$	$u - o - u - o - u - o - u.$
31.	$(i = 14)$	$9_{\text{inner}} \longrightarrow 9_{\text{outer}}$	$u - o - u - o - u - o - u.$
32.	$(i = 15)$	$1_{\text{inner}} \longleftarrow 8_{\text{outer}}$	$u - o - u - o - u - o - u - o.$

The circular appliqué braid in Fig. 298 has the same string-run as the previous one, however, its coding is a 2-pass Gaucho-coding. Its algorithm diagram gives the following half-cycle algorithms:

1.		$0_{\text{inner}} \longrightarrow 0_{\text{outer}}$	Free run.
2.	$(i = 0)$	$8_{\text{inner}} \longleftarrow 15_{\text{outer}}$	$u.$
3.	$(i = 0)$	$7_{\text{inner}} \longrightarrow 7_{\text{outer}}$	$u.$
4.	$(i = 1)$	$15_{\text{inner}} \longleftarrow 6_{\text{outer}}$	$u.$
5.	$(i = 1)$	$14_{\text{inner}} \longrightarrow 14_{\text{outer}}$	$u.$
6.	$(i = 2)$	$6_{\text{inner}} \longleftarrow 13_{\text{outer}}$	$u - o.$
7.	$(i = 2)$	$5_{\text{inner}} \longrightarrow 5_{\text{outer}}$	$u - o.$
8.	$(i = 3)$	$13_{\text{inner}} \longleftarrow 4_{\text{outer}}$	$u - o.$
9.	$(i = 3)$	$12_{\text{inner}} \longrightarrow 12_{\text{outer}}$	$u - o.$
10.	$(i = 4)$	$4_{\text{inner}} \longleftarrow 11_{\text{outer}}$	$u - o - u.$
11.	$(i = 4)$	$3_{\text{inner}} \longrightarrow 3_{\text{outer}}$	$u - o - u.$
12.	$(i = 5)$	$11_{\text{inner}} \longleftarrow 2_{\text{outer}}$	$u - o - u.$
13.	$(i = 5)$	$10_{\text{inner}} \longrightarrow 10_{\text{outer}}$	$u - o - u.$
14.	$(i = 6)$	$2_{\text{inner}} \longleftarrow 9_{\text{outer}}$	$u - o - u - o.$
15.	$(i = 6)$	$1_{\text{inner}} \longrightarrow 1_{\text{outer}}$	$u - o - u - o.$
16.	$(i = 7)$	$9_{\text{inner}} \longleftarrow 0_{\text{outer}}$	$u - o - u - o.$

3.	$(i = A, 0)$	$13_{\text{inner}} \longrightarrow 13_{\text{outer}}$	$u - o - 3u.$
4.	$(i = A, 1)$	$5_{\text{inner}} \longleftarrow 12_{\text{outer}}$	$u - 4o.$
5.	$(i = A, 1)$	$3_{\text{inner}} \longrightarrow 3_{\text{outer}}$	$u - o - 3u.$
6.	$(i = A, 2)$	$11_{\text{inner}} \longleftarrow 2_{\text{outer}}$	$u - 2o - u - 2o.$
7.	$(i = A, 2)$	$9_{\text{inner}} \longrightarrow 9_{\text{outer}}$	$u - o - 2u - o - u.$
8.	$(i = A, 3)$	$1_{\text{inner}} \longleftarrow 8_{\text{outer}}$	$u - 2o - u - 2o.$
9.	$(i = A, 3)$	$15_{\text{inner}} \longrightarrow 15_{\text{outer}}$	$u - o - 2u - o - u.$
10.	$(i = A, 4)$	$7_{\text{inner}} \longleftarrow 14_{\text{outer}}$	$u - 2o - u - 2o.$
11.	$(i = A, 4)$	$5_{\text{inner}} \longrightarrow 5_{\text{outer}}$	$u - o - 2u - o - u.$
12.	$(i = A, 5)$	$13_{\text{inner}} \longleftarrow 4_{\text{outer}}$	$u - o - u - o - u - 2o.$
13.	$(i = A, 5)$	$11_{\text{inner}} \longrightarrow 11_{\text{outer}}$	$u - o - u - o - u - o - u.$
14.	$(i = A, 6)$	$3_{\text{inner}} \longleftarrow 10_{\text{outer}}$	$u - o - u - o - u - 2o.$
15.	$(i = A, 6)$	$1_{\text{inner}} \longrightarrow 1_{\text{outer}}$	$u - o - u - o - u - o - u.$
16.	$(i = A, 7)$	$9_{\text{inner}} \longleftarrow 0_{\text{outer}}$	$u - o - u - o - u - o - u - o.$

The circular appliqué braid depicted in Fig. 300 has the same string-run as the circular appliqué braid in Fig. 299, however, its coding is a 2-pass Gaucho-coding.

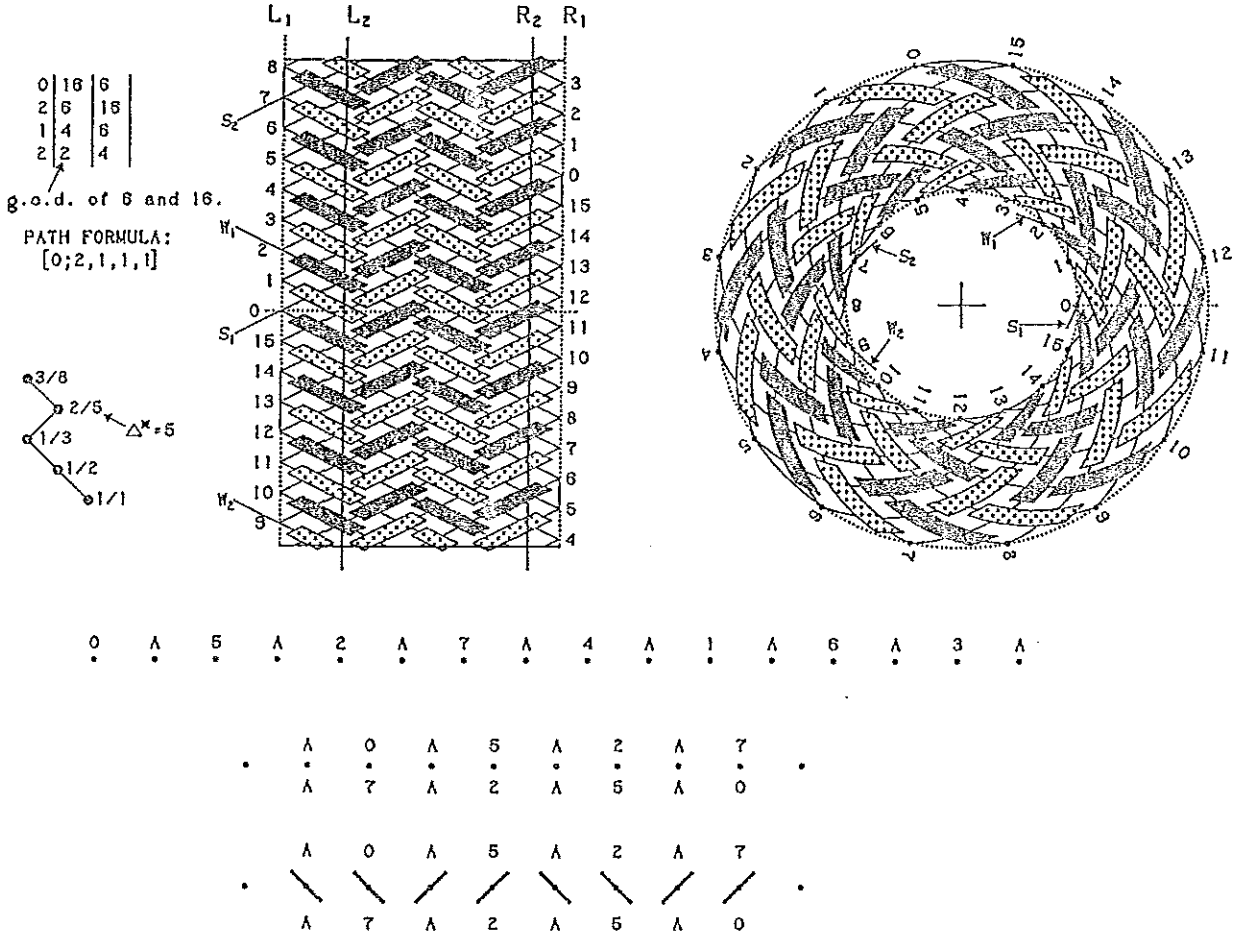


Fig. 300 — A 2-string 2-pass Gaucho-coded $p/b = 9/16$ circular appliqué braid.

The half-cycle algorithms for the two components are again obtained from its algorithm diagram in the same way as shown for the previous example.

THE BRAIDER

APPENDIX 1995

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A quarterly publication for the braiding artisan

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Appendixes

In order to ensure that an uniform publication interval can be maintained, it is essential to have several consecutive issues completed well before their due publication dates. We have now reached the stage where we are four issues (a year's subscription) ahead. Although this should ensure a trouble free uniform publication interval, it does however create some undesirable side effects. Since each issue will have been completed a year before its publication date, it would be impossible to address queries from subscribers and announce new publications promptly. To avoid this undesirable side effect, we shall make use of appendixes, the pages of which will, for the year concerned, be numbered in Roman numerals in ascending order.

If applicable, the final appendix for a year will be supplied with the first issue of the following year.

Isometric Graph paper

An isometric graph paper which is available in the U.S.A.:

No. 1000HP; Clearprint Paper Company of Emeryville, California 94608.

It comes in a "Calculation and Sketch Pad" of 50 sheets 279×432 mm. (11" × 17"). It is also available in 216×279 mm. ($8\frac{1}{2}$" × 11").

It can, for example, be obtained from Scott Blueprint Company.

A New Publication for Braiders

Pamphlet No. 10 Pt. 2 — *Special Braid Forms Pt. 2* — **The Braiding of CFC Braids and CWH Braids.** A.G. Schaake.

This Pamphlet in A4 format (295 × 210 mm.), soft cover, stapled and taped spine, contains 196 pages.

Price: NZ$17.00; Weight: 0.53 kg.

Although this Pamphlet has been available since the beginning of September 1995, its availability has not been announced earlier since its weight brings it in the weight-category of 0.50 – 1.00 kg. for postage. It will therefore be more economical and hence advisable, for those who want to obtain a copy, to delay ordering it until Pamphlet No. 11 (**Braiding Application – Bridle and Reins**) has been published, since these two publications together, will fall in the same weight-category. The publication of Pamphlet No. 11 will hopefully take place shortly.

The CFC Braids consist of three integrated components, namely two Cylindrical Braids which are joined by a Flat Braid. The CWH Braids are Cylindrical Braid with a hole in their cylindrical surface. Any CWH Braid is also a CFC Braid, however the converse is not true. Although there are several applications for these braids, the most common one for CFC Braids is as wear-braids for the protection against wear

of important braid components. The most common application for CWH Braids is as rudder-position indicators on the steeringwheels of yachts, in which case they are normally braided from round string in multi-ply form.

The main purpose of this Pamphlet is to provide a small data base for the more commonly encountered Regular $C_1F^*C_1$ Knots (single string CFC Braids in which the two Cylindrical Braids are Regular Cylindrical Braids with identical string-runs, and in which the Flat Braid is a Regular Flat Braid with either no end-bights or end-bights placed symmetrically relative to the length-wise centre-line of the Flat Braid), and the 1-string and 2-string Regular CWH* Braids (Regular Cylindrical Braids with a central hole in their cylindrical surface). The data for the Regular $C_1F^*C_1$ Knots are presented in general string-run diagrams, while the data for the Regular 1-string and 2-string CWH* Braids are given in the form of tables.

The Pamphlets No. 10 Pt. 1 and Pt. 2 are essential references for Pamphlet No. 11 in which the construction of a bridle and reins are described.

A Reminder

The November 1995 issue of *The Braider* was the first issue which contained questions. This will be a regular feature in future issues of this publication. The questions are intended to encourage the braider along the path of discovery, and hence we hope that readers will attempt to solve the questions posed, rather then just wait for the next issue to see the solution. Although waiting for the presentation of the solution is the soft option, little if any real knowledge will be gained by doing so. Only by comparing one's solution with the presented one can the reader channel his or her thought-patterns into the discovery-mode.

The most important basic tool is the grid-diagram and especially the string-run diagram. If isometric graph-paper is locally unobtainable, one can draw up such a grid (see *The Braider*, issue No. 1, pg. 3), and get it photo-copied in red or green in order to obtain a convenient base-grid which does not interfere with the diagrams which can then be drawn in black on top of it.

From the diagrams we derive our relationships between the parameters involved. Mathematics is the basic tool which allows these relationships to be presented in an easy communicable form. Some readers might require a certain amount of "assistence" with such presentations, and if so, should not hesitate to find a suitable way of obtaining it. One does not achieve anything by sitting back and let it all float by. After all, a mathematical expression is nothing but a form of writing which conveys relationships in an unambiguous manner, and hence is considerably easier to master than ordinary writing which may have several meanings.

The Braider is a publication which aims to be of a high *technical* standard. Consequently, grid-diagrams, string-run diagrams and mathematical expressions are a common feature. This publication is therefore *not* intended for those who adhere to the doctrine that truth consists not in correspondence with the facts but in successful coherence with experience only; hence such people should *not* subscribe to *The Braider*.

Please don't forget that we like to receive your solutions, which will of course be acknowledged.
